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RESEARCH ARTICLE

ANALYSIS OF NON-SUCCESSIVE OCCURRENCE OF DIGIT 4 IN PRIME NUMBERS TILL 1 TRILLION

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ABSTRACT

The ways in which digit 4 occurs non-successively in prime numbers till one trillion is considered in this work. All multiple non-successive occurrences of 4 are analyzed. The first, i.e., the smallest, and the last, i.e., the largest, prime with all possible repetitions of non-successive 4's are determined within ranges of growing powers of 10.

Keywords : Prime numbers, digit 4, non-successive occurrences

2010 Mathematical Subject Classification: 11Y35, 11Y60, 11Y99

1. INTRODUCTION

Usable Mathematics starts with counting and counting requires numbers. This was the reason that early explorations of human races in different parts of the world have dealt with numbers in their own ways [2].

As we progressed, our knowledge about numbers advanced by leaps and bounds. But some numbers, by their very nature, are difficult to understand fully or even to satisfactory level. In the list of such types, primes lead undoubtedly. Failing to precisely formulate the properties of primes, mathematicians have ultimately started either approximating them to possible precisions [1] or do a manual hunt by rigorously checking each prime in higher and higher ranges [4].

Number 4 is special. It is a digit and first composite number. It is first even perfect square integer. It is equal to sum of 2 2's and also their product. ($4 = 2 + 2 = 2 \times 2$). The famous four-colour theorem states that to colour a planar graph such that adjacent regions are always of different colours, 4 colours will be required and lesser would not suffice. In fact, there are many properties peculiar about 4.

The ways in which zero occurs in positive integers have been analyzed earlier in detail [5], [6], [7] and the ways in which digit 1, in fact, any non-zero digit, occurs in positive integers have also been analyzed in detail [11], [12], [13], which is clearly applicable to 4 also.

All and successive occurrences of 4's in prime numbers are presented in [23], [24]. The analysis of all, successive as well as non-successive occurrences of digit 0 [8], [9], [10], digit 1 [14], [15], [16], digit 2 [17], [18], [19] and digit 3 [20], [21], [22] in primes are also available.

2. OCCURRENCE OF SINGLE NON-SUCCESSIVE DIGIT 4 IN PRIME NUMBERS

How single digit 4 comes in prime numbers can be found in [23] and [24]. But as single occurrence is always treated as successive, there cannot be non-successive occurrence of single digit 4 in any numbers, including primes.

3. OCCURRENCE OF MULTIPLE NON-SUCCESSIVE 4'S IN PRIME NUMBERS

Running a special program on many computers parallely, following values are obtained for the number of primes in ranges $1 - 10^n$, $1 \le n \le 12$, containing 2 or more number of non-successive 4's in their digits.

Table 1: Number of Primes in Various Ranges with Multiple Non-successive 4's in Their Digits

Sr. No.	Number Range <	Number of Primes with		
		2 Non-successive 4's	3 Non-successive 4's	4 Non-successive 4's
1.	10^4	9	0	0
2.	10^5	231	15	0
3.	10^6	3,423	417	27
4.	10^7	43,522	7,554	671
5.	10^8	507,544	112,795	12,811
6.	10^9	5,657,229	1,495,706	215,898
7.	10^{10}	60,806,336	18,561,814	3,210,703
8.	10^{11}	637,151,633	220,043,710	44,267,498
9.	10^{12}	6,549,114,184	2,520,543,238	577,585,698

Table 1: Continued …

Sr. No.	Number Range <	Number of Primes with		
		5 Non-successive 4's	6 Non-successive 4's	7 Non-successive 4's
1.	10^7	29	0	0
2.	10^8	847	23	0
3.	10^9	19,088	934	38
4.	10^{10}	359,977	26,187	1,277
5.	10^{11}	5,952,324	549,978	34,956
6.	10^{12}	90,592,197	10,074,698	794,621

Table 1: Continued …

Sr. No.	Number Range <	Number of Primes with			
		8 Non-successive 4's	9 Non-successive 4's	10 Non-successive 4's	11 Non-successive 4's
1.	10^{10}	34	0	0	0
2.	10^{11}	1,390	30	0	0
3.	10^{12}	43,541	1,512	28	0

The number of primes with various number of non-successive digit 4's in them in ranges of $1 - 10^n$ shows following plot when vertical axis is taken on logarithmic scale.

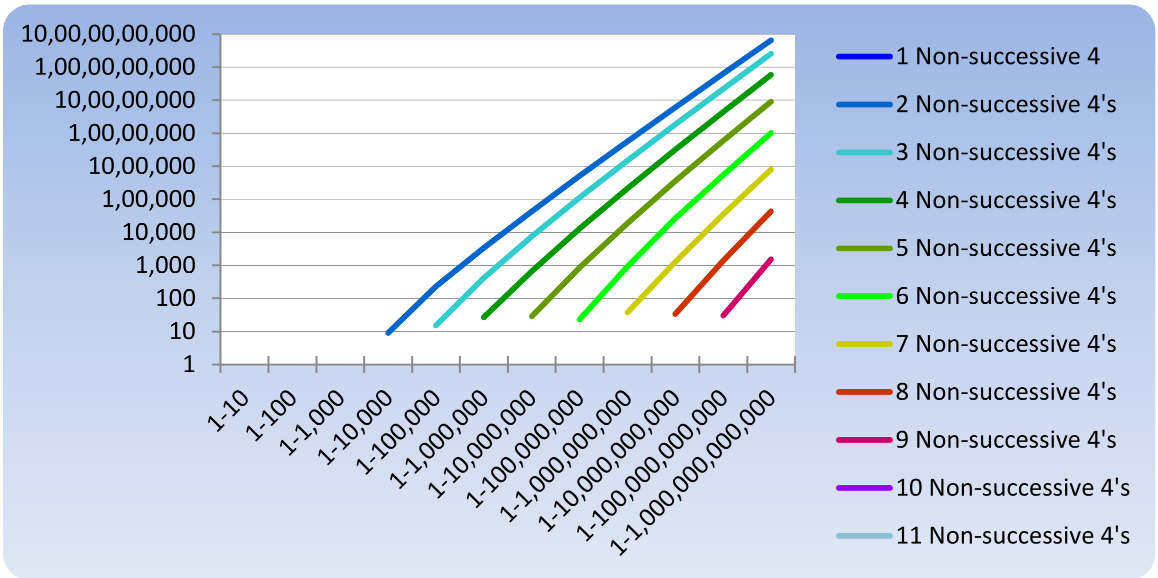


Figure 1: Number of Primes in Various Ranges with Multiple Non-successive 4's in Their Digits

The percentage of this number of primes with different number of non-successive 4' in them in various ranges with respect to number of integers with equal number of non-successive 4's [13] in corresponding ranges is as following.

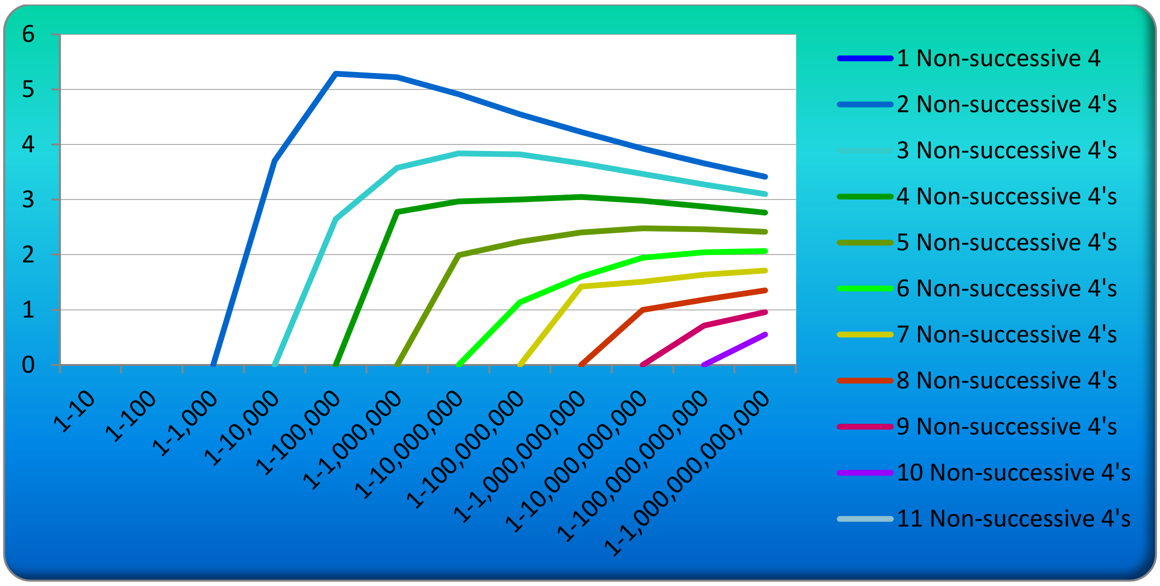


Figure 2: Percentage of Primes in Different Ranges with Multiple Non-successive 4's in Their Digits with Respect to All Such Positive Integers in Respective Ranges

Differences of number of multiple non-successive occurrences of digits 1, 2 and 3 in primes with those of 4 in them in our ranges are plotted graphically by dividing them in two blocks – first block of differences with 1 and 3 and the other with 2; the first ones occupy units place and the second one, except in unique case of 2, doesn't. Differences with digit 0 is not considered because 0 doesn't occupy units and leading n^{th} places in any n digit prime number.

We drop the graphs of differences for presence of single digit as there is no case of presence of single non-successive any digit whatsoever!

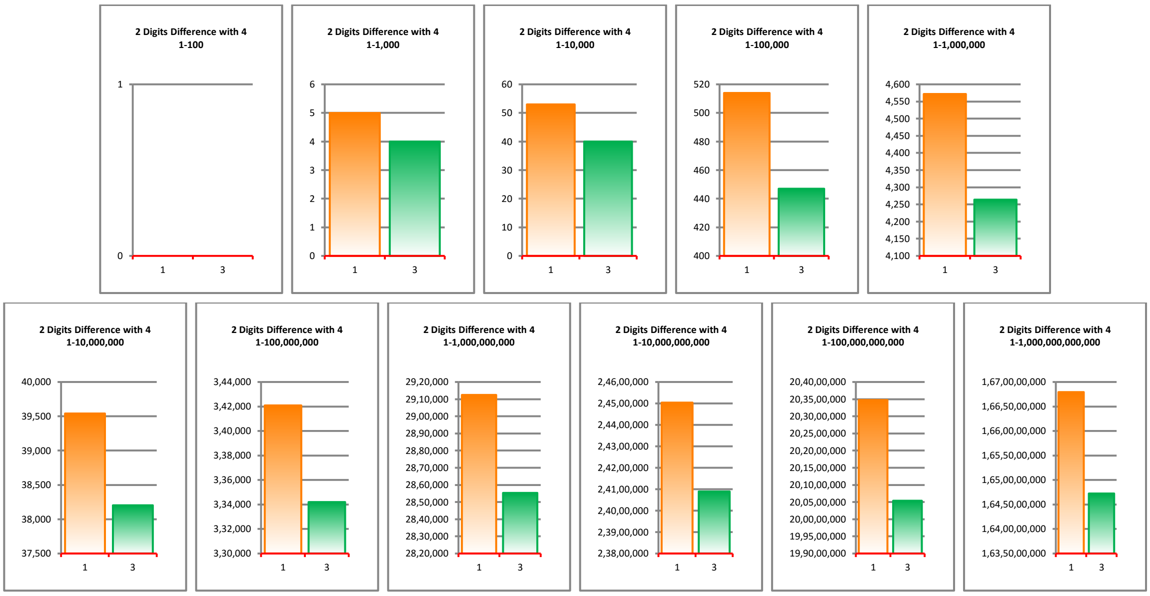


Figure 3: Differences of Number of Primes having Two Non-successive 1's and 3's in their Digits with those having Two Non-successive 4's in them in Ranges of 1 – 10^n.

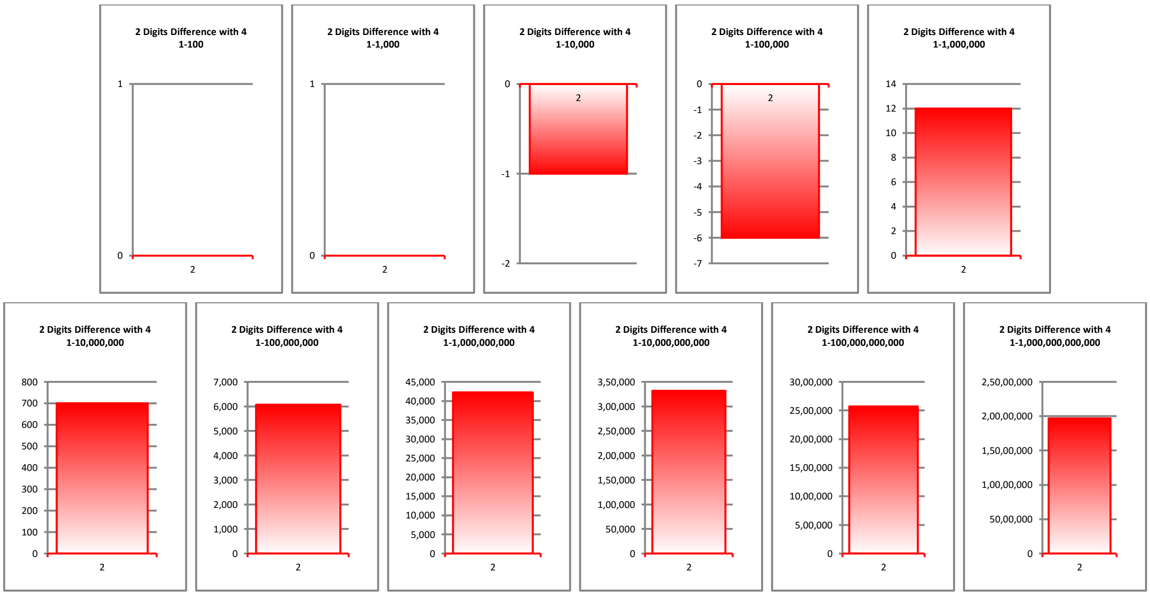
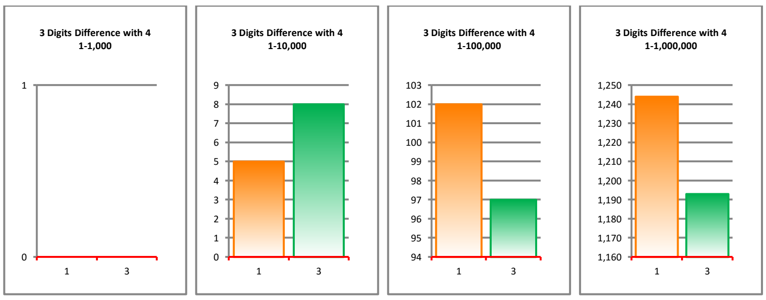


Figure 4: Difference of Number of Primes having Two Non-successive 2's in their Digits with those having Two Non-successive 4's in them in Ranges of 1 – 10^n.



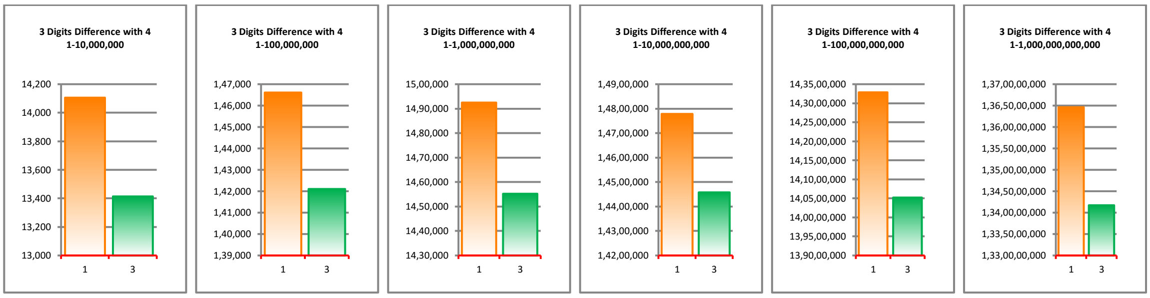


Figure 5: Differences of Number of Primes having Three Non-successive 1's and 3's in their Digits with those having Three Non-successive 4's in them in Ranges of $1 - 10^n$.

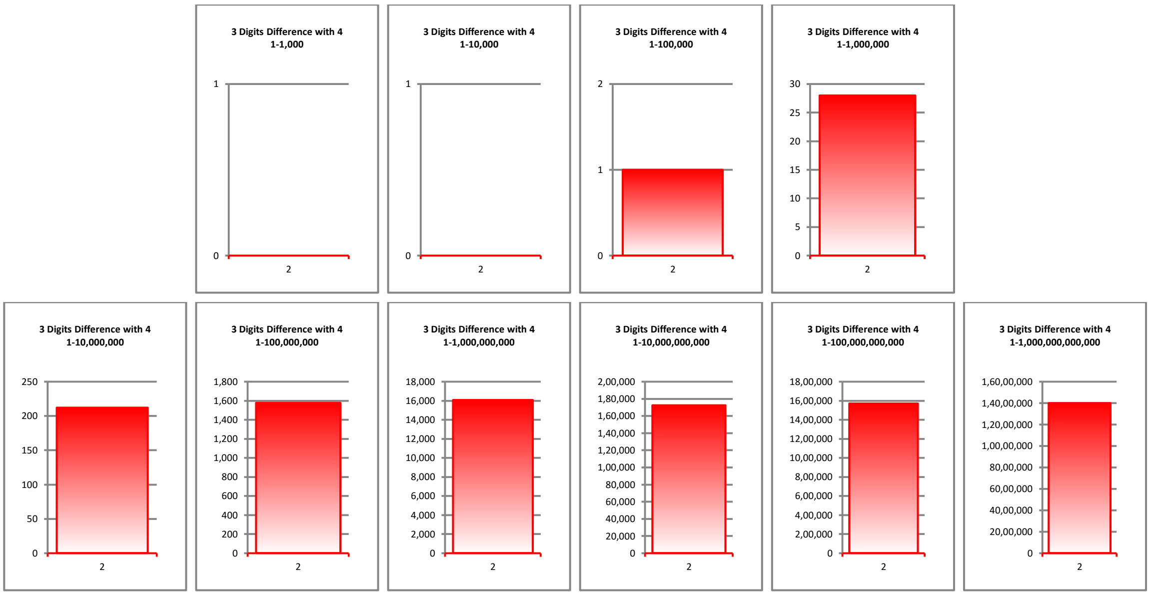


Figure 6: Difference of Number of Primes having Three Non-successive 2's in their Digits with those having Three Non-successive 4's in them in Ranges of $1 - 10^n$.

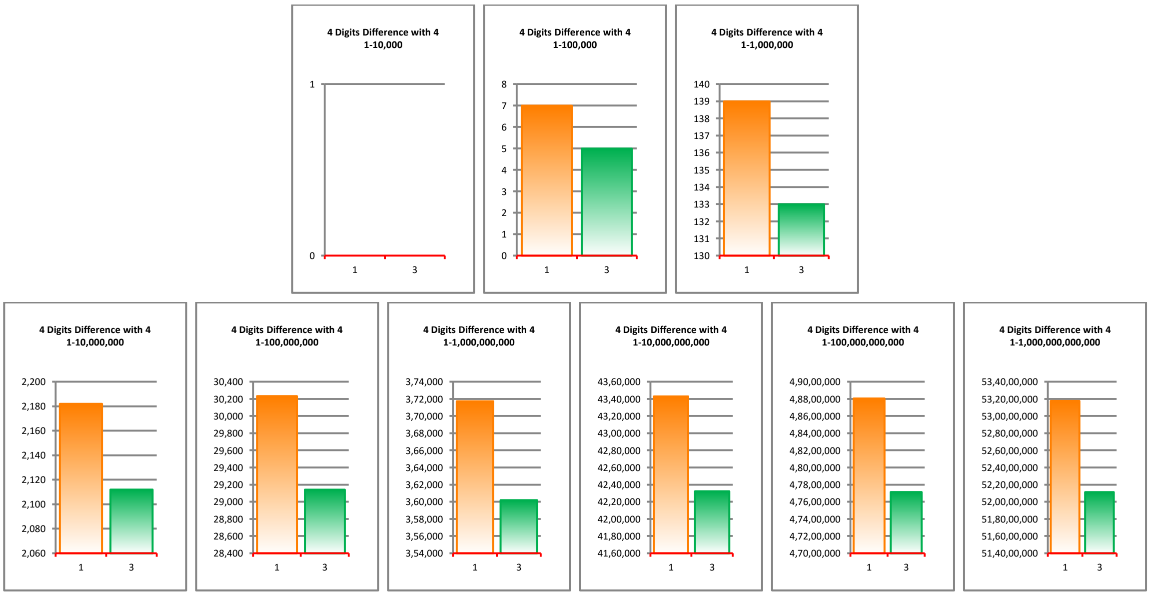


Figure 7: Differences of Number of Primes having Four Non-successive 1's and 3's in their Digits with those having Four Non-successive 4's in them in Ranges of $1 - 10^n$.

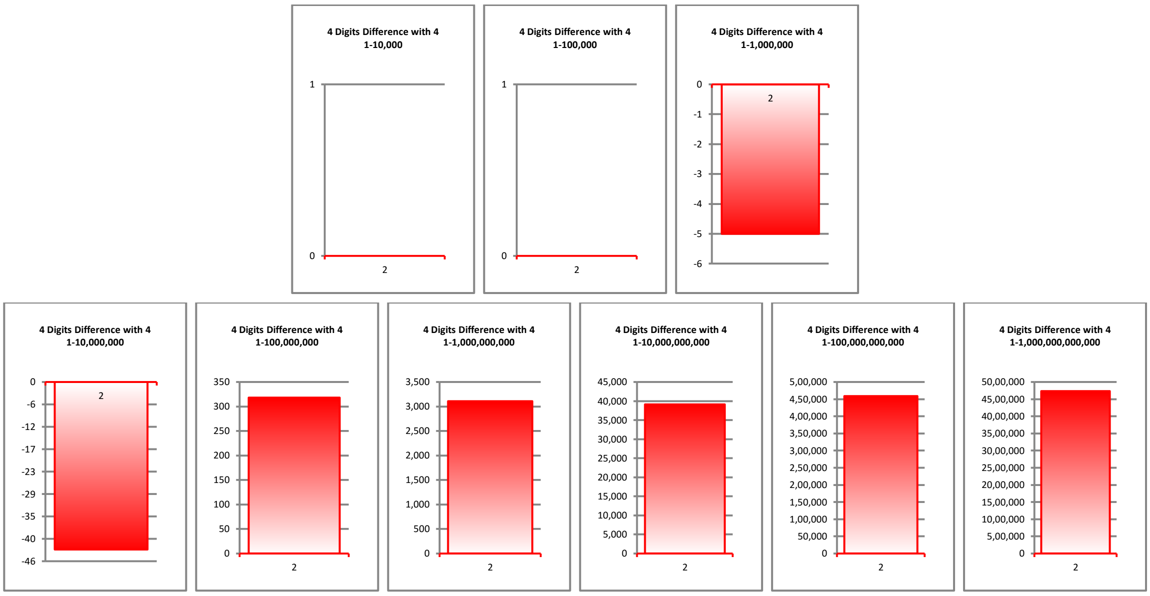


Figure 8: Difference of Number of Primes having Four Non-successive 2's in their Digits with those having Four Non-successive 4's in them in Ranges of 1 – 10^n.

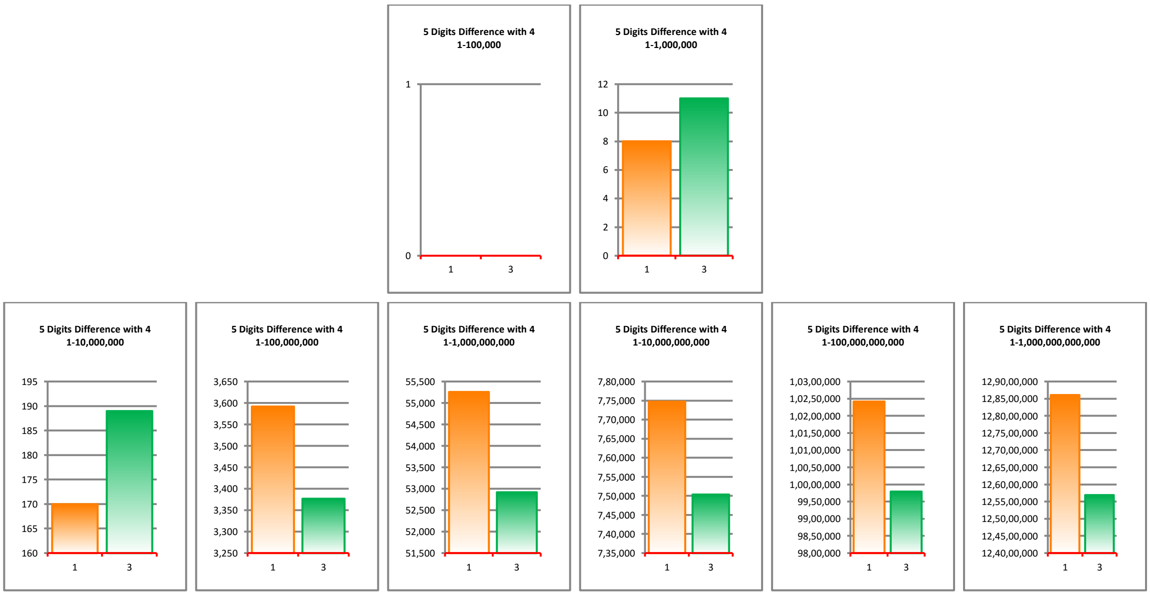
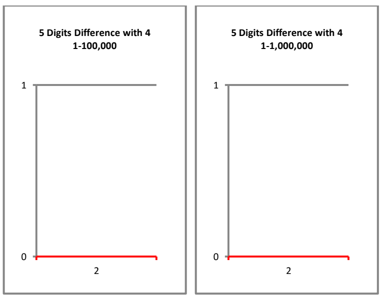


Figure 9: Differences of Number of Primes having Five Non-successive 1's and 3's in their Digits with those having Five Non-successive 4's in them in Ranges of 1 – 10^n.



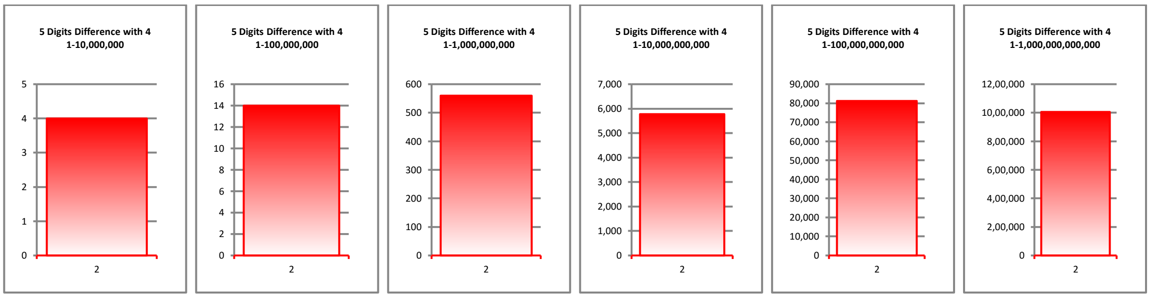


Figure 10: Difference of Number of Primes having Five Non-successive 2's in their Digits with those having Five Non-successive 4's in them in Ranges of $1 - 10^n$.

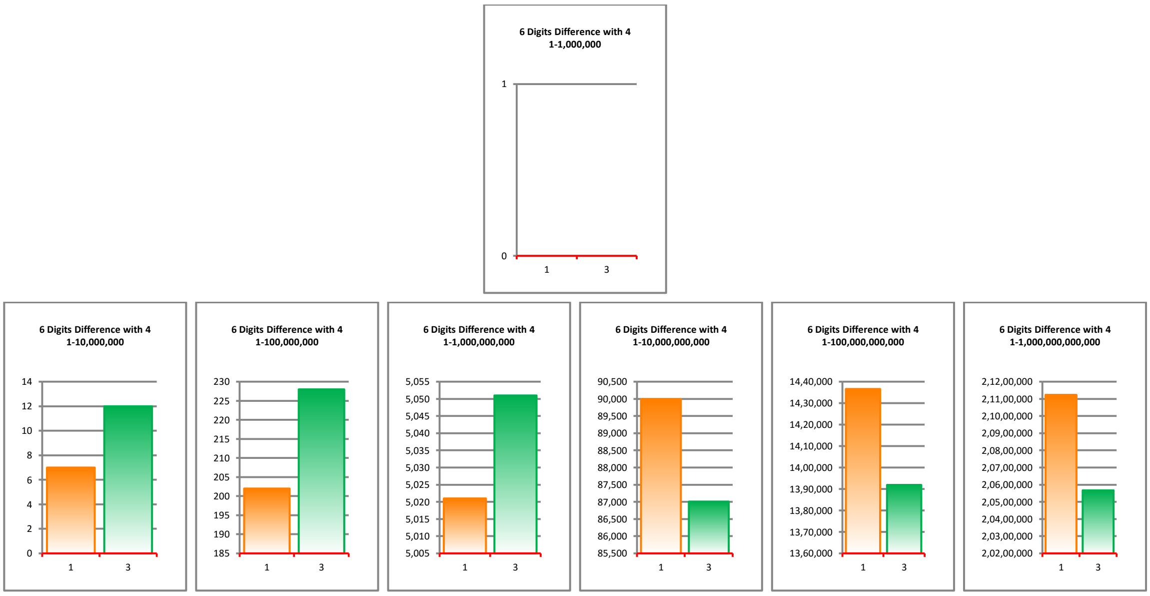


Figure 11: Differences of Number of Primes having Six Non-successive 1's and 3's in their Digits with those having Six Non-successive 4's in them in Ranges of $1 - 10^n$.

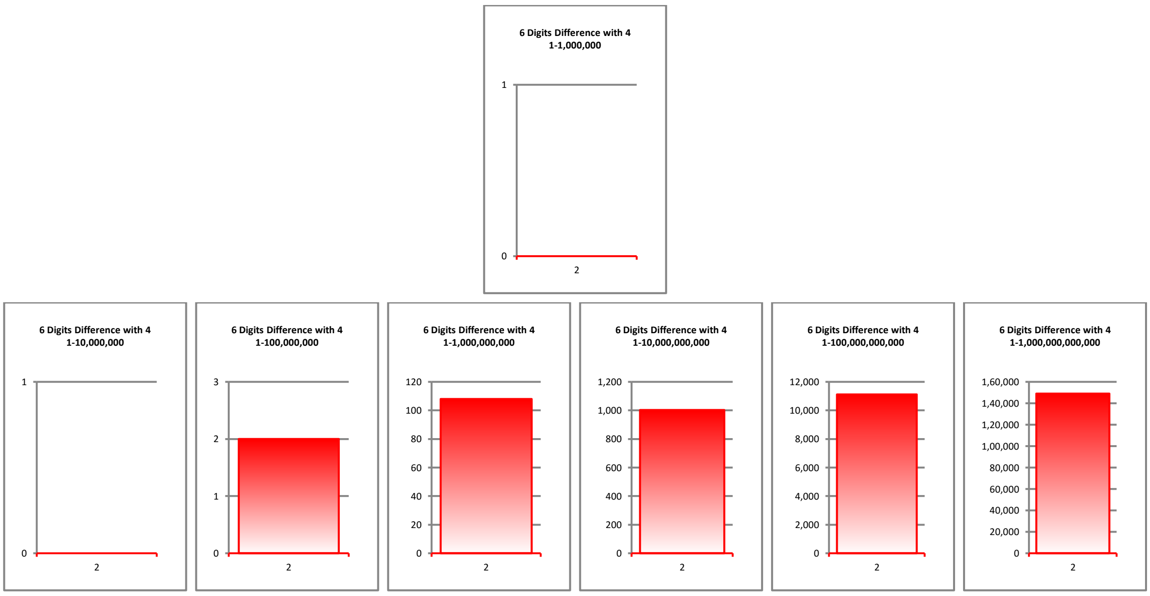


Figure 12: Difference of Number of Primes having Six Non-successive 2's in their Digits with those having Six Non-successive 4's in them in Ranges of $1 - 10^n$.

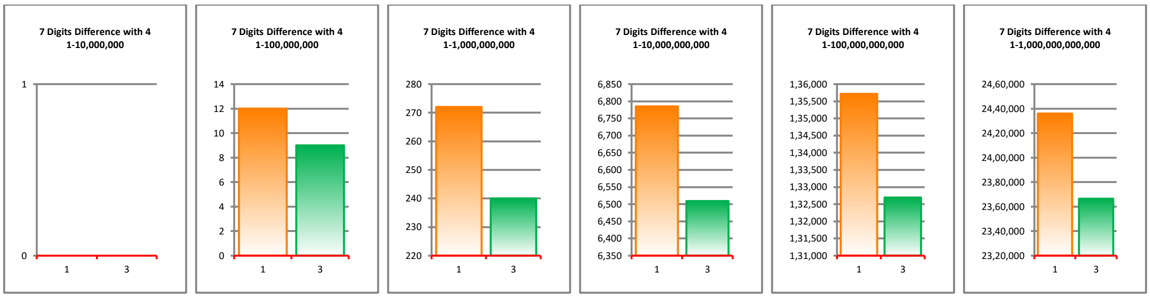


Figure 13: Differences of Number of Primes having Seven Non-successive 1's and 3's in their Digits with those having Seven Non-successive 4's in them in Ranges of 1 – 10^n.

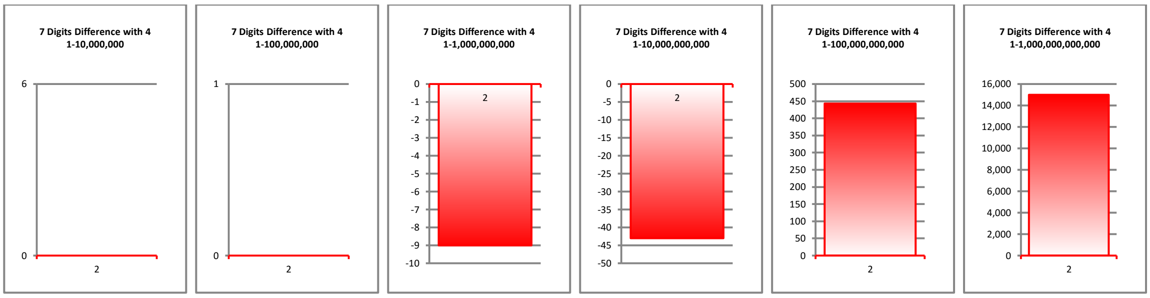


Figure 14: Difference of Number of Primes having Seven Non-successive 2's in their Digits with those having Seven Non-successive 4's in them in Ranges of 1 – 10^n.

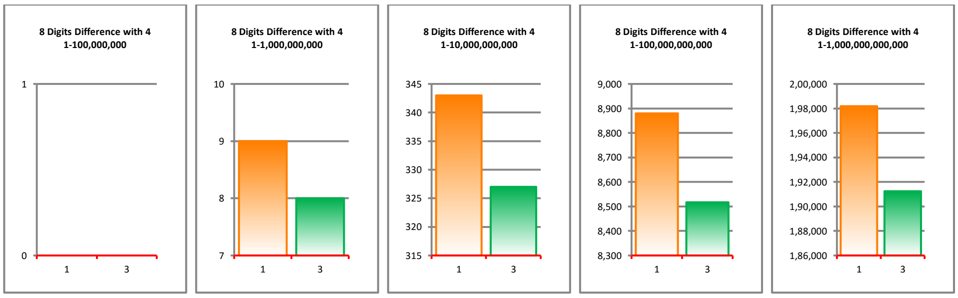


Figure 15: Differences of Number of Primes having Eight Non-successive 1's and 3's in their Digits with those having Eight Non-successive 4's in them in Ranges of 1 – 10^n.

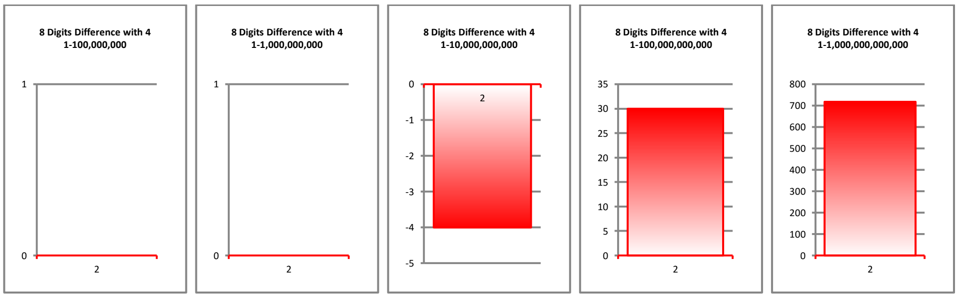


Figure 16: Difference of Number of Primes having Eight Non-successive 2's in their Digits with those having Eight Non-successive 4's in them in Ranges of 1 – 10^n.

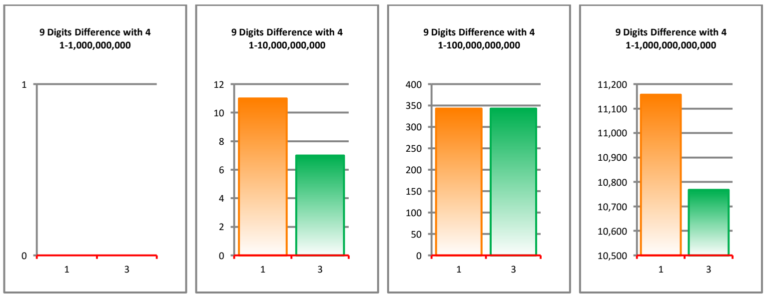


Figure 17: Differences of Number of Primes having Nine Non-successive 1's and 3's in their Digits with those having Nine Non-successive 4's in them in Ranges of $1 - 10^n$.

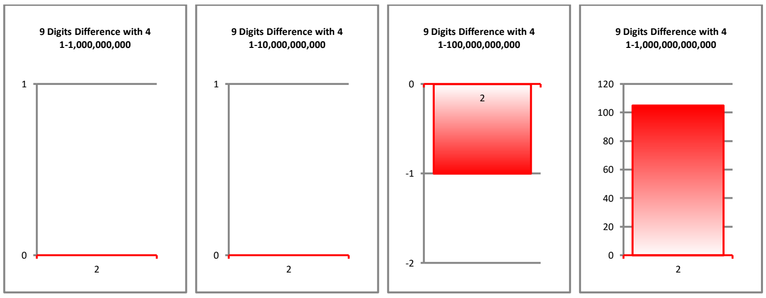


Figure 18: Difference of Number of Primes having Nine Non-successive 2's in their Digits with those having Nine Non-successive 4's in them in Ranges of $1 - 10^n$.

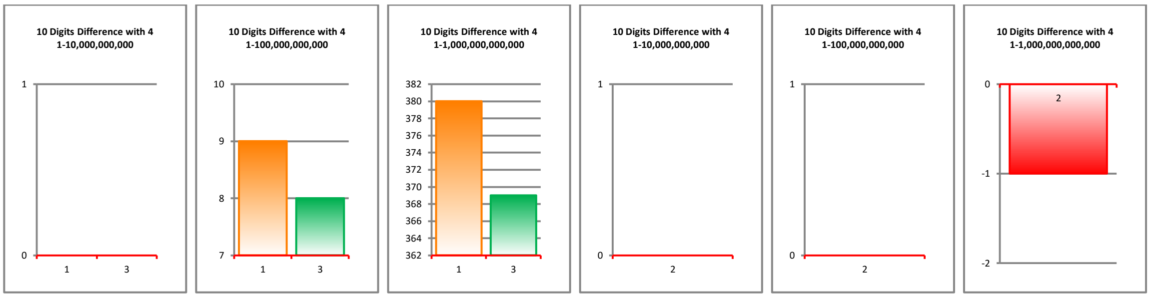


Figure 19: Differences of Number of Primes having Ten Non-successive 1's & 3's and having Ten Non-successive 2's in them with those having Ten Non-successive 4's in them in Ranges of $1 - 10^n$.

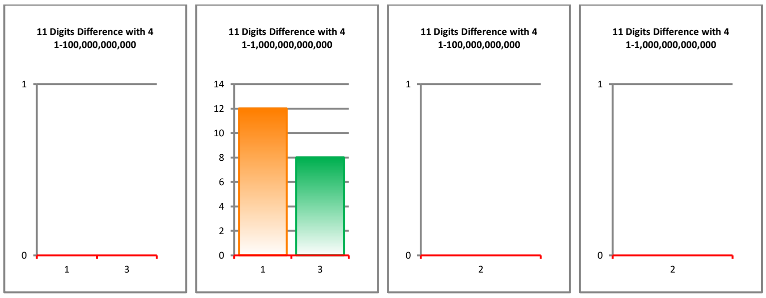


Figure 20: Differences of Number of Primes having 11 Non-successive 1's & 3's & having 11 Non-successive 2's in them with those having 11 Non-successive 4's in them in Ranges of $1 - 10^n$.

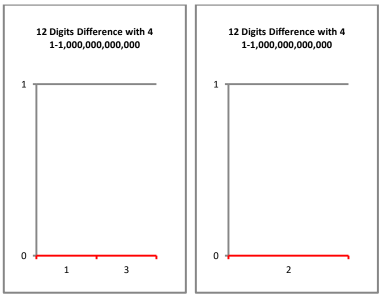


Figure 21: Differences of Number of Primes having 12 Non-successive 1's & 3's & having 12 Non-successive 2's in them with those having 12 Non-successive 4's in them in Ranges of $1 - 10^n$.

4. FIRST OCCURRENCE OF NON-SUCCESSIVE DIGIT 4'S IN PRIME NUMBERS

The first positive integer in different ranges containing various number of non-successive digit 4's in it is given by

Formula 1 [13] : If n and r are natural numbers, then the first occurrence of r number of non-successive 4's in numbers in range 1 ≤ m < 10^n is f = $\begin{cases} - & \text{, if } r<2 \text{ or } r \geq n \\ \sum\limits_{\substack{j=0 \\ j\neq r-1}}^{r} \left(4\times 10^{j}\right), & \text{if } r\geq 2 \text{ and } r<n \end{cases}$ ·

First such primes with multiple non-successive 4's are determined by ingenuous but rigorous computations.

Table 2: First Primes in Various Ranges with Multiple Non-successive 4's in Their Digits

Sr. No.	Range	First Prime Number in Range with r Non-successive 4's					
		r = 1	r = 2	r = 3	r = 4	r = 5	r = 6
1.	$1 - 10^1$	-	-	-	-	-	-
2.	$1 - 10^2$	-	-	-	-	-	-
3.	$1 - 10^3$	-	-	-	-	-	-
4.	$1 - 10^4$	-	4,049	-	-	-	-
5.	$1 - 10^5$	-	4,049	41,443	-	-	-
6.	$1 - 10^6$	-	4,049	41,443	404,449	-	-
7.	$1 - 10^7$	-	4,049	41,443	404,449	4,044,449	-
8.	$1 - 10^8$	-	4,049	41,443	404,449	4,044,449	40,444,447
9.	$1 - 10^9$	-	4,049	41,443	404,449	4,044,449	40,444,447
10.	$1 - 10^{10}$	-	4,049	41,443	404,449	4,044,449	40,444,447
11.	$1 - 10^{11}$	-	4,049	41,443	404,449	4,044,449	40,444,447
12.	$1 - 10^{12}$	-	4,049	41,443	404,449	4,044,449	40,444,447

Table 2: Continued …

Sr. No.	Range	First Prime Number in Range with r Non-successive 4's				
		r = 7	r = 8	r = 9	r = 10	r = 11
1.	$1 - 10^1$	-	-	-	-	-
2.	$1 - 10^2$	-	-	-	-	-
3.	$1 - 10^3$	-	-	-	-	-
4.	$1 - 10^4$	-	-	-	-	-
5.	$1 - 10^5$	-	-	-	-	-
6.	$1 - 10^6$	-	-	-	-	-
7.	$1 - 10^7$	-	-	-	-	-
8.	$1 - 10^8$	-	-	-	-	-
9.	$1 - 10^9$	424,444,441	-	-	-	-
10.	$1 - 10^{10}$	424,444,441	4,144,444,441	-	-	-
11.	$1 - 10^{11}$	424,444,441	4,144,444,441	42,444,444,443	-	-
12.	$1 - 10^{12}$	424,444,441	4,144,444,441	42,444,444,443	424,444,444,441	-

Some of them also happen to be overall first occurrences of those many 4's while others are not as successive 4's come in before them.

5. LAST OCCURRENCE OF NON-SUCCESSIVE DIGIT 4'S IN PRIME NUMBERS

The largest integer in ranges 1 – 10^n with r number of non-successive 4's is as in

Formula 2 [13] : If n and r are natural numbers, then last occurrence of r number of non-successive 4's in numbers in range $1 \le m < 10^n$ is $l = \begin{cases} - & \text{, if } r<2 \text{ or } r \ge n \\ \sum\limits_{\substack{j=0 \\ j\ne r-1}}^{r} \left(4\times 10^j\right) + \sum\limits_{\substack{j=r-1 \\ j\ne r}}^{n-1} \left(9\times 10^j\right), & \text{if } r\ge 2 \text{ and } r<n \end{cases}$.

The largest prime in these ranges with all possible non-successive 4's in its digits is determined computationally.

Table 3: Last Primes in Various Ranges with Multiple Non-successive 4's in Their Digits

Sr. No.	Number of Non-Successive 4's	Last Prime Number in Range 1 –							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
1.	1	-	-	-	-	-	-	-	-
2.	2	-	-	-	4,943	94,949	994,949	9,994,349	99,994,849
3.	3	-	-	-	-	48,449	949,441	9,949,441	99,949,441
4.	4	-	-	-	-	-	494,443	9,494,447	99,454,441
5.	5	-	-	-	-	-	-	4,844,443	94,844,443
6.	6	-	-	-	-	-	-	-	46,444,441
7.	7	-	-	-	-	-	-	-	-
8.	8	-	-	-	-	-	-	-	-
9.	9	-	-	-	-	-	-	-	-
10.	10	-	-	-	-	-	-	-	-
11.	11	-	-	-	-	-	-	-	-

Table 3: Continued …

Sr. No.	Number of Non-Successive 4's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
1.	1	-	-	-
2.	2	999,994,843	9,999,994,141	99,999,994,747
3.	3	999,944,549	9,999,949,441	99,999,944,741
4.	4	999,464,449	9,999,464,441	99,999,484,441
5.	5	994,544,443	9,994,844,443	99,994,644,443
6.	6	944,744,441	9,949,444,441	99,946,444,441
7.	7	494,444,441	9,494,444,447	99,494,444,449
8.	8	-	4,844,444,443	94,944,444,449
9.	9	-	-	48,444,444,443
10.	10	-	-	-
11.	11	-	-	-

Remark : The maximum number of non-successive digits 4's in any prime number in the range 1 - 10^n is at most $n - 2$.

All numbers coming in each section of this work constitute interesting integer sequences and are worth exploring further.

6. ACKNOWLEDGMENT

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