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RESEARCH ARTICLE

SOLVING FUZZY MATRIX GAME PROBLEM INVOLVING HEXAGONAL FUZZY NUMBERS

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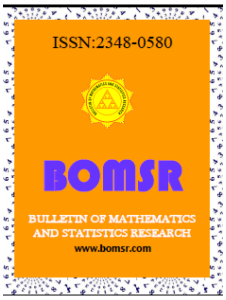

ABSTRACT

Fuzzy set theory has been applied in many fields such as operations research, control theory, management science and matrix theory etc. In this paper, we consider a solution of fuzzy game with pay-off matrix elements as imprecise numbers instead of crisp numbers. In particular Hexagonal fuzzy numbers have been employed in the work. The solution of such fuzzy games with pure strategies by minimax-maximin principle is discussed. The various methods of fuzzy games have been solved by the general rules to follow, after the pure strategy is fails, such a methods are 2×2 fuzzy game method, Oddment method and Dominance principle. A relevant numerical examples have been given in support of the proposed methods.

Keywords: Fuzzy Number, Hexagonal Fuzzy Number(HFN), Pay-off Matrix, Fuzzy Game, Max-Min Principle.

1. INTRODUCTION

The problem of game theory [2, 3] is defined as a body of knowledge thats deals with making decision when two or more intelligent and rational opponents are involved under conditions of conflict and competition. In practical situation, it is the utility of decisions in a competing real life situation that there are two or more opposite teams with conflicting interest and the outcome is controlled by the decision of all parties concerned. Such problems occur frequently in economic business administration sociology, political science and military operations. The mathematical treatment of the game the game theory was made available in 1944, when John von Newmann and Oscar Morgenstern [9] published the famous article, Theory of games and economics behavior. However, in real life situations, an inherent degree of vagueness or uncertainty present in the system is available in imprecise nature the information.

Fuzzy numbers play an important aspects in many applications, including decision making, approximate reasoning, optimization etc, of all shapes of fuzzy number, triangular fuzzy number, trapezoidal fuzzy number and now we proposed the hexagonal fuzzy number can be applied in optimization problems. When we apply the game theory to model some practical situation, we have to know the values of pay-off exactly. However, it is difficult situation of decision making to know the exact values of payoffs and we could know the values of payoffs approximately. In such situations, it is useful to model the problems as games with fuzzy payoffs.

In this work, we have concentrated on the solution of fuzzy games with payoff matrix as hexagonal fuzzy number which is characterized in various methods. The methods like 2×2 fuzzy games, matrix oddment method and the dominance method. These methods are generated to solve the fuzzy game problem depends upon, if there is no saddle point.

In Narayanan.et.al[5], probability and possibility approaches have been used to solve a 2×2 interval game problem. Nishizaki and Sakawa [8] shows the max-min solution for fuzzy multi objective matrix games. Zimmermann [14] shows that α−cut of a fuzzy number is an interval number. Dubois and Prade [1] proposed the operations on fuzzy number. In Nayak.et.al [6, 7] the concept of the matrix game with triangular pay-off using score function and solution to the rectangular fuzzy games have been studied. Selvakumari and Lavanya [10, 11] proposed to solve the fuzzy game problem for the shapes of fuzzy numbers. Dinagar and Porchelvi [13] established the concept of the fuzzy dominance theory. Loganathan and Christi [4] proposed the fuzzy game value of the interval matrix. In Dinagar and Harinarayanan [12] the concept of hexagonal fuzzy number with convexity condition have been proposed.

The paper organized as follows. In section 2, basic definitions of hexagonal fuzzy numbers and its operations are described. In section 3, the basic building blocks of game theory definitions are discussed. In section4, the concept 2×2 fuzzy game problem is defined and their illustration is given. In section 5, the method of oddment n×n fuzzy matrix and their example are given. In section 6, the concept of dominance principle and their proper examples are given. In section 7, conclusion has been drawn.

2.PRELIMINARIES

In this section, some basic definitions are recalled for hexagonal fuzzy number.

Definition 2.1. (Fuzzy Set)

A fuzzy set A in X is characterized by its membership function $\mu_A: X \to [0,1]$ and $\mu_A(x)$ is the degree of membership of elements$x$ in fuzzy set A for each $x \in X$.

Definition 2.2. (Convex Fuzzy Set)

A fuzzy set $A = \{(x, \mu_A(x))\} \subseteq X$ is called convex fuzzy set if all A_α are convex set (i.e) for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ for every $\alpha \in [0,1]$. $\lambda x_1 + (1-\lambda)x_2 \in A_\alpha$for all $\lambda \in [0,1]$. Otherwise the fuzzy set is called non-convex fuzzy set.

Definition 2.3. (Fuzzy Number)

A fuzzy set Ã, defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

i. Ã is normal
ii. Ã is convex set
iii. The support of Ã is closed and bounded then Ã is called fuzzy number.

Definition 2.4. (Hexagonal Fuzzy Number)

A fuzzy number on $\tilde{A}_h$ is hexagonal fuzzy number denoted by

$\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$, where $(a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6)$ are real number satisfying $a_2 - a_1 \le a_3 - a_2$ and $a_5 - a_4 \ge a_6 - a_5$ and its membership function $\mu_{\tilde{A}_h}(x)$ is given by

$$\mu_{\tilde{A}_h}(x) = \begin{cases} 0 & , x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & , a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & , a_2 \le x \le a_3 \\ 1 & , a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & , a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & , a_5 \le x \le a_6 \\ 0 & , x > a_6 \end{cases}$$

Remark 2.5

The hexagonal fuzzy number $\tilde{A}_h$ becomes trapezoidal fuzzy number if $a_2 - a_1 = a_3 - a_2$ and $a_5 - a_4 = a_6 - a_5$.

The hexagonal fuzzy number $\tilde{A}_h$ becomes non-convex fuzzy number if $a_2 - a_1 > a_3 - a_2$ and $a_5 - a_4 < a_6 - a_5$.

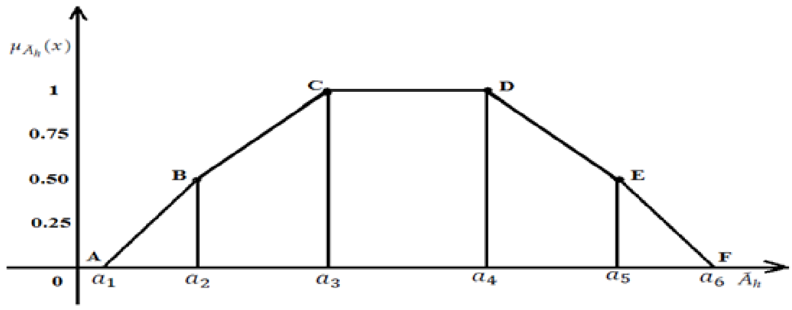

Fig 1. Hexagonal Fuzzy Number $\tilde{A}_h$

2.6. Arithmetic Operations On Hexagonal Fuzzy Numbers (HFNs)

The arithmetic operations between hexagonal fuzzy number (HFNs) are proposed given below.

Let us consider $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B}_h = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two hexagonal fuzzy numbers then,

(i) Addition

$$\tilde{A}_h(+)\tilde{B}_h = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6).$$

(ii) Subtraction

$$\tilde{A}_h(-)\tilde{B}_h = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1).$$

(iii) Multiplication

$$\tilde{A}_h(\times)\tilde{B}_h = \left(\frac{a_1}{6}\sigma_b, \frac{a_2}{6}\sigma_b, \frac{a_3}{6}\sigma_b, \frac{a_4}{6}\sigma_b, \frac{a_5}{6}\sigma_b, \frac{a_6}{6}\sigma_b\right).$$

Where $\sigma_b = b_1 + b_2 + b_3 + b_4 + b_5 + b_6$.

(iv) Division

$$\tilde{A}_h(\div)\tilde{B}_h = \left(\frac{6a_1}{\sigma_b}, \frac{6a_2}{\sigma_b}, \frac{6a_3}{\sigma_b}, \frac{6a_4}{\sigma_b}, \frac{6a_5}{\sigma_b}, \frac{6a_6}{\sigma_b}\right).$$

Where $\sigma_b = b_1 + b_2 + b_3 + b_4 + b_5 + b_6$.

(v) Scalar Multiplication

If $k \ne 0$ is scalar $k\tilde{A}_h$ is defined as

$$k\tilde{A}_h = \begin{cases} (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6) & if\ k \ge 0 \\ (ka_6, ka_5, ka_4, ka_3, ka_2, ka_1) & if\ k \le 0 \end{cases}$$

Definition 2.7. Ranking Function

We define a ranking function $\breve{R}: F(R) \to R$ which maps each fuzzy numbers to real line $F(R)$ represent the set of all hexagonal fuzzy numbers. If R be any linear ranking functions.

$$\breve{R}(\tilde{A}_h) = \left(\frac{a_1+a_2+a_3+a_4+a_5+a_6}{6}\right).$$

Also we defined orders on $F(R)$ by

$\breve{R}(\tilde{A}_h) \geq \breve{R}(\tilde{B}_h)$if and only if $\tilde{A}_h \underset{\breve{R}}{\geq} \tilde{B}_h$,

$\breve{R}(\tilde{A}_h) \leq \breve{R}(\tilde{B}_h)$if and only if $\tilde{A}_h \underset{\breve{R}}{\leq} \tilde{B}_h$ and

$\breve{R}(\tilde{A}_h) = \breve{R}(\tilde{B}_h)$if and only if $\tilde{A}_h \underset{\breve{R}}{=} \tilde{B}_h$

Definition 2.8. Zero Hexagonal fuzzy number

If $\tilde{A}_h = (0,0,0,0,0,0)$ then $\tilde{A}_h$ is said to be zero hexagonal fuzzy number. It is denoted by 0.

Definition 2.9. Zero-Equivalent Hexagonal fuzzy number

A hexagonal fuzzy number $\tilde{A}_h$ is said to be a zero-equivalent hexagonal fuzzy number if $\breve{R}(\tilde{A}_h) = 0$. It is denoted by $\tilde{0}$.

Definition 2.10. Unit Hexagonal fuzzy number

If $\tilde{A}_h = (1,1,1,1,1,1)$ then $\tilde{A}_h$ is said to be unit hexagonal fuzzy number. It is denoted by 1.

Definition 2.11. Unit-Equivalent Hexagonal fuzzy number

A hexagonal fuzzy number $\tilde{A}_h$ is said to be a unit-equivalent hexagonal fuzzy number if $\breve{R}(\tilde{A}_h) = 1$. It is denoted by $\tilde{1}$.

Definition 2.12. Inverse Hexagonal fuzzy number

If $\tilde{a}_h$ is hexagonal fuzzy number and $\tilde{a}_h \neq \tilde{0}$ then we define ${\tilde{a}_h}^{-1} = \frac{\tilde{1}}{\tilde{a}_h}$.

3.HexagonalFuzzy GameMatrix

In this section, we give some basic definitions of fuzzy matrix game problem. These concepts form the basic building blocks of game theory.

3.1Two Person Zero Sum Games

A game of two persons in which gains of one player are losses of other is called a two person zero sum game, i.e., in two person zero sum the algebraic sum of gains to both players after a play is bounded to be zero.

3.2 Pay-off Matrix

The table showing how payments should be made at the end of the game is called a pay-off matrix. If the player A has m strategies available to him, then the pay-off for various strategies is represented by m×n pay-off matrix whose entries are hexagonal fuzzy numbers as

$$\hat{A} = \begin{pmatrix} \tilde{a}_{h11} & \cdots & \tilde{a}_{h1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{hm1} & \cdots & \tilde{a}_{hmn} \end{pmatrix}$$

Here it is assumed that when player A chooses the strategy A_i and player B selects strategy B_i.

3.3 Pure Strategy

Pure strategy is a decision making rule in which one particular course of action is selected. For fuzzy games the Max-Min principle is described by Nishizaki. The course of the fuzzy game is determined by the desire of A to maximize his gain and that of restrict his loss to a minimum.

Now for Hexagonal fuzzy matrix game,

$$\bar{v}_h = max - min = \bigvee_i \{\bigwedge_j (\tilde{a}_{hij})\}$$

$$\underline{v}_h = min - max = \bigwedge_j \{\bigvee_i (\tilde{a}_{hij})\}$$

Where $\tilde{a}_{hij} = \left(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5}, a_{ij6}\right)$be hexagonal fuzzy number (HFN).

Based on HFN order, for such games, we define the concept of max-min equilibrium strategies.

Definition 3.1

The concept of saddle point in classical form is introduced by Newmann. The (*k, r*)thposition of the pay-off fuzzy matrix will be called a fuzzy saddle point if and only if,

$\tilde{V}_{hkr} = \vee_i\{\wedge_j(\tilde{a}_{hij})\} = \wedge_j\{\vee_i(\tilde{a}_{hij})\}$

where$\tilde{V}_{hkr}$be the value of the fuzzy game and $\tilde{a}_{hij}$be hexagonal fuzzy number.

(i,e)., If in a fuzzy game the maximin value is equal to the minimax value, then the point is called fuzzy saddle point and fuzzy equilibrium point and the corresponding strategies of the fuzzy saddle point are called optimal strategies. The pay-off at fuzzy saddle point is called value of fuzzy matrix game whose elements are hexagonal fuzzy number.

Example 3.2.

Consider the matrix,

$$\hat{A} = \begin{pmatrix} (0,2,4,6,8,10) & (-1,0,1,2,4,6) \\ (-1,0,1,3,6,9) & (-2,-1,0,1,2,6) \end{pmatrix}$$

Then, for$\hat{A}$ *is*

Row min=*(-1,0,1,2,4,6),(-2,-1,0,1,2,6)*

Column max=*(0,2,4,6,8,10),(-1,0,1,2,4,6)*

Now,

$\bar{v}_h = \vee_i\{\wedge_j(\tilde{a}_{hij})\}$ = (-1,0,1,2,4,6).

$\underline{v}_h = \wedge_j\{\vee_i(\tilde{a}_{hij})\}$ = (-1,0,1,2,4,6).

∴ The value of fuzzy game $\tilde{V}_{hkr} = (-1,0,1,2,4,6)$.

3.4 MixedStrategy

whenmax-min≠min-max,thenpurestrategyfails. Thereforeeachplayerwithcertain probabilistic fixation. This type of strategy

(i,e) $\vee_i\{\wedge_j(\tilde{a}_{hij})\} \neq \wedge_j\{\vee_i(\tilde{a}_{hij})\}$

3.5 Game Without SaddlePoint

In the case of fuzzy game problem with no saddle point. We consider am×n fuzzy game value in various methods like,

(i) Solution of all 2×2 fuzzy matrixgame
(ii) Matrixoddmentmethodforn×ngames
(iii) Dominanceproperty

4. Solution of all 2×2 Fuzzy MatrixGame

The simplest case is a 2×2 fuzzy game with no saddle point. We discuss the first method of, solution of all 2× 2 fuzzy matrix game and find the value of game. General rules for finding the fuzzy game using hexagonal fuzzy number in following manner.

4.1 Rules For Solution of all 2×2 Fuzzy Matrix Game

Consider the general 2×2 fuzzy matrix game using hexagonal fuzzy number

$$\hat{A} = \begin{pmatrix} \tilde{a}_{h11} & \tilde{a}_{h12} \\ \tilde{a}_{h21} & \tilde{a}_{h22} \end{pmatrix}$$

To solve this game we proceed as follows

(i) Test for a fuzzy saddle point.

(ii) If there is no fuzzy saddle point, solve by finding equalizingstrategies.

The optimum mixed strategies for Player A=$(\tilde{p}_{h1}, \tilde{p}_{h2})$ and for Player B =$(\tilde{q}_{h1}, \tilde{q}_{h2})$ where,

$$\tilde{p}_{h1} = \frac{\tilde{a}_{h22} - \tilde{a}_{h12}}{(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21})} \; ; \; \tilde{p}_{h2} = 1 - \tilde{p}_{h1}$$

$$\tilde{q}_{h1} = \frac{\tilde{a}_{h22} - \tilde{a}_{h21}}{(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21})} \; ; \; \tilde{q}_{h2} = 1 - \tilde{q}_{h1}$$

The value of fuzzy game,

$$\tilde{V}_h = \frac{\tilde{a}_{h11}.\tilde{a}_{h22} - \tilde{a}_{h12}.\tilde{a}_{h21}}{(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21})}$$

Example 4.1.

Consider the following fuzzy game problem with pay-off matrix as hexagonal fuzzy number. Let us to solve it by 2×2 fuzzy game method.

$$\hat{A} = \begin{pmatrix} (-1,0,1,2,4,6) & (-1,0,1,8,10,12) \\ (-1,1,4,5,6,9) & (-2,-1,0,1,2,6) \end{pmatrix}$$

Then,for$\hat{A}$is

Rowmin= (-1,0,1,2,4,6),(-2,-1,0,1,2,6)

Column max= (-1,1,4,5,6,9),(-1,0,1,8,10,12)

$\bar{v}_h = \vee_i\{\wedge_j(\tilde{a}_{hij})\} = (-1,0,1,2,4,6)$

$$\underline{v}_h = \bigwedge_j \{\bigvee_i (\tilde{a}_{hij})\} = (-1,1,4,5,6,9)$$

(i,e) $\vee_i\{\wedge_j(\tilde{a}_{hij})\} \neq \wedge_j\{\vee_i(\tilde{a}_{hij})\}$

∴ There is no fuzzy saddle point. To find the optimum mixed strategy by calculating the above method.

Strategy of A=$(\tilde{p}_{h1}, \tilde{p}_{h2})$

Strategy of B=$(\tilde{q}_{h1}, \tilde{q}_{h2})$

Now,

$$(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21}) = (-24,-17,-12,2,5,14)$$

$$\tilde{p}_{h1} = \frac{\tilde{a}_{h22} - \tilde{a}_{h12}}{(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21})}$$

$$\tilde{p}_{h1} = \frac{(-2,-1,0,1,2,6) - (-1,0,1,8,10,12)}{(-24,-17,-12,2,5,14)}$$

$$\tilde{p}_{h1} = \left(\frac{-7}{6}, \frac{-1}{3}, 0, \frac{4}{3}, \frac{11}{6}, \frac{7}{3}\right)$$

$$\tilde{p}_{h2} = 1 - \tilde{p}_{h1}$$

$$\tilde{p}_{h2} = 1 - \left(\frac{-7}{6}, \frac{-1}{6}, 0, \frac{4}{6}, \frac{11}{6}, \frac{7}{6}\right)$$

$$\tilde{p}_{h2} = \left(\frac{-4}{3}, \frac{-5}{6}, \frac{-1}{3}, 1, \frac{4}{3}, \frac{13}{6}\right).$$

Now,

$$\tilde{q}_{h1} = \frac{\tilde{a}_{h22} - \tilde{a}_{h21}}{(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21})}$$

$$\tilde{q}_{h1} = \frac{(-2,-1,0,1,2,6) - (-1,1,4,5,6,9)}{(-24,-17,-12,2,5,14)}$$

$$\tilde{q}_{h1} = \left(\frac{-7}{6}, \frac{-1}{6}, \frac{3}{6}, \frac{5}{6}, \frac{7}{6}, \frac{11}{6}\right).$$

$$\tilde{q}_{h2} = 1 - \tilde{q}_{h1}$$

$$\tilde{q}_{h2} = 1 - \left(\frac{-7}{6}, \frac{-1}{6}, \frac{3}{6}, \frac{5}{6}, \frac{7}{6}, \frac{11}{6}\right)$$

$$\tilde{q}_{h2} = \left(\frac{-5}{6}, \frac{-1}{6}, \frac{1}{6}, \frac{3}{6}, \frac{7}{6}, \frac{13}{6}\right).$$

Strategy of A=$\left(\left(\frac{-7}{6}, \frac{-1}{3}, 0, \frac{4}{3}, \frac{11}{6}, \frac{7}{3}\right), \left(\frac{-4}{3}, \frac{-5}{6}, \frac{-1}{3}, 1, \frac{4}{3}, \frac{13}{6}\right)\right)$

Strategy of B=$\left(\left(\frac{-7}{6}, \frac{-1}{6}, \frac{3}{6}, \frac{5}{6}, \frac{7}{6}, \frac{11}{6}\right), \left(\frac{-5}{6}, \frac{-1}{6}, \frac{1}{6}, \frac{3}{6}, \frac{7}{6}, \frac{13}{6}\right)\right)$

$$\tilde{V}_h = \frac{\tilde{a}_{h11}.\tilde{a}_{h22} - \tilde{a}_{h12}.\tilde{a}_{h21}}{(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21})}$$

$$= \frac{(-1,0,1,2,4,6).(-2,-1,0,1,2,6) - (-1,0,1,8,10,12).(-1,1,4,5,6,9)}{(-24,-17,-12,2,5,14)}$$

$$\tilde{V}_h = \left(\frac{-5}{3}, \frac{-2}{3}, \frac{1}{3}, \frac{31}{6}, \frac{20}{3}, \frac{49}{6}\right).$$

∴ The value of the fuzzy game $\tilde{V}_h = \left(\frac{-5}{3}, \frac{-2}{3}, \frac{1}{3}, \frac{31}{6}, \frac{20}{3}, \frac{49}{6}\right)$.

5.Matrix Oddment Method For n×n FuzzyGames

In this section, we discuss the matrix oddment method for n×n fuzzy games using the hexagonal fuzzy number.

5.1 General Rules For Matrix Oddment Method For n×n FuzzyGames

Step 1: Let $\hat{A} = (\tilde{a}_{hij})$be $n \times n$ pay-off matrix. Obtain a new matrix $\hat{C}$, whose first column is obtained from $\hat{A}$ by subtracting 2nd column from 1st, second column is obtained by subtracting $\hat{A}'s$ 3rd column from 2nd and so on till the last column of $\hat{A}$ is taken care of. Thus $\hat{C}$ is a $n \times (n-1)$ matrix.

Step 2: Obtain a new matrix $\hat{R}$, from $\hat{A}$,by subtracting its successive row from the proceeding ones, in exactly the same manner as was done for columns in step 1. Thus $\hat{R}$ is a $(n-1) \times n$ matrix.

Step 3: Determine the magnitude of oddments corresponding to ith row of $\hat{A}$ is defined as the determinant $|\hat{C}_i|$ where $\hat{C}_i$ is obtain from $\hat{C}$ by deleting ith row. Similarly oddment corresponding jth column of $\hat{A}$=$|\hat{R}_j|$, defined as determinant where $\hat{R}_j$ is obtained from $\hat{R}$ by deleting its jth column.

Step 4: Write the magnitude of oddments (after ignoring negative sign, if any) against their respective rows and columns.

Step 5: Check whether the sum of the row oddments (S.R.O) is equal to the sum of the column oddments (S.C.O) using in the ranking function of Hexagonal fuzzy number. If choose any one of the value of (S.R.O) or (S.C.O). If so, the oddments expressed as fraction of the grand total yields the optimum strategies. If not, the method fails.

Step 6: Calculate the expressed value of the fuzzy game corresponding to the optimum mixed strategy determined above for the row player (against any move of the column player)

Example:5.1

Consider the following game problem with pay-off matrix as hexagonal fuzzy numbers. Solve it by oddment method.

$$\hat{A} = \begin{pmatrix} (-6,-2,-1,0,1,2) & (-1,0,1,2,4,6) & (-2,-1,0,1,2,6) \\ (-2,-1,0,1,2,6) & (-6,-4,-2,-1,0,1) & (-1,0,1,2,4,6) \\ (-1,0,1,3,6,9) & (1,2,3,5,6,7) & (-9,-6,-3,-1,0,1) \end{pmatrix}$$

Then, for $\hat{A}$ is

Row min= $((-6,-2,-1,0,1,2),(-6,-4,-2,-1,0,1),(-9,-6,-3,-1,0,1))$

Column max=$((-1,0,1,3,6,9),(1,2,3,5,6,7),(-1,0,1,2,4,6))$

$$\bar{v}_h = \bigvee_i \{\bigwedge_j (\tilde{a}_{hij})\} = (-6,-2,-1,0,1,2)$$

$$\underline{v}_h = \bigwedge_j \{\bigvee_i (\tilde{a}_{hij})\} = (-1,0,1,2,4,6)$$

(i,e) $\vee_i\{\wedge_j(\tilde{a}_{hij})\} \neq \wedge_j\{\vee_i(\tilde{a}_{hij})\}$, It has no saddle point.

To find the optimum strategy by calculating the above method.

Step 1:

A new matrix $\hat{C}$, due to the successive subtraction of columns for a matrix $\hat{A}$ and we get,

$$\hat{C} = \begin{pmatrix} (-3,-1,1,3,6,12) & (-8,-5,-2,0,2,7) \\ (-12,-6,-3,-1,1,3) & (-2,0,2,4,8,12) \\ (-8,-4,0,4,6,8) & (-16,-12,-8,-4,-2,0) \end{pmatrix}$$

Step 2:

A new matrix $\hat{R}$, by the same procedure in step 1 and we get,

$$\hat{R} = \begin{pmatrix} (-4,-2,0,2,4,12) & (-12,-8,-4,-2,0,2) & (-7,-2,0,2,5,8) \\ (-7,-2,0,3,7,11) & (0,2,4,7,10,13) & (-15,-10,-5,-2,0,2) \end{pmatrix}$$

Step 3:

To determine $|\hat{C}_i|$ values,

$|\hat{C}_1| = (-53,-31,-9,21,58,116)$.

$|\hat{C}_2| = -(-29,-13,3,21,46,92)$.

$$|\hat{C}_3| = (-24,-10,1,11,25,51).$$

Step 4:

To determine $|\hat{R}_i|$ values,

$|\hat{R}_1| = (-23,-10,3,16,38,60)$.

$|\hat{R}_2| = -(-27,-12,0,13,27,71)$.

$$|\hat{R}_3| = (-52,-20,0,24,52,116).$$

Step 5:

To check the sum of the row oddments (S.R.O) is equal to sum of column oddments (S.C.O) by using ranking function as follows.

$$\begin{pmatrix} (-6,-2,-1,0,1,2) & (-1,0,1,2,4,6) & (-2,-1,0,1,2,6) \\ (-2,-1,0,1,2,6) & (-6,-4,-2,-1,0,1) & (-1,0,1,2,4,6) \\ (-1,0,1,3,6,9) & (1,2,3,5,6,7) & (-9,-6,-3,-1,0,1) \end{pmatrix}$$

Row

Oddments=$((-53,-31,-9,21,58,116),(-29,-13,3,21,46,92),(-24,-10,1,11,25,51))$.

Column

Oddments=$\big((-23,-10,3,16,38,60),(-27,-12,0,13,27,71),(-52,-20,0,24,52,116)\big)$.

Sum of Row oddments (S.R.O)=(-106,-54,-5,53,129,259)

Sum of Column oddments (S.C.O)=(-102,-42,3,53,,117,247)

To find the (S.R.O) and (S.C.O) are equal by using ranking function,

(i,e).,$\breve{R}$(S.R.O)=$\breve{R}$ (S.C.O)=46.

Here, we choose any of the values of (S.R.O) or (S.C.O) to find the values of fuzzy game.

Step 6:

The optimum strategies, to choose the value of S.R.O then,

Player A=$\left(\frac{(-53,-31,-9,21,58,116)}{(-106,-54,-5,53,129,259)},\frac{(-29,-13,3,21,46,92)}{(-106,-54,-5,53,129,259)},\frac{(-24,-10,1,11,25,51)}{(-106,-54,-5,53,129,259)}\right)$.

Player B=$\left(\frac{(-23,-10,3,16,38,60)}{(-102,-42,3,53,,117,247)},\frac{(-27,-12,0,13,27,71)}{(-102,-42,3,53,,117,247)},\frac{(-52,-20,0,24,52,116)}{(-102,-42,3,53,,117,247)}\right)$.

$$\tilde{V}_h = \frac{(-53,-31,-9,21,58,116)}{(-106,-54,-5,53,129,259)}(-6,-2,-1,0,1,2) + \frac{(-29,-13,3,21,46,92)}{(-106,-54,-5,53,129,259)}(-2,-1,0,1,2,6) + \frac{(-24,-10,1,11,25,51)}{(-106,-54,-5,53,129,259)}(-1,0,1,3,6,9)$$

$= \left(\frac{-217}{46},\frac{-101}{46},\frac{-15}{46},\frac{63}{46},\frac{71}{46},\frac{149}{46}\right)$.

$\therefore$ The value of the fuzzy game $\tilde{V}_h = \left(\frac{-217}{46},\frac{-101}{46},\frac{-15}{46},\frac{63}{46},\frac{71}{46},\frac{149}{46}\right)$.

Remark 5.2

If we choose the value of S.C.O and proceed to follow as step 6 to find the value of fuzzy game.If the value of the fuzzy game is same in both the S.R.O and S.C.O to defuzzify the number into crisp number.

6. Dominance Principle

In this section, we introduce the method of dominance principle using Hexagonal fuzzy number to find the value of fuzzy game obtain by the rules as follows and the example to justify.

6.1 General Rules For Dominance

Step 1: If all the elements of the i^{th}row are lessthan or equal to the corresponding elements of any other row say r^{th}row then i^{th}row is dominated by the r^{th}row.

Step 2: If all the elements of the j^{th}column are greaterthan or equal to the corresponding elements of any other column say k^{th}column then j^{th}column is dominated by the k^{th}column.

Step 3: Dominated columns or rows may be deleted to reduce the size of the pay-off matrix as optimal strategies will remain unaffected.

Step 4: A given strategy can also said to be dominated if it is inferior to an average of two or more than pure strategies. More generally if some convex linear combination of some rows dominates the i^{th}row, then i^{th}row will deleted. Similar arguments follow and columns.

Step 5: Thus the given matrix can be reduced to a simple matrix for which the fuzzy game value can be evaluated.

Example 6.1.

Consider the following fuzzy game problem with pay-off matrix as hexagonal fuzzy number. Let us solve it by dominance method.

$$\hat{A} = \begin{pmatrix} (-1,0,1,3,6,9) & (-1,0,1,2,4,6) & (-1,0,1,6,8,10) & (-1,0,1,8,10,12) \\ (-1,0,1,10,12,14) & (-1,0,1,14,16,18) & (-1,0,1,8,10,12) & (-1,0,1,12,14,16) \\ (-1,0,1,2,4,6) & (-1,0,1,3,6,9) & (-1,0,1,14,16,18) & (-1,0,1,16,18,20) \end{pmatrix}$$

Then, for $\hat{A}$ is

Rowmin= $((-1,0,1,2,4,6),(-1,0,1,8,10,12),(-1,0,1,2,4,6))$

Column max=$((-1,0,1,10,12,14),(-1,0,1,14,16,18),(-1,0,1,14,16,18),(-1,0,1,16,18,20))$

$$\bar{v}_h = \bigvee_i \{\bigwedge_j (\tilde{a}_{hij})\} = (-6,-2,-1,0,1,2)$$

$$\underline{v}_h = \bigwedge_j \{\bigvee_i (\tilde{a}_{hij})\} = (-1,0,1,2,4,6)$$

(i,e) $\bar{v}_h \neq \underline{v}_h$, It has no saddle point.

To find the optimum strategy by calculating the above method as dominance principle.

Here, Row I is dominated by Row II, so we omit Row I and we get,

$$\begin{pmatrix} (-1,0,1,10,12,14) & (-1,0,1,14,16,18) & (-1,0,1,8,10,12) & (-1,0,1,12,14,16) \\ (-1,0,1,2,4,6) & (-1,0,1,3,6,9) & (-1,0,1,14,16,18) & (-1,0,1,16,18,20) \end{pmatrix}$$

Now, column II is dominated by column I , so we omit column II and we get,

$$\begin{pmatrix} (-1,0,1,10,12,14,) & (-1,0,1,8,10,12) & (-1,0,1,12,14,16) \\ (-1,0,1,2,4,6) & (-1,0,1,14,16,18) & (-1,0,1,16,18,20) \end{pmatrix}$$

Column IV is dominated by column III, so we omit column IV,

$$\begin{pmatrix} (-1,0,1,10,12,14) & (-1,0,1,8,10,12) \\ (-1,0,1,2,4,6) & (-1,0,1,14,16,18) \end{pmatrix}$$

Rowmin= $((-1,0,1,8,10,12),(-1,0,1,2,4,6))$

Column max=$((-1,0,1,10,12,14),(-1,0,1,14,16,18))$

$$\bar{v}_h = \bigvee_i \{\bigwedge_j (\tilde{a}_{hij})\} = (-1,0,1,8,10,12)$$

$$\underline{v}_h = \bigwedge_j \{\bigvee_i (\tilde{a}_{hij})\} = (-1,0,1,10,12,14)$$

(i,e) $\bar{v}_h \neq \underline{v}_h$, It has no saddle point.

To find the optimum strategy and value of fuzzy game,

Here $\tilde{a}_{h11} = (-1,0,1,10,12,14)$; $\tilde{a}_{h12} = (-1,0,1,8,10,12)$

$\tilde{a}_{h21} = (-1,0,1,2,4,6)$; $\tilde{a}_{h22} = (-1,0,1,14,16,18)$

Now,

$$(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21}) = (-20,-14,-8,22,28,34)$$

$$\tilde{p}_{h1} = \frac{\tilde{a}_{h22} - \tilde{a}_{h12}}{(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21})}$$

$$\tilde{p}_{h1} = \frac{(-1,0,1,14,16,18) - (-1,0,1,8,10,12)}{(-20,-14,-8,22,28,34)}$$

$$\tilde{p}_{h1} = \left(\frac{-13}{7},\frac{-10}{7},\frac{-7}{7},\frac{13}{7},\frac{16}{7},\frac{19}{7}\right)$$

$$\tilde{p}_{h2} = 1 - \tilde{p}_{h1}$$

$$\tilde{p}_{h2} = 1 - \left(\frac{-13}{7},\frac{-10}{7},\frac{-7}{7},\frac{13}{7},\frac{16}{7},\frac{19}{7}\right)$$

$$\tilde{p}_{h2} = \left(\frac{-12}{7},\frac{-9}{7},\frac{-6}{7},\frac{14}{7},\frac{17}{7},\frac{20}{7}\right).$$

Now,

$$\tilde{q}_{h1} = \frac{\tilde{a}_{h22} - \tilde{a}_{h21}}{(\tilde{a}_{h11} + \tilde{a}_{h22}) - (\tilde{a}_{h12} + \tilde{a}_{h21})}$$

$$\tilde{q}_{h1} = \frac{(-1,0,1,14,16,18) - (-1,0,1,2,4,6)}{(-20,-14,-8,22,28,34)}$$

$\tilde{q}_{h1} = \left(\frac{-7}{7},\frac{-4}{7},\frac{-1}{7},\frac{13}{7},\frac{16}{7},\frac{19}{7}\right)$.

$$\tilde{q}_{h2} = 1 - \tilde{q}_{h1}$$

$$\tilde{q}_{h2} = 1 - \left(\frac{-7}{7},\frac{-4}{7},\frac{-1}{7},\frac{13}{7},\frac{16}{7},\frac{19}{7}\right)$$

$\tilde{q}_{h2} = \left(\frac{-12}{7},\frac{-9}{7},\frac{-6}{7},\frac{8}{7},\frac{11}{7},\frac{14}{7}\right)$.

Strategy of A=$\left(0,\left(\frac{-13}{7},\frac{-10}{7},\frac{-7}{7},\frac{13}{7},\frac{16}{7},\frac{19}{7}\right),\left(\frac{-12}{7},\frac{-9}{7},\frac{-6}{7},\frac{14}{7},\frac{17}{7},\frac{20}{7}\right)\right)$

Strategy of B=$\left(\left(\frac{-7}{7},\frac{-4}{7},\frac{-1}{7},\frac{13}{7},\frac{16}{7},\frac{19}{7}\right),0,\left(\frac{-12}{7},\frac{-9}{7},\frac{-6}{7},\frac{8}{7},\frac{11}{7},\frac{14}{7}\right),0\right)$

$$\tilde{V}_h = \frac{\tilde{a}_{h11}.\tilde{a}_{h22} - \tilde{a}_{h12}.\tilde{a}_{h21}}{(\tilde{a}_{h11}+\tilde{a}_{h22}) - (\tilde{a}_{h12}+\tilde{a}_{h21})}$$

$$= \frac{(-1,0,1,10,12,14).(-1,0,1,14,16,18) - (-1,0,1,8,10,12).(-1,0,1,2,4,6)}{(-20,-14,-8,22,28,34)}$$

$\tilde{V}_h = \left(\frac{-32}{7},\frac{-20}{7},\frac{-8}{7},\frac{78}{7},\frac{96}{7},\frac{114}{7}\right)$.

$\therefore$ The value of the fuzzy game $\tilde{V}_h = \left(\frac{-32}{7},\frac{-20}{7},\frac{-8}{7},\frac{78}{7},\frac{96}{7},\frac{114}{7}\right)$.

7.CONCLUSION

In this paper, we have used Hexagonal fuzzy number as elements of pay-off matrix. Here pay-off is consider as imprecise number instead of crisp numbers which takes care of the uncertaintyandvaguenessinherentinsuchproblem.Wediscusssolutionoffuzzygamesin various methods to justify with proper examples and obtained the optimum solutions by the proposed rules. This work can be extended to some additional methods like graphical method and linear programming method to find the optimum values in game problems under the domain of fuzzy environment.

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