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SOME CHARACTERIZATIONS OF VAGUE 'N' GROUPS

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ABSTRACT
In this paper we introduce the concepts of Vague N set, Vague N group and we studied their properties.These concepts are used in the development of some important results and theorems in vague algebra.

Keywords: Vague set, Vague group, Vague N set, Vague N group.
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1. Introduction

Gau.W.L and Bueher D.J. [1] have initiated the study of vague sets as an improvement over the theory of fuzzy sets to interpret and solve real life problems which are in general vague. Ranjit Biswas [5] initiated the study of vague groups. In this paper we introduce the concept of vague N set, Vague N group and we studied the some of their properties.

2. Preliminaries: We discuss here a review of some definitions and results which are in Gau. W. L. and Buehrer [1], Ranjit Biswas [5].

Definition 2.1: A vague set A in the universe of discourse U is a pair (t_A, f_A) where $t_A : U \to [0,1]$, $f_A : U \to [0,1]$,are mappings such that $t_A(u)+ f_A(u) \le 1$,for all $u \in U$. The functions t_A and f_A are called true membership function and false membership function respectively.

Definition 2.2: The interval $[t_A(u),1- f_A(u)]$ is called the vague value of u in A , and it is denoted by $V_A(u)$. i.e. $V_A(u)$ = $[t_A(u),1- f_A(u)]$.

Definition 2.3 : Let (G , $*$) be a group. A vague set A of G is called a vague group of G if for all x,y in G $V_A(xy) \ge imin\{V_A(x),V_A(y)\}$ and $V_A(x^{-1}) \ge V_A(x)$ for all x in G.

i.e. $t_A(xy) \ge min\{t_A(x),t_A(y)\}$, $f_A(xy) \le max\{ f_A(x), f_A(y)\}$

and $t_A(x^{-1}) \ge t_A(x)$, $f_A(x^{-1}) \le f_A(x)$.

Here the element xy stands for x $*$ y.

Notation 2.3: Let I[0,1] denotes the family of all closed subinterval of [0,1]. If $I_1 = [a_1,b_1]$ and $I_2 = [a_2,b_2]$ be two elements of I[0,1]. We call $I_1 \ge I_2$ if $a_1 \ge a_2$ and $b_1 \ge b_2$, with the order in I[0,1] is a lattice with operations min., or inf and max. or sup given by imin. $\{I_1, I_2\}$ = $[min.(a_1,a_2), min.(b_1,b_2)]$, $imax\{I_1, I_2\}$ = $[max.(a_1,a_2), max.(b_1,b_2)]$

3. Vague N set and Vague N group

Now we introducing the following

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Definition 3.1 : A vague N set A_N in the universe of discourse X is a pair (t_{A_N}, f_{A_N}) where $t_{A_N} : X\times N \to [0,1]$, $f_{A_N} : X\times N \to [0,1]$,are mappings such that $t_{A_N}(x,n) + f_{A_N}(x,n) \le 1$ for all $(x,n)\in X\times N$. The functions t_{A_N} and f_{A_N} are called true membership function and false membership function respectively.

Definition 3.2 : If $A_N = (t_{A_N}, f_{A_N})$, $B_N = (t_{B_N}, f_{B_N})$ are two vague N sets of X then their intersection is defined as $V_{(A_N\cap B_N)}(x,n) = imin.\{V_{A_N}(x,n), V_{B_N}(x,n)\}$.

i.e. $t_{(A_N\cap B_N)}(x,n) = min.\{t_{A_N}(x,n), t_{B_N}(x,n)\}$ and

$f_{(A_N\cap B_N)}(x,n) = max.\{f_{A_N}(x,n), f_{B_N}(x,n)\}$, for x in G and n in N.

Definition 3.3: Let (G ,∗) be a group. A vague N set A_N of G is called a vague N group of G if for all x, y in G ,n in N, $V_{A_N}(xy,n) \ge imin\{V_{A_N}(x,n), V_{A_N}(y,n)\}$ and $V_{A_N}(x^{-1},n) \ge V_{A_N}(x,n)$ for all x in G, n in N . i.e. $t_{A_N}(xy,n) \ge min\{t_{A_N}(x,n), t_{A_N}(y,n)\}$, $f_{A_N}(xy,n) \le max\{f_{A_N}(x,n), f_{A_N}(y,n)\}$ and $t_{A_N}(x^{-1},n) \ge t_{A_N}(x,n)$, $f_{A_N}(x^{-1},n) \le f_{A_N}(x,n)$.

Here the element x y stands for x∗y.

Theorem 3.4: If A_N is a vague N group of group G then for all x $\in G$, $n\in N$, $V_{A_N}(x^{-1},n) = V_{A_N}(x,n)$

Proof: Let A_N be a vague N group of a group G ,we have

$V_{A_N}(x^{-1},n) \ge V_{A_N}(x,n)$, for all x $\in G$, n $\in N$. Since $x^{-1}\in G$

we have $V_{A_N}(x,n) = V_{A_N}((x^{-1})^{-1},n) \ge V_{A_N}(x^{-1},n)$.

This implies $V_{A_N}(x^{-1},n) = V_{A_N}(x,n)$.

Theorem 3.5: If A_N is a vague N group of group G then for all x $\in G$, $n\in N$, $V_{A_N}(e,n) \ge V_{A_N}(x,n)$.

Proof: Let A_N be a vague N group of a group G ,

For all x $\in G$, n $\in N$ we have

$V_{A_N}(e,n) = V_{A_N}((xx^{-1},n) \ge imin\{V_{A_N}(x,n), V_{A_N}(x^{-1},n)\} = imin\{V_{A_N}(x,n), V_{A_N}(x,n)\} = V_{A_N}(x,n)$.

This implies $V_{A_N}(e,n) \ge V_{A_N}(x,n)$, for all x $\in G$, $n\in N$.

Theorem 3.6 : A necessary and sufficient condition for a vague N set of a group G to be a vague N group of G is that $V_{A_N}(xy^{-1},n) \ge imin.\{V_{A_N}(x,n), V_{A_N}(y,n)\}$.

Proof: Let A_N be a vague N set. Suppose $V_{A_N}(xy^{-1},n) \ge imin.\{V_{A_N}(x,n), V_{A_N}(y,n)\}$ for all x $\in G$, $n\in N$ .We have by theorem 3.4, $V_{A_N}(e,n) \ge V_{A_N}(x,n)$.

Now $V_{A_N}(x^{-1},n) = V_{A_N}(ex^{-1},n) \ge imin\{V_{A_N}(e,n), V_{A_N}(x,n)\}$

$= V_{A_N}(x,n)\}$. Thus $V_{A_N}(x^{-1},n) \ge V_{A_N}(x,n)$.

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Let x, y $\in G, n \in N, V_{A_N}(xy,n) = V_{A_N}(x(y^{-1})^{-1},n)$

$\geq imin.\{V_{A_N}(x,n), V_{A_N}(y^{-1},n)\} \geq imin.\{V_{A_N}(x,n), V_{A_N}(y,n)\}$. This gives

A_N is a vague N group of G.

Conversely, A_N is vague N group of G. Let x, y $\in G, n \in N$ we have

$V_{A_N}(xy^{-1},n) \geq imin.\{V_{A_N}(x,n), V_{A_N}(y^{-1},n)\} = imin.\{V_{A_N}(x,n), V_{A_N}(y,n)\}$.

This completes the proof.

Theorem 3.7: If A_N, B_N are two vague N groups of a group G then $A_N \cap B_N$ is also a vague N group of G.

Proof : A_N, B_N are two vague N groups of a group G, we have

$t_{(A_N \cap B_N)}(xy^{-1},n) = min.\{t_{A_N}(xy^{-1},n), t_{B_N}(xy^{-1},n)\}$

$\geq min\{min\{t_{A_N}(x,n), t_{A_N}(y,n)\}, min\{t_{B_N}(x,n), t_{B_N}(y,n)\}\}$.

$= min\{min\{t_{A_N}(x,n), t_{B_N}(x,n)\}, min\{t_{A_N}(y,n), t_{B_N}(y,n)\}\}$

$= min\{t_{(A_N \cap B_N)}(x,n), t_{(A_N \cap B_N)}(y,n)\}$.

Thus $t_{(A_N \cap B_N)}(xy^{-1},n) \geq min\{t_{(A_N \cap B_N)}(x,n), t_{(A_N \cap B_N)}(y,n)\}$.

Similarly we have $f_{(A_N \cap B_N)}(xy^{-1},n) \leq max\{f_{(A_N \cap B_N)}(x,n), f_{(A_N \cap B_N)}(y,n)\}$.

Hence $V_{(A_N \cap B_N)}(xy^{-1},n) \geq imin\{V_{(A_N \cap B_N)}(x,n), V_{(A_N \cap B_N)}(y,n)\}$.

Thus $A \cap B$ is a vague N group of group G.

Theorem 3.8: Let A_N be a vague N group of a group G .Then $V_{A_N}(xy^{-1},n) = V_{A_N}(e,n)$ implies $V_{A_N}(x,n) = V_{A_N}(y,n)$ for any x and y in G, n in N.

Proof : Suppose $V_{A_N}(xy^{-1},n) = V_{A_N}(e,n)$. Consider

$V_{A_N}(x,n) = V_{A_N}(x.e,n) = V_{A_N}(x.y^{-1}.y,n) \geq imin.\{V_{A_N}(x.y^{-1},n), V_{A_N}(y,n)\} = imin.\{V_{A_N}(e,n), V_{A_N}(y,n)\} = V_{A_N}(y,n)$

since $V_{A_N}(e,n) \geq V_{A_N}(y,n)$ for all y in G.

This gives $V_{A_N}(x,n) \geq V_{A_N}(y,n)$, since $V_{A_N}(z,n) = V_{A_N}(z^{-1},n)$, we get

$V_{A_N}(yx^{-1},n) = V_{A_N}(e,n)$ and now interchange the roles of x and y then we get $V_{A_N}(y,n) \geq V_{A_N}(x,n)$ This implies $V_{A_N}(x,n) = V_{A_N}(y,n)$.

Theorem 3.9: Let G be a group and A_N be a vague N group of G and if for a fixed y in G, if for all x in G, n in N $V_{A_N}(x,n) \leq V_{A_N}(y,n)$ $then V_{A_N}(xy,n) = V_{A_N}(x,n) = V_{A_N}(yx,n)$.

Proof: $V_{A_N}(xy,n) \geq imin.\{V_{A_N}(x,n), V_{A_N}(y,n)\} = V_{A_N}(x,n)$

implies $V_{A_N}(xy,n) \geq V_{A_N}(x,n)$, since by hypothesis $V_{A_N}(y,n) \geq V_{A_N}(x,n)$ for all x, we in particular have $V_{A_N}(x,n) \geq V_{A_N}(xy,n)$ by taking xy in place of x now

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$V_{A_N}(x,n)=V_{A_N}(x.e,n)=V_{A_N}(xy.y^{-1},n)\geq imin.\{V_{A_N}(xy,n),V_{A_N}(y^{-1},n)\}=imin.\{V_{A_N}(xy,n),V_{A_N}(y,n)\}=V_{A_N}(xy,n)$ which implies $V_{A_N}(x,n)\geq V_{A_N}(xy,n)$ .Thus

$V_{A_N}(xy,n)=V_{A_N}(x,n)$. In a similar fashion , we have $V_{A_N}(yx,n)=V_{A_N}(x,n)$.This completes the theorem.

Theorem 3.10 : Let A_N be a Vague N group of group G and $x\in G$,n $\in N$. Then $V_{A_N}(xy,n)=V_{A_N}(y,n)\ for\ all\ y\in G$ iff $V_{A_N}(x,n)=V_{A_N}(e,n)$.

Proof: Suppose $V_{A_N}(xy,n)=V_{A_N}(y,n)\ for\ all\ y\in G, n\in N$. Chose $y=e$ in this equality then we have $V_{A_N}(x.e,n)=V_{A_N}(e,n)$ implies $V_{A_N}(x,n)=V_{A_N}(e,n)$. Conversely, suppose

$V_{A_N}(x,n)=V_{A_N}(e,n)$.

For any $y\in G$, $V_{A_N}(y,n)\leq V_{A_N}(e,n)$ implies $V_{A_N}(y,n)\leq V_{A_N}(x,n)$.

Now, $V_{A_N}(xy,n)\geq imin\{V_{A_N}(x,n),V_{A_N}(y,n)\}=V_{A_N}(y,n)$ by

(1). This implies $V_{A_N}(xy,n)\geq V_{A_N}(y,n)\ \ for\ all\ y\in G.$

But $V_{A_N}(y,n)=V_{A_N}(e.y,n)=V_{A_N}(x^{-1}x.y,n)\geq imin\{V_{A_N}(x^{-1},n),V_{A_N}(xy,n)\}$

= imin $\{V_{A_N}(x,n),V_{A_N}(xy,n)\}$ = imin $\{V_{A_N}(e,n),V_{A_N}(xy,n)\}$ = $V_{A_N}(xy,n)$

This implies $V_{A_N}(y,n)\geq V_{A_N}(xy,n)$ .Thus $V_{A_N}(xy,n)=V_{A_N}(y,n)$.

Conclusion : Group theory has many applications in computer Science, Space Physics, Analytical Chemistry, etc, In this paper we introduced and studied the properties of vague N set and vague N group.

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