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RESEARCH ARTICLE

WEAK CONTRACTION IN CONE METRIC SPACES

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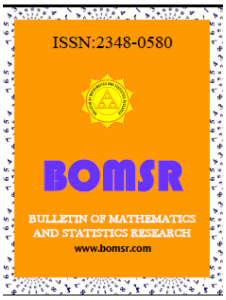

ABSTRACT

The purpose of this article is to introduced the concept of weak contraction in cone metric space and also establish a coincidence and common fixed point result for weak contractions in cone metric spaces. Our result proper generalizes previous known results in this direction.

Keywords :- Cone metric spaces, weak contraction, weak contraction, coincidence point, common fixed point.

2000 AMS Subject Classification :- 47H10, 47H09.

INTRODUCTION

It is quite natural to consider generalization of the notion of metric $d : X \times X \to [0, \infty)$. The question was, what must $[0, \infty)$ be raplace by E. In 1980 Bogdan Rzepecki [6]in 1987 Shy- Der Lin [5]and in 2007 Huang and Zhang [4] gave the same answer; Replace the real numbers with a Banach ordered by a cone, resulting in the so called cone metric.

Cone metric space are generalizations of metric space, in which each pair of points of domain is assigned to a member of real Banach space with a cone. This cone naturally induces a partial order in a Banach space.

Recently , Choudhary and Metiya [3] established a fixed point result for a weak contractions in cone metric spaces. Sintunavarat and Kumam [7] give the notion of f- contractions and establish a coincidence and common fixed point result for f –weak contraction in cone metric space.

In this paper, we introduce the notion of $(C - f) -$ weak contraction condition on cone metric space and prove common fixed point theorem for $(C - f) -$ weak contraction mapping. Our results are proper generalizations of [7].

In next section we give some previous and known results which are used to prove of our main theorem.

Priliminaries

In 1972, the concept of $\mathrm{C}-$ contraction was introduced by Chatterjea [1] as follows,

Definition1:- Let (X, d) be a metric space. A mapping $T : X \to X$ is called a Chatterjea type contraction if there exists $k \in \left(0, \frac{1}{2}\right)$ such that for all $x, y \in X$ the following inequality holds:

$$d(Tx, Ty) \leq k\,[\max\{d(x, y), d(x, Ty), d(y, Tx)\}] \qquad 2.1$$

Later, Chouddhury [2] introduced the generalization of Chatterjea type construction as follows,

Definition 2:- A self mapping $T : X \to X$ is said to be weak C- contraction if for all $x, y \in X$,

$$d(Tx, Ty) \leq \frac{1}{2}[d(x, Ty) + d(y, Tx)] - \psi\left(d(x, Ty), d(y, Tx)\right) \qquad 2.2$$

where $\psi : [0, \infty)^2 \to [0, \infty)$ is a continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$.

Now we introduced the following definition of $(C - f) -$ weak contraction which is proper generalization of Definition 2

Definition 3:- Let (X, d) be a metric space and $f : X \to X$. A mapping $T : X \to X$ is said to be $(C - f) -$ weak contraction if

$$d(Tx, Ty) \leq \frac{1}{2}[d(fx, Ty) + d(fy, Tx)] - \psi\,(d(fx, Ty), d(fy, Tx)\,) \qquad 2.3$$

for $x, y \in X$ where $\psi : [0, \infty)^2 \to [0, \infty)$ is a continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$.

Remark 4:- If we take $\psi(x, y) = k(x + y)$ where $0 < k < \frac{1}{2}$ then 2.2 reduces to 2.1, that is weak $\mathrm{C}-$ contraction are generalization of C- contraction.

Remark 5:- If we take $f = I$ (identity mapping) then 2.3 reduced to 2.2, that is $C - f) -$ weak contraction are generalization of weak C- contraction.

Remark 6:- If we take $f = I$ (identity mapping) and $\psi(x, y) = k(x + y)$ where $0 < k < \frac{1}{2}$ then 2.3 reduced to 2.1, that is $(C - f) -$ weak contraction are generalization of C- contraction.

Definition 7:- Let E be a real Banach space and P a subset of E. P is called a cone if and only if

i. P is closed non empty and $P \neq \{0\}$,

ii. $a, b \in R, a, b \geq 0, x, y \in P \to ax + by \in P$,

iii. $x \in P$ and $-x \in P \to x = 0$.

Given a cone $P \subset E$, define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x \leq y$ to indicate that $x \leq y$, but $x \neq y$, while $x \ll y$ will stand for $y - x \in \operatorname{int} P$, with $\operatorname{int} P$ denoting the interior of P.

The cone P is called normal if there is a number $k > 0$ such that for all $x, y \in E$,

$$0 \leq x \leq y \to \| x \| \leq K \| y \|.$$

The least positive number satisfying the above inequality is called the normal constant of P.

The cone P is called regular if every increasing sequence bounded form above is convergent. That is, if $\{x_n\}$ is a sequence such that

$$x_1 \leq x_2 \leq \ldots\ldots \leq x_n \leq \ldots\ldots \leq y$$

for some $y \in E$, then there is $x \in E$ such that $\| x_n - x \| \to 0$ as $n \to \infty$. Equivalently, the cone P is regular if and only if every decreasing sequence bounded from below is convergent. It is well known that a regular cone is a normal cone.

In the following we always suppose E is a Banach space, P is a cone in E with $\operatorname{int} P \neq \phi$ and $\leq$ is a partial ordering with respect to P.

Definition 8:- Let X be a non empty set. Suppose that the mapping $d: X \times X \to E$ satisfies

i. $0 \leq d(x, y)$, for all $x, y \in X$, and $d(x, y) = 0$ if and only if $x = y$,

ii. $d(x, y) = d(y, x)$, for all $x, y \in X$,

iii. $d(x,y) \leq d(x,z) + d(z,y)$, for all $x, y, z \in X$.

Then, d is called a cone metric on X, and (X, d) is called a cone metric space.

Definition 9 :- Let (X, d) be a cone metric space. Let $\{x_n\}$ be a sequence in X and $x \in X$. If for every $c \in E$ with $0 \ll c$ there exists $n > N$, $d(x\_n, x) \ll c$, then $\{x_n\}$ is said to be convergent and $\{x_n\}$ converges to x, and x is the limit of $\{x_n\}$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$, as $n \to \infty$.

Definition 10:- Let (X, d) be a cone metric space and $\{x_n\}$ be a sequence in X. If for any $c \in E$ with $0 \ll c$, there exists $m, n > N$ such that $d(x_n, x_m) \ll c$, then $\{x_n\}$ is called a Cauchy sequence in X.

Definition 11:- Let (X, d) be a cone metric space and $\{x_n\}$ be a sequence in X. If every Cauchy sequence is convergent in X, then X called a complete cone metric space.

Lemma 12:- Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $d(x_n, x) \to 0$, as $n \to \infty$.

Lemma 13:- Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. If $\{x_n\}$ converges to x and $\{x_n\}$ converges to y, then $x = y$, that is the limit of $\{x_n\}$ is unique.

Lemma 14:- Let (X, d) be a cone metric space and $\{x_n\}$ be a sequence in X. If $\{x_n\}$ converges to x, then $\{x_n\}$ is Cauchy sequence.

Lemma 15:- Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $d(x\_n, x\_m) \to 0$, as $m, n \to \infty$.

Lemma 16:- Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X and $x_n \to x, y_n \to y$, as $n \to \infty$. Then, $d(x_n, y_n) \to d(x, y)$ as $n \to \infty$.

Lemma 17:- If P is a normal cone in E, then

i. if $0 \leq x \leq y$ and $a \geq 0$, where a is real number, then $0 \leq ax \leq ay$,

ii. if $0 \leq x_n \leq y_n$, for $n \in N$ and $x_n \to x, y_n \to y$, then $0 \leq x \leq y$.

Lemma 18:- Let E is a real Banach space with cone P in E, then for $a, b, c \in E$,

i. if $a \leq b$ and $b \ll c$, then $a \ll c$,

ii. if $a \ll b$ and $b \ll c$, then $a \ll c$.

Definition 19:- Let $(Y, \leq)$ be a partially ordered set. Then, a function $F: Y \to Y$ is said to be monotone increasing if it preserves ordering.

Definition 20:- Let f and T be self mappings of a nonempty set X. If $w = fx = Tx$ for some $x \in X$, then x is called a coincidence point of f and T, and w is called a point of coincidence of f and T. If $w = x$, then x is called a common fixed point of f and T.

In [7], Sintunavarat and Kumam prove following,

Theorem 21:- Let (X, d) be a cone metric space with a regular cone P such that $d(x, y) \in \operatorname{int} P$ for $x, y \in X$ with $x \neq y$. Let $f : X \to X$ and $T : X \to X$ be mappings satisfying the inequality

$$d(Tx, Ty) \leq \frac{1}{2}[d(fx, fy)] - \psi\left(d(fx, fy)\right) \qquad 2.4$$

for $x, y \in X$, where $\psi : \operatorname{int} P \cup \{0\} \to \operatorname{int} P \cup \{0\}$ is continuous mapping such that

i. $\psi(t) = 0$ if and only if $t = 0$,

ii. $\psi(t) \ll t$ for $t \in \operatorname{int} P$,

iii. either $\psi(t) \leq d(fx, fy)$ or $\psi(t) \geq d(fx, fy)$ for $t \in \operatorname{int} P \cup \{0\}$.

If $T(X) \subseteq f(X)$ and $f(X)$ is a complete subspace of X, then f and T have a unique point of coincidence in X. Moreover, f and T have a common fixed point in X if $ffz = fz$ for the coincidence point z.

Main Results

Theorem22:- Let (X, d) be a cone metric space with a regular cone P such that $d(x, y) \in \text{int } P$ for $x, y \in X$ with $x \neq y$. Let $f : X \to X$ and $T : X \to X$ be mappings satisfying the inequality

$$d(Tx, Ty) \leq \frac{1}{2}[d(fx, Ty) + d(fy, Tx)] - \psi\left(d(fx, Ty), d(fy, Tx)\right) \quad 3.1$$

for $x, y \in X$, where $\psi : (\text{int } P \cup \{0\})^2 \to \text{int } P \cup \{0\}$ is continuous mapping such that

i. $\psi(t_1, t_2) = 0$ if and only if $t_1 = t_2 = 0$,

ii. $\psi(t_1, t_2) \ll \min\{t_1, t_2\}$ for $t_1, t_2 \in \text{int } P$,

iii. either $\psi(t_1, t_2) \leq d(fx, fy)$ or $\psi(t_1, t_2) \geq d(fx, fy)$ for $t_1, t_2 \in \text{int } P \cup \{0\}$.

If $T(X) \subseteq f(X)$ and $f(X)$ is a complete subspace of X, then f and T have a unique point of coincidence in X. Moreover, f and T have a common fixed point in X if $ffz = fz$ for the coincidence point z.

Proof:- Let $x_0 \in X$. Since $T(X) \subseteq f(X)$, we construct the sequence $\{fx_n\}$ where $fx_n = Tx_{n-1}$, $n \geq 1$. If $fx_{n+1} = fx_n$, for some n, then trivially f and T have coincidence point in X. If $fx_{n+1} \neq fx_n$, for $n \in N$ then, from (3.1)we have

$$d(fx_n, fx_{n+1}) = d(Tx_{n-1}, Tx_n)$$

$$\leq \frac{1}{2}[d(fx_{n-1}, Tx_n) + d(fx_n, Tx_{n-1})] - \psi\left(d(fx_{n-1}, Tx_n), d(fx_n, Tx_{n-1})\right)$$

By the property of ψ, that is $\psi(t_1, t_2) \geq 0$ for all $t_1, t_2 \in \text{int } P \cup \{0\}$, we have

$d(fx_n, fx_{n+1}) \leq d(fx_{n-1}, fx_n)$.

Its follows that the sequence $\{d(fx_n, fx_{n+1})\}$ is monotonically decreasing. Since cone P is regular and $0 \leq d(fx_n, fx_{n+1})$, for all $n \in N$, there exists $r \geq 0$ such that

$$d(fx_n, fx_{n+1}) \to r \text{ as } n \to \infty.$$

Since ψ is continuous and

$$d(fx_n, fx_{n+1}) \leq \frac{1}{2}[d(fx_{n-1}, Tx_n) + d(fx_n, Tx_{n-1})] - \psi\left(d(fx_{n-1}, Tx_n), d(fx_n, Tx_{n-1})\right)$$

by taking $n \to \infty$, we get

$$r \leq r - \psi(r, r)$$

which is contradiction, unless $r = 0$. Therefore, $d(fx_n, fx_{n+1}) \to r$ as $n \to \infty$.

Let $c \in E$ with $0 \ll c$ be arbitrary. Since $d(fx_n, fx_{n+1}) \to r$ as $n \to \infty$, there exists $m \in N$ such that

$$d(fx\_m, fx\_(m+1)) \ll \psi\left(\psi\left(\frac{c}{2}, \frac{c}{2}\right), \psi\left(\frac{c}{2}, \frac{c}{2}\right)\right).$$

Let $B(fx_m, c) = \{fx \in X : d(fx_m, fx) \ll c\}$. Clearly, $x_m \in B(fx_m, c)$. Therefore, $B(fx_m, c)$ is nonempty. Now we will show that $Tx \in B(fx_m, c)$, for $fx \in B(fx_m, c)$.

Let $x \in B(fx_m, c)$. By property (3) of ψ, we have the following two possible cases.

Case (i): $d(fx, fx_m) \leq \psi\left(\frac{c}{2}, \frac{c}{2}\right)$,

Case (ii): $\psi\left(\frac{c}{2}, \frac{c}{2}\right) < d(fx, fx_m) \ll c$.

We have,

Case (i): $d(Tx, fx_m) \leq d(Tx, Tx_m) + d(Tx_m, fx_m)$

$$\leq \frac{1}{2}[d(fx, Tx_m) + d(fx_m, Tx)] - \psi\left(d(fx, Tx_m), d(fx_m, Tx)\right) + d(Tx_m, fx_m)$$

$$\leq \frac{1}{2}[d(fx, fx_{m-1}) + d(fx_m, Tx)] - \psi\left(d(fx, fx_{m-1}), d(fx_m, Tx)\right) + d(fx_{m+1}, fx_m)$$

$$\leq \psi\left(\frac{c}{2}, \frac{c}{2}\right) + \psi\left(\psi\left(\frac{c}{2}, \frac{c}{2}\right), \psi\left(\frac{c}{2}, \frac{c}{2}\right)\right)$$

$$\ll \frac{c}{2} + \frac{c}{2} \ll c.$$

Case (ii): $d(Tx, fx_m) \leq d(Tx, Tx_m) + d(Tx_m, fx_m)$

$$\leq \frac{1}{2}[d(fx, Tx_m) + d(fx_m, Tx)] - \psi\big(d(fx, Tx_m), d(fx_m, Tx)\big) +$$
$$d(Tx_m, fx_m)$$
$$\leq \frac{1}{2}[d(fx, fx_{m-1}) + d(fx_m, Tx)] - \psi\big(d(fx, fx_{m-1}), d(fx_m, Tx)\big)$$
$$+ d(fx_{m+1}, fx_m)$$
$$\leq \frac{1}{2}[d(fx, fx_{m-1}) + d(fx_m, Tx)] - \psi\left(\psi\left(\frac{c}{2}, \frac{c}{2}\right), \psi\left(\frac{c}{2}, \frac{c}{2}\right)\right)$$
$$+ \psi\left(\psi\left(\frac{c}{2}, \frac{c}{2}\right), \psi\left(\frac{c}{2}, \frac{c}{2}\right)\right)$$
$$\ll c.$$

Therefore, T is a self mapping of $B(fx_m, c)$. Since $fx_m \in B(fx_m, c)$ and $fx_n = Tx_{n-1}, n \geq 1$, it follows that $x_m \in B(fx_m, c)$, for all $n \geq m$. Again, c is arbitrary. This establishes that $\{ fx_n \}$ is a Cauchy sequence in f(X). It follows from completeness of f(X) that $fx_n \to fx$, for some $x \in X$. Now, we observe that

$$d(fx_m, Tx) = d(Tx_{n-1}, Tx)$$
$$\leq \frac{1}{2}[d(fx_{n-1}, fx) + d(fx, fx_{n-1})] - \psi\left(d(fx_{n-1}, fx), d(fx, fx_{n-1})\right).$$

By making $n \to \infty$, we have $d(fx, Tx) \leq 0$. Therefore, $d(fx, Tx) = 0$, that is, $fx = Tx$. Hence, x is a coincidence point of f and T.

For uniqueness of the coincidence point of f and T, let, if possible, $y \in X\,(x \neq y)$ be another coincidence point of f and T.

We note that

$$d(fx, fy) = d(Tx, Ty)$$
$$\leq \frac{1}{2}[d(fx, Ty) + d(fy, Tx)] - \psi\left(d(fx, Ty), d(fy, Tx)\right)$$
$$\leq \frac{1}{2}[d(fx, fy) + d(fy, fx)] - \psi\,(d(fx, fy), d(fy, fx)\,).$$

Hence $\psi\left(d(fx, fy), d(fy, fx)\right) \leq 0$, which contradiction, by the property of ψ. Therefore, f and T have a common unique point of coincidence of X.

Let z be a coincidence point of f and T. It follows from $ffx = fz$ and z being a coincidence point of f and T that $ffz = fz = Tz$.

From 3.1, we get

$$d(Tfz, Tz) \leq \frac{1}{2}[d(fz, Tz) + d(fz, Tfz)] - \psi\left(d(fz, Tz), d(fz, Tfz)\right)$$
$$\leq d(fz, Tfz).$$

Which contradiction. Therefore $Tfz = fz$, that is $ffz = fz = Tz$. Hence fz is a common fixed point of f and T. The uniqueness of the common fixed point is easy to establish from 3.1. This complete the proof.

It is easy to see that if $f = I$ (identity mapping) in Theorem 22 then we get following Corollary.

Corollary 23:- Let (X, d) be a cone metric space with a regular cone P such that $d(x, y) \in \text{int}\, P$ for $x, y \in X$ with $x \neq y$. Let $T : X \to X$ be a mapping satisfying the inequality

$$d(Tx, Ty) \leq \frac{1}{2}[d(x, Ty) + d(y, Tx)] - \psi\left(d(x, Ty), d(y, Tx)\right) \qquad 3.2$$

for $x, y \in X$, where $\psi : (\text{int}\, P \cup \{0\})^2 \to \text{int}\, P \cup \{0\}$ is continuous mapping such that

i. $\psi(t_1, t_2) = 0$ if and only if $t_1 = t_2 = 0$,

ii. $\psi(t_1, t_2) \ll \min\{t_1, t_2\}$ for $t_1, t_2 \in \text{int}\, P$,

iii. either $\psi(t_1, t_2) \leq d(fx, fy)$ or $\psi\,(t_1, t_2) \geq d(fx, fy)$ for $t_1, t_2 \in \text{int}\, P \cup \{0\}$.

If $T(X) \subseteq f(X)$ and f(X) is a complete subspace of X, then T has a unique point in X.

If we take $\psi\,(t_1, t_2) = k(t_1 + t_2)$ for $0 < k < \frac{1}{2}$ in Corollary 23 then we get following result.

Corollary24:- Let (X, d) be a cone metric space with a regular cone P such that $d(x, y) \in \text{int } P$ for $x, y \in X$ with $x \neq y$. Let $T: X \rightarrow X$ be a mapping satisfying the inequality

$$d(Tx, Ty) \leq \frac{1}{2}[d(x, Ty) + d(y, Tx)] \tag{3.3}$$

for $x, y \in X$. If $T(X) \subseteq f(X)$ and $f(X)$ is a complete subspace of X, then T has a unique point in X.

If we take $\psi(t_1, t_2) = (\alpha - k)(t_1 + t_2)$ for $\alpha \in \left[\frac{1}{4}, \frac{1}{2}\right)$, $0 < k < \frac{1}{2}$ in Theorem 22 then we get following result.

Corollary 25:- Let (X, d) be a cone metric space with a regular cone P such that $d(x, y) \in \text{int } P$ for $x, y \in X$ with $x \neq y$. Let $f: X \rightarrow X$ and $T: X \rightarrow X$ be a mapping satisfying the inequality

$$d(Tx, Ty) \leq k[d(fx, Ty) + d(fy, Tx)] \tag{3.4}$$

for $x, y \in X$. If $T(X) \subseteq f(X)$ and $f(X)$ is a complete subspace of X, then f and T have a unique point of coincidence in X. Moreover, f and T have a common fixed point in X if $ffz = fz$ for the coincidence point z.

Example 26:- Let $X = [0,1]$, $E = R \times R$, with usual norm, be a real Banach space, $P = \{(x, y) \in E : x, y \geq 0\}$ be a regular cone and the partial ordering $\leq$ with respect to the cone P be the usual partial ordering in E. Define $d: X \times X \rightarrow E$ as :

$$d(x, y) = (|x - y|, |x - y|), \text{ for } x, y \in X.$$

Then (X, d) is a complete cone metric space with $d(x, y) \in \text{int } P$, for $x, y \in X$ with $x \neq y$. Let us define $\psi : (\text{int } P \cup \{0\})^2 \rightarrow \text{int } P \cup \{0\}$ such that $\psi(t_1, t_2) = \frac{t_1 + t_2}{3}$ for all $t_1, t_2 \in \text{int } P \cup \{0\}$, $fx = 2x$ and $Tx = \frac{x}{7}$ for $x \in X$ then, Theorem 22 is true and $0 \in X$ is the unique common fixed point of f and T.

Corollary 27:- Let (X, d) be a cone metric space with a regular cone P such that $d(x, y) \in \text{int } P$ for $x, y \in X$ with $x \neq y$. Let $f: X \rightarrow X$ and $T: X \rightarrow X$ be mappings satisfying the inequality

$$\int_0^{d(Tx,Ty)} \rho(s)ds \leq \beta \in \int_0^{d(fx,Ty)+ d(fy,Tx)} \rho(s)ds \tag{3.5}$$

for $x, y \in X, \beta \in \left[\frac{0,1}{2}\right)$ and $\rho: [0, \infty) \rightarrow [0, \infty)$ is a Lebesgue integrable mapping satisfying $\in t_0^{\epsilon} \rho(s)\, ds$ for $\epsilon > 0$. If $T(X) \subseteq f(X)$ and $f(X)$ is a complete subspace of X, then f and T have a unique point of coincidence in X. Moreover, f and T have a common fixed point in X if $ffz = fz$ for the coincidence point z.

Corollary 28 :- Let (X, d) be a cone metric space with a regular cone P such that $d(x, y) \in \text{int } P$ for $x, y \in X$ with $x \neq y$. Let $T: X \rightarrow X$ be mapping satisfying the inequality

$$\int_0^{d(Tx,Ty)} \rho(s)ds \leq \beta \int_0^{d(x,Ty) + d(y,Tx)} \rho(s)ds \tag{3.6}$$

for $x, y \in X, \beta \in \left[\frac{0,1}{2}\right)$ and $\rho: [0, \infty) \rightarrow [0, \infty)$ is a Lebesgue integrable mapping satisfying $\int_0^{\epsilon} \rho(s)\, ds$ for $\epsilon > 0$. Then T has a fixed point in X.

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