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SOME THEOREMS IN Q-TS-FUZZY IDEALS OF A RING

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ABSTRACT

In this paper, we have introduced some theorems in Q-TS-fuzzy ideal of a ring and prove some results on these.

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KEY WORDS: Q-fuzzy subset, Q-TS-fuzzy ideal, Q-fuzzy relation, Product of Q-fuzzy subsets.

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R ; + ; .). After the introduction of fuzzy sets by L.A.Zadeh[15], several researchers explored on the generalization of the concept of fuzzy sets. Azriel Rosenfeld[3] defined a fuzzy groups. Asok Kumer Ray[2] defined a product of fuzzy subgroups and A.Solairaju and R.Nagarajan[12, 13] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. In this paper, we introduce the some Theorems in Q-TS-fuzzy ideal of a ring and established some results.

1.PRELIMINARIES

1.1 Definition: A T-norm is a binary operations T: $[0, 1]\times[0, 1] \to [0, 1]$ satisfying the following requirements;

(i) T(0, x)= 0, T(1, x) = x (boundary condition)

(ii) T(x, y) = T(y, x) (commutativity)

(iii) T(x, T(y, z))= T (T(x,y), z)(associativity)

(iv) if $x \leq y$ and $w \leq z$, then $T(x, w) \leq T(y, z)$(monotonicity).

1.2 Definition: A S-norm is a binary operation S: $[0, 1]\times[0, 1] \to [0, 1]$ satisfying the following requirements;

(i) 0 S x = x, 1 S x = 1 (boundary condition)

(ii) x S y = y S x (commutativity)

(iii) x S (y S z) = (x S y) S z (associativity)

(iv) if $x \leq y$ and $w \leq z$, then x S $w \leq y$ S z (monotonicity).

1.3 Definition: Let X be a non-empty set and Q be a non-empty set. A **Q-fuzzy subset** A of X is a function A: X×Q $\rightarrow$ [0, 1].

1.4 Definition: Let (R, +, $\cdot$) be a ring and Q be a non empty set. A Q-fuzzy subset A of R is said to be a **Q-TS-fuzzy ideal** of R if the following conditions are satisfied:

(i) $A(x+y, q) \geq T(A(x, q), A(y, q))$,

(ii) $A(-x, q) \geq A(x, q)$,

(iii) $A(xy, q) \geq S(A(x, q), A(y, q))$, for all x and y in R and q in Q.

1.5 Definition: Let A and B be any two Q-fuzzy subsets of sets R and H, respectively. The product of A and B, denoted by A×B, is defined as A×B = {$\langle$ ((x, y), q), A×B((x, y), q) $\rangle$ / for all x in R and y in H, q in Q }, where A×B((x, y), q) = min {A(x, q), B(y, q) }.

1.6 Definition: Let A be a Q-fuzzy subset in a set S, the **strongest Q-fuzzy relation** on S, that is a Q-fuzzy relation V with respect to A given by V((x, y), q) = min{ A(x, q), A(y, q) }, for all x and y in S and q in Q.

2 – PROPERTIES OF Q-TS-FUZZY IDEALS OF A RING:

2.1 Theorem: If A is a Q-TS-fuzzy ideal of a ring (R, +, $\cdot$), then $A(x, q) \leq A(e, q)$, for x in R, the identity e in R and q in Q.

Proof: For x in R, q in Q and e is the identity element of R. Now, $A(e, q) = A(x-x, q) \geq T(A(x, q), A(-x, q)) = A(x, q)$. Therefore, $A(e, q) \geq A(x, q)$, for x in R and q in Q.

2.2 Theorem: If A is a Q-TS-fuzzy ideal of a ring (R, +, $\cdot$), then $A(x-y, q) = A(e, q)$ gives $A(x, q) = A(y, q)$, for x and y in R, e in R and q in Q.

Proof: Let x and y in R, the identity e in R and q in Q. Now, $A(x, q) = A(x-y+y, q) \geq T(A(x-y, q), A(y, q)) = T(A(e, q), A(y, q)) = A(y, q) = A(x-(x-y), q) \geq T(A(x-y, q), A(x, q)) = T(A(e, q), A(x, q)) = A(x, q)$. Therefore, $A(x, q) = A(y, q)$, for x and y in R and q in Q.

2.3 Theorem: If A is a Q-TS-fuzzy ideal of a ring (R, +, $\cdot$), then H = { x / x$\in$R: A(x, q) = 1} is either empty or is a ideal of R.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $A(x-y, q) \geq T(A(x, q), A(-y, q)) \geq T(A(x, q), A(y, q)) = T(1, 1) = 1$. Therefore, $A(x-y, q) = 1$. We get x–y in H. And $A(xy, q) \geq S(A(x, q), A(y, q)) = S(1, 1) = 1$. Therefore, $A(xy, q) = 1$. We get xy in H. Therefore, H is a ideal of R. Hence H is either empty or is a ideal of R.

2.4 Theorem: If A is a Q-TS-fuzzy ideal of a ring (R, +, $\cdot$), then H = { x$\in$R: A(x, q) = A(e, q) } is a ideal of R.

Proof: Let x and y be in H. Now, $A(x-y, q) \geq T(A(x, q), A(-y, q)) \geq T(A(x, q), A(y, q)) = T(A(e, q), A(e, q)) = A(e, q)$. Therefore $A(x-y, q) \geq A(e, q)$ ---------(1)

And $A(e, q) = A((x-y) - (x-y), q) \geq T(A(x-y, q), A(-(x-y), q) \geq T(A(x-y, q), A(x-y, q)) = A(x-y, q)$.
Therefore $A(e, q) \geq A(x-y, q)$ ---------(2)

From (1) and (2), we get $A(e, q) = A(x-y, q)$. Therefore x–y in H. Now $A(xy, q) \geq S(A(x, q), A(y, q)) = S(A(e, q), A(e, q)) = A(e, q)$. Therefore $A(xy, q) \geq A(e, q)$ --------- (3)

And clearly, $A(e, q) \geq A(xy, q)$ --------- (4)

From (3), (4), we get $A(e, q) = A(xy, q)$. Therefore, xy in H. Hence H is a ideal of R.

2.5 Theorem: Let A be a Q-TS-fuzzy ideal of a ring (R, +, $\cdot$). If $A(x-y, q) = 1$, then $A(x, q) = A(y, q)$, for x and y in R and q in Q.

Proof: Let x and y in R and q in Q. **N**ow A(x, q) = A(x–y+y, q) ≥ T(A(x–y, q), A(y, q)) = T(1, A(y, q)) = A(y, q) = A(–y, q) = A(–x+x–y, q) ≥ T(A(–x, q), A(x–y, q)) = T(A(–x, q), 1) = A(–x, q) = A(x, q). Therefore A(x, q) = A(y, q), for x and y in R, q in Q.

2.6 Theorem: Let A be a Q-TS-fuzzy ideal of a ring (R, +, ·). If A(x–y, q) = 0, then either A(x, q) = 0 or A(y, q) = 0, for all x and y in R and q in Q.

Proof: Let x and y in R and q in Q. By the definition A(x–y, q) ≥ T(A(x, q), A(y, q))
which implies that 0 ≥ T (A(x, q), A(y, q)). Therefore, either A(x, q) = 0 or A(y, q) = 0.

2.7 Theorem: Let (R, +, ·) be a ring and Q be a non-empty set. If A is a Q-TS-fuzzy ideal of R, then A(x+y, q) = T(A(x, q), A(y, q)) with A(x, q) ≠ A(y, q), for each x and y in R and q in Q.

Proof: Let x and y belongs to R and q in Q. Assume that A(x, q) > A(y, q). Now A(y, q) = A(–x+x+y, q) ≥ T(A(–x, q), A(x+y, q)) ≥ T(A(x, q), A(x+y, q)) ≥ T(A(y, q), A(x+y, q)) = A(y, q). And A(y, q) = T(A(x, q), A(x+y, q)) = A(x+y, q). Therefore, A(x+y, q) = A(y, q) = T(A(x, q), A(y, q)), for all x and y in R and q in Q.

2.8 Theorem: If A and B are two Q-TS-fuzzy ideals of a ring R, then their intersection A∩B is a Q-TS-fuzzy ideal of R.

Proof: Let x and y belong to R and q in Q, A = { 〈 (x, q), A(x, q) 〉 / x in R and q in Q } and B = { 〈 (x, q), B(x, q) 〉 / x in R and q in Q }. Let C= A∩B and C = {〈 (x, q), C(x, q) 〉 / x in R, q in Q}. (i) C(x+y, q) = min(A(x+y, q), B(x+y, q) ≥ min(T(A(x, q), A(y, q)), T(B(x, q), B(y, q))) ≥ T(min(A(x, q), B(x, q)), min(A(y, q), B(y, q))) = T(C(x, q), C(y, q)). Therefore C(x+y, q) ≥ T(C(x, q), C(y, q)), for all x and y in R and q in Q. (ii) C(–x, q) = T(A(–x, q), B(–x, q)) ≥ T(A(x, q), B(x, q)) = C(x, q). Therefore C(–x, q) ≥ C(x, q), for all x in R, q in Q. (iii) C(xy, q) = min(A(xy, q), B(xy, q)) ≥ min(S(A(x, q), A(y, q)), S(B(x, q), B(y, q))) ≥ S(min(A(x, q), B(x, q)), min(A(y, q), B(y, q))) = S(C(x, q), C(y, q)). Therefore C(xy, q) ≥ S(C(x, q), C(y, q)), for all x and y in R and q in Q. Hence A∩B is a Q-TS-fuzzy ideal of the ring R.

2.9 Theorem: The intersection of a family of Q-TS-fuzzy ideals of a ring R is a Q-TS-fuzzy ideal of R.

Proof: By Theorem 2.8, we can prove easily.

2.10 Theorem: Let A be a Q-TS-fuzzy ideal of a ring R. If A(x, q) < A(y, q), for some x and y in R and q in Q, then A(x+y, q) = A(x, q) = A(y+x, q), for all x and y in R, q in Q.

Proof: Let A be a Q-TS-fuzzy ideal of a ring R. Also we have A(x, q) < A(y, q), for some x and y in R, q in Q, A(x+y, q) ≥ T(A(x, q), A(y, q)) = A(x, q); and A(x, q) = A(x+y–y, q) ≥ T(A(x+y, q), A(–y, q)) ≥ T(A(x+y, q), A(y, q)) = A(x+y, q). Therefore A(x+y, q) = A(x, q), for all x and y in R, q in Q. Hence A(x+y, q) = A(x, q) = A(y+x, q), for all x and y in R and q in Q.

2.11 Theorem: Let A be a Q-TS-fuzzy ideal of a ring R. If A(x, q) > A(y, q), for some x and y in R, q in Q, then A(x+y, q) = A(y, q) = A(y+x, q), for all x and y in R, q in Q.

Proof: It is trivial.

2.12 Theorem: Let A be a Q-TS-fuzzy ideal of a ring R such that Im A = {α}, where α in L. If A = B∪C, where B and C are Q-TS-fuzzy ideals of R, then either B ⊆ C or C ⊆ B.

Proof: Let A = B∪C = { 〈 (x , q), A(x, q) 〉 / x in R and q in Q}, B = { 〈 (x, q), B(x, q) 〉 / x in R and q in Q } and C = { 〈 (x, q), C(x, q) 〉 / x in R and q in Q }. Suppose that neither B ⊆ C nor C ⊆ B. Assume that B(x, q) > C(x, q) and B(y, q) < C(y, q), for some x and y in R, q in Q. Then α = A(x, q) = (B∪C)(x, q) = max(B(x, q), C(x, q)) = B(x, q) > C(x, q). Therefore α > C(x, q). And α = A(y, q) = (B∪C)(y, q) = max(B(y, q), C(y, q)) = C(y, q) > B(y, q). Therefore α > B(y, q). So that C(y, q) > C(x, q) and B(x, q) > B(y, q). Hence B(x+y, q) = B(y, q) and C(x+y, q) = C(x, q), by Theorem 2.10 and 2.11. But then α = A(x+y, q) = (B∪C)(x+y, q) = max (B(x+y, q), C(x+y, q)) = max(B(y, q), C(x, q)) < α ---------(1).
It is a contradiction by (1). Therefore, either B ⊆ C or C ⊆ B is true.

2.13 Theorem: If A and B are Q-TS-fuzzy ideals of the rings R and H, respectively, then A×B is a Q-TS-fuzzy ideal of R×H.

Proof: Let A and B be Q-TS-fuzzy ideals of the rings R and H respectively. Let x_1 and x_2 be in R, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in R×H and q in Q. Now $A\times B[(x_1, y_1)+(x_2, y_2), q] = A\times B((x_1+x_2, y_1+y_2), q) = \min(A(x_1+x_2, q), B(y_1+y_2, q)) \geq \min(T(A(x_1, q), A(x_2, q)), T(B(y_1, q), B(y_2, q))) \geq T(\min(A(x_1, q), B(y_1, q)), \min(A(x_2, q), B(y_2, q))) = T(A\times B((x_1, y_1), q), A\times B((x_2, y_2), q))$. Therefore $A\times B[(x_1, y_1)+(x_2, y_2), q] \geq T(A\times B((x_1, y_1), q), A\times B((x_2, y_2), q))$. And $A\times B[-(x_1, y_1), q] = A\times B((-x_1, -y_1), q) = \min (A(-x_1, q), B(-y_1, q)) \geq \min (A(x_1, q), B(y_1, q)) = A\times B((x_1, y_1), q)$. Therefore $A\times B[-(x_1, y_1), q] \geq A\times B((x_1, y_1), q)$. Now $A\times B[(x_1, y_1)(x_2, y_2), q] = A\times B((x_1x_2, y_1y_2), q) = \min(A(x_1x_2, q), B(y_1y_2, q)) \geq \min(S(A(x_1, q), A(x_2, q)), S(B(y_1, q), B(y_2, q))) \geq S(\min(A(x_1, q), B(y_1, q)), \min(A(x_2, q), B(y_2, q))) = S(A\times B((x_1, y_1), q), A\times B((x_2, y_2), q))$. Therefore $A\times B[(x_1, y_1)(x_2, y_2), q] \geq S(A\times B((x_1, y_1), q), A\times B((x_2, y_2), q))$. Hence A×B is a Q-TS-fuzzy ideal of R×H.

2.14 Theorem: Let A and B be Q-fuzzy subsets of the rings R and H, respectively. Suppose that e and e^l are the identity element of R and H, respectively. If A×B is a Q-TS-fuzzy ideal of R×H, then at least one of the following two statements must hold.

(i) $B(e^l, q) \geq A(x, q)$, for all x in R and q in Q,

(ii) $A(e, q) \geq B(y, q)$, for all y in H and q in Q.

Proof: Let A×B be a Q-TS-fuzzy ideal of R×H. By contra positive, suppose that none of the statements (i) and (ii) holds. Then we can find a in R and b in H such that $A(a, q) > B(e^l, q)$ and $B(b, q) > A(e, q)$, q in Q. We have $A\times B((a, b), q) = \min(A(a, q), B(b, q)) > \min(A(e, q), B(e^l, q)) = A\times B((e, e^l), q)$. Thus A×B is not a Q-TS-fuzzy ideal of R×H. Hence either $B(e^l, q) \geq A(x, q)$, for all x in R and q in Q or $A(e, q) \geq B(y, q)$, for all y in H and q in Q.

2.15 Theorem: Let A and B be Q-fuzzy subsets of the rings R and H, respectively and A×B is a Q-TS-fuzzy ideal of R×H. Then the following are true:

(i) if $A(x, q) \leq B(e^l, q)$, then A is a Q-TS-fuzzy ideal of R.

(ii) if $B(x, q) \leq A(e, q)$, then B is a Q-TS-fuzzy ideal of H.

(iii) either A is a Q-TS-fuzzy ideal of R or B is a Q-TS-fuzzy ideal of H.

Proof: Let A×B be a Q-TS-fuzzy ideal of R×H, x and y in R and q in Q. Then (x, e^l) and (y, e^l) are in R×H. Now, using the property $A(x, q) \leq B(e^l, q)$, for all x in R and q in Q, we get, $A(x-y, q) = \min(A(x-y, q), B(e^le^l, q)) = A\times B(((x-y), (e^le^l)), q) = A\times B[(x, e^l) +(-y, e^l), q] \geq T (A\times B((x, e^l), q), A\times B((-y, e^l), q)) = T (\min(A(x, q), B(e^l ,q)), \min(A(-y, q), B(e^l, q))) = T(A(x, q), A(-y, q)) \geq T(A(x, q), A(y, q))$. Therefore $A(x-y, q) \geq T(A(x, q), A(y, q))$, for all x, y in R, q in Q. And $A(xy, q) = \min(A(xy, q), B(e^le^l, q)) = A\times B(((xy), (e^le^l)), q) = A\times B[(x, e^l)(y, e^l), q] \geq S(A\times B((x, e^l), q), A\times B((y, e^l), q)) = S (\min(A(x, q), B(e^l ,q)), \min(A(y, q), B(e^l, q))) = S(A(x, q), A(y, q))$. Therefore $A(xy, q) \geq S(A(x, q), A(y, q))$ for all x, y in R and q in Q. Hence A is a Q-TS-fuzzy ideal of R. Thus (i) is proved. Now, using the property $B(x, q) \leq A(e, q)$, for all x in H and q in Q, we get, $B(x-y, q) = \min(B(x-y, q), A(ee, q)) = A\times B(((ee), (x-y)), q) = A\times B[(e, x)+(e, -y), q] \geq T(A\times B((e, x), q), A\times B((e, -y), q)) = T(\min (B(x, q), A(e, q)), \min(B(-y, q), A(e, q))) = T(B(x, q), B(-y, q)) \geq T(B(x, q), B(y, q))$. Therefore $B(x-y, q) \geq T(B(x, q), B(y, q))$ for all x and y in H and q in Q. And $B(xy, q) = \min(B(xy, q), A(ee, q)) = A\times B(((ee), (xy)), q) = A\times B[(e, x)(e, y), q] \geq S(A\times B((e, x), q), A\times B((e, y), q)) = S (\min(B(x, q), A(e, q)), \min(B(y, q), A(e, q))) = S(B(x, q), B(y, q))$. Therefore $B(xy, q) \geq S(B(x, q), B(y, q))$ for all x and y in H and q in Q. Hence B is a Q-TS-fuzzy ideal of H. Thus (ii) is proved. (iii) is clear.

2.16 Theorem: Let A be a Q-fuzzy subset of a ring R and V be the strongest Q-fuzzy relation of R with respect to A. Then A is a Q-TS-fuzzy ideal of R if and only if V is a Q-TS-fuzzy ideal of R×R.

Proof: Suppose that A is a Q-TS-fuzzy ideal of R. Then for any x= (x_1, x_2) and y=(y_1, y_2) are in R×R and q in Q. We have V(x–y, q) = V[(x_1, x_2)–(y_1, y_2), q] = V($(x_1–y_1, x_2–y_2)$, q) = min(A($(x_1–y_1)$, q), A($(x_2–y_2)$, q)) ≥ min(T(A(x_1, q), A(–y_1, q)), T(A(x_2, q), A(–y_2, q))) ≥ min (T(A(x_1, q), A(x_2, q)), T(A(–y_1, q), A(–y_2, q))) ≥ T(min(A(x_1, q), A(x_2, q)), min(A(y_1, q), A(y_2, q))) = T(V((x_1, x_2), q), V((y_1, y_2), q)) = T(V(x, q), V(y, q)). Therefore V((x–y), q) ≥ T(V(x, q), V(y, q)) for all x and y in R×R and q in Q. And we have V(xy, q) = V[(x_1, x_2)(y_1, y_2), q] = V((x_1y_1 , x_2y_2), q) = min(A((x_1y_1), q), A((x_2y_2), q)) ≥ min(S(A(x_1, q), A(y_1, q)), S (A(x_2, q), A(y_2, q))) ≥ S(min(A(x_1, q), A(x_2, q)), min(A(y_1, q), A(y_2, q))) = S(V((x_1, x_2), q), V((y_1, y_2), q)) = S(V(x, q), V(y, q)). Therefore V((xy), q) ≥ S(V (x, q), V(y, q)), for all x and y in R×R and q in Q. This proves that V is a Q-TS-fuzzy ideal of R×R. Conversely, assume that V is a Q-TS-fuzzy ideal of R×R, then for any x = (x_1, x_2) and y= (y_1, y_2) are in R×R, we have min (A(x_1–y_1, q), A(x_2–y_2, q)) = V((x_1–y_1, x_2–y_2), q) = V[(x_1, x_2)–(y_1, y_2), q] = V(x–y, q) ≥ T(V(x, q), V(y, q)) = T(V((x_1, x_2), q), V((y_1, y_2), q)) = T(min(A(x_1, q), A(x_2, q)), min(A(y_1, q), A(y_2, q))). If we put x_2 = y_2 = e, where e is the identity element of R. We get, A((x_1–y_1), q) ≥ T(A(x_1, q), A(y_1, q)), for all x_1 and y_1 in R and q in Q. And min(A(x_1y_1, q), A(x_2y_2, q)) = V((x_1y_1, x_2y_2), q) = V[(x_1, x_2)(y_1, y_2), q] = V(xy, q) ≥ S(V(x, q), V(y, q)) = S(V((x_1, x_2), q), V((y_1, y_2), q)) = S(min(A(x_1, q), A(x_2, q)), min(A(y_1, q), A(y_2, q)). If we put x_2 = y_2 = e, where e is the identity element of R. We get, A(x_1y_1, q) ≥ S(A(x_1, q), A(y_1, q)), for all x_1 and y_1 in R and q in Q. Hence A is a Q-TS-fuzzy ideal of R.

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