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RESEARCH ARTICLE

FIXED POINT THEOREMS FOR CONTRACTION TYPE MAPPINGS IN MODULAR METRIC SPACES

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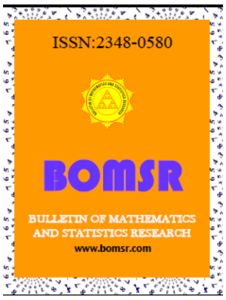

ABSTRACT

In this paper, some existence theorems of fixed points for contraction type mappings in modular metric spaces are proved. The result generalizes recent results of Mongkolkeha et al. [10, 11] and Chistyakov [4].

AMS: 47H09; 47H10.

Keywords: Modular metric spaces, common fixed point, contraction mappings

1. INTRODUCTION

The metric fixed point theory and its variations are far reaching developments of Banach's contraction principle, where metric conditions on the underlying space and maps under consideration play a fundamental role. Let (X,d) be a metric space. A mapping $T: X \to X$ is a contraction if $d(Tx, Ty) \leq kd(x, y)$, for all $x, y \in X$, where $0 \leq k < 1$. The Banach's contraction mapping principle appeared in explicit form in Banach's thesis in 1922 [1]. Since its simplicity and usefulness, it has become a very popular tool in solving existence problems in many branches of mathematical analysis. Banach contraction principle has been extended in many different directions, see [5, 14]. The notion of modular spaces, as a generalization of metric spaces, was introduced by Nakano [13] and was intensively developed by Koshi and Shimogaki [6], Yamamuro [15] and others. We begin with a certain motivation of the definition of a (metric) modular, introduced axiomatically in [2, 3] .The main idea behind this new concept is the physical interpretation of the modular. Informally speaking whereas a metric on a set represent finite nonnegative distances between two points of the set, a modular on a set attributes a non-negative (possibly, infinite valued) 'field of (generalized) velocities': to each 'time' $\lambda > 0$ (the absolute value of), an average velocity $\omega_\lambda(x, y)$ is associated in such way that in order to cover the 'distance' between points $x, y \in X$, it takes time λ to move from X to Y with velocity $\omega_\lambda(x, y)$. A lot of mathematicians are interested fixed points of modular spaces. Further the most complete development of these theories are due to Luxemburg [7], Mosielak, and Orlicz [8], Musielak and Orlicz [9], Mazur [12], Turpin [14] and there collaborators.

The notion of a (metric) modular on an arbitrary set and the corresponding modular space, more general than a metric space, were introduced and studied recently by Chistyakov [2, 3, 4]. In 2008, Chistyakov [2] introduced the notion of modular metric spaces generated by F-modular and developed the theory of this space. In 2010 Chistyakov [3] defined the notion of modular on an arbitrary set and develop the theory of metric spaces generated by modular such that called the modular metric spaces.

2. Basic Definitions and Preliminaries. We will start with a brief recollection of basic concepts and facts in modular spaces and modular metric spaces (see [2, 3, 4]).

Definition 2.1. Let X be a vector space over R (or C). A functional $\rho : X \to [0, \infty]$ is called a modular if for arbitrary x and y, elements of X satisfying the following three conditions:

(A.1) $\rho(x)=0$ *if and only if* $x = 0$.

(A.2) $\rho(\alpha x)= \rho(x)$ *for all scalar* α *with* $|\alpha|=1$;

(A.3) $\rho(\alpha x+\beta y) \le \rho(x) + \rho(y)$, *whenever* $\alpha,\beta \ge 0$, $\alpha+\beta=1$.

If we replace (A.3) by

(A.4) $\rho(\alpha x+\beta y)\le \alpha^s \rho(x)+\beta^s \rho(y)$, *for* $\alpha,\beta \ge 0, \alpha^s+\beta^s=1$ *with an* $s\in (0,1]$,

then the modular ρ is called s-convex modular, and if s = 1, ρ is called a convex modular.

If ρ is modular in X, then the set defined by

$$X_\rho=\{x \in X: \rho(\lambda x)\to 0 \text{ as } \lambda\to 0^+\} \tag{2.1}$$

is called a modular space. X_ρ is a vector subspace of X it can be equipped with an F - norm defined by setting

$$\|x\|_\rho= \inf \{\lambda > 0 : \rho(\tfrac{x}{\lambda}) \le \lambda\},\ x\in X_\rho. \tag{2.2}$$

In addition, if ρ is convex, then the modular space X_ρ coincides with

$$X^*_\rho=\{x \in X: \exists\lambda=\lambda(x) > 0 \text{ such that } \rho(\lambda x)< \infty\} \tag{2.3}$$

and the functional $\|x\|^*_\rho = \inf \{\lambda >0 : \rho\left(\frac{x}{\lambda}\right) \le 1\}$ is an ordinary norm on X^*_ρ which is equivalence to $\|x\|_\rho$ (see [8]).

Let X be a non-empty set, $\lambda \in(0,\infty)$and due to the disparity of the arguments, function $\omega:(0, \infty) \times X \times X \to[0, \infty]$ will be written as $\omega_\lambda(x,y) = \omega(\lambda,x,y)$ for all $\lambda > 0$ and $x,y \in X$.

Definition 2.2. Let X be a non-empty set. A function $\omega : (0, \infty) \times X \times X \to[0, \infty]$ is said to be a metric modular on X if it satisfies the following three axioms:

(i) given $x,y \in X$, $\omega_\lambda(x,y)=0$ *for all* $\lambda>0$ *if and only if* $x=y$;

(ii) $\omega_\lambda(x,y) = \omega_\lambda(y,x)$ *for all* $\lambda>0$ and $x,y \in X$;

(iii) $\omega_{\lambda+\mu}(x,y) \le \omega_\lambda(x,z) + \omega_\mu(z,y)$ *for all* $\lambda, \mu > 0$ and $x,y,z \in X$.

If instead of (i), we have only the condition

(i') $\omega_\lambda(x,x)=0$ *for all* $\lambda >0$ and $x \in X$, then ω is said to be a (metric) pseudo modular on X and if ω satisfies (i') and

(is) given x, y $\in$ X, if there exists a number $\lambda>0$, possibly depending on x and y, such that $\omega_\lambda(x,y)=0$, then x = y, with this condition ω is called a strict modular on X.

A modular (pseudo modular, strict modular) ω on X is said to be convex if, instead of (iii), we replace the following condition:

(iv) for all $\lambda > 0$, $\mu > 0$ and x, y, z ∈ X it satisfies the inequality

$$\omega_{\lambda+\mu}(x,y) = \frac{\lambda}{\lambda+\mu}\omega_\lambda(x,z) + \frac{\mu}{\lambda+\mu}\omega_\mu(z,y) \text{ for all } \lambda,\mu > 0 \text{and } x,y,z \in X$$

Clearly, if ω is a strict modular, then ω is a modular, which in turn implies ω is a pseudo modular on X, and similar implications hold for convex ω.

The essential property of a (pseudo) modular ω on a set X is a following given $x, y \in X$, the function $0 < \lambda \to \omega_\lambda(x,y) \in [0, \infty]$ is non increasing on $(0, \infty)$. In fact, if $0 < \mu < \lambda$, then (iii), (i') and (ii) imply

$$\omega_\lambda(x,y) \le \omega_{\lambda-\mu}(x,x)+\omega_\mu(x,y)=\omega_\mu(x,y) \tag{2.4}$$

It follows that at each point $\lambda > 0$ the right limit $\omega_{\lambda+0}(x,y) := \lim_{\varepsilon\to+0}\omega_{\lambda+\varepsilon}(x,y)$and the left limit $\omega_{\lambda-0}(x,y) := \lim_{\varepsilon\to+0}\omega_{\lambda-\varepsilon}(x,y)$ exist in $[0, \infty]$ and the following two inequalities hold:

$$\omega_{\lambda+0}(x,y) \le \omega_\lambda(x,y) \le \omega_{\lambda-0}(x,y) \tag{2.5}$$

From [2,3], we know that, if $x_0 \in X$, the set

$$X_\omega = \{x \in X : \lim_{\lambda\to\infty}\omega_\lambda(x,x_0) = 0\}$$

is a metric space, called a modular space, whose metric is given by

$$d_\omega^0(x,y) = \inf\{\lambda > 0 : \omega_\lambda(x,y) \le \lambda\} \, for \, all \, x,y \in X_\omega.$$

Moreover, if ω is convex, the modular set $X\omega$ is equal to

$$X_\omega^* = \{x \in X : \exists\, \lambda = \lambda(x) > 0 \text{ such that } \omega_\lambda(x,x_0) < \infty\}.$$

and metrizable by $d_\omega^*(x,y) = \inf\{\lambda > 0: \omega_\lambda(x,y) \le 1\}\, for\, all\, x,y \in X_\omega^*$.

We know that if X is a real linear space, $\rho: X \to [0, \infty]$ and

$$\omega_\lambda(x,y) = \rho\left(\frac{x-y}{\lambda}\right) for \, all \, \lambda > 0 \, and \, x,y \in X, \tag{2.6}$$

then ρ is modular (convex modular) on X in the sense of (A.1) - (A.4) if and only if ω is metric modular (convex metric modular, respectively) on X. On the other hand, if ω satisfy the following two conditions:

(i) $\omega_\lambda(\mu x,0)=\omega_{\lambda/\mu}(x,0)\, for\, all\, \lambda,\mu > 0\, and\, x \in X$,

(ii) $\omega_\lambda(x+z, y+z)= \omega_\lambda(x,y) for\, all\, \lambda > 0\, and\, x, y, z \in X$, if we set

$\rho(x) = \omega_1(x,0)$ with (2.6) holds, where $x \in X$, then

(a) $X_\rho=X_\omega$ is a linear subspace of X and the functional $\|x\|_\rho=d_\omega^0(x,0), x \in X_\rho$, is an F-norm on X_ρ;

(b) If ω is convex, $X_\rho^* \equiv X_\omega^*(0) = X_\rho$is a linear subspace of X and the functional $\|x\|_\rho=d_\omega^*(x,0)$, $x \in X_\rho^*$, is an norm on X_ρ^*.

Similar assertions hold if replace the word modular by pseudo modular. If ω is metric modular in X, we called the set X_ω is modular metric space.

By the idea of property in metric spaces and modular spaces, we defined the following:

Definition 2.3. [10] Let X_ω be a modular metric space.

(1) The sequence $(x_n)_{n\in N}$ in X_ω is said to be convergent to $x \in X_\omega$ if

$\omega_\lambda(x_n,x)\to 0\, as\, n\to\infty\, for\, all\, \lambda > o$.

(2) The sequence $(x_n)_{n\in N}$in X_ω is said to be Cauchy if

$\omega_\lambda(x_m,x_n)\to 0\, as\, m,n\to\infty\, for\, all\, \lambda > o$.

(3) A subset C of X_ω is said to be closed if the limit of the convergent sequence of C always belong to C.

(4) A subset C of X_ω is said to be complete if any Cauchy sequence in C is a convergent sequence and its limit in C.

(5) A subset C of X_ω is said to be bounded if for all $\lambda>o$

$$\delta_\omega(C) = \sup\{\omega_\lambda(x,y); x,y \epsilon C\} < \infty.$$

Recently, Mongkolkeha et al.[10,11] has introduced some notions and established some fixed point results in modular metric spaces. In this paper, we study and prove the existence of fixed point theorems for contraction mappings in modular metric spaces and generalize the result of Mongkolkeha et al. [10, 11] and Chistyakov [4].

3. Main results

Definition 3.1. Let ω be a metric modular on X and X_ω be a modular metric space induced by ω and $T: X_\omega \to X_\omega$ be an arbitrary mapping. A mapping T is called a contraction if for each x,y ϵX_ω and for all $\lambda > o$ there exist $0 \le k < 1$ such that $\omega_\lambda (Tx,Ty) \le k\, \omega_\lambda (x,y)$.

Theorem 3.1. Let ω be a metric modular on X and X_ω be a modular metric space induced by ω. If X_ω is complete modular metric space and S,T : $X_\omega \to X_\omega$ be mappings satisfying

(i) T(X_ω) ⊂ S(X_ω), and the inequality

(ii) $\omega_\lambda (Tx, Ty) \le k\, \omega_\lambda(Sx,Sy)$,

for all $x,y \epsilon X_\omega$, where k ϵ [0,1). Suppose that there exist x ϵ X such that $\omega_\lambda (x,Sx)<\infty$ for all $\lambda > 0$. Then S and T have a unique fixed common point in X_ω. Moreover, for any $x \epsilon X_\omega$, iterative sequence $\{S^n x\}$ and $\{T^n x\}$ converges to x.

Proof. Let x_0 be an arbitrary point in X_ω and since T(X_ω) ⊂ S(X_ω) ∃ a point $x_1 \epsilon X_\omega$ such that we write $x_1=Tx_0=Sx_1$, and for $x_2 \epsilon X_\omega$, $x_2=Tx_1=Sx_2$, thus inductively we can define for all n ϵN, $x_n=Tx_{n-1}=Sx_n$.

Consider, $\omega_\lambda (x_n, x_{n+1}) = \omega_\lambda (Tx_{n-1},Tx_n) \le k\omega_\lambda(Sx_n,Sx_{n+1})$.

$$\begin{aligned} &\le k\omega_\lambda(Sx_{n+1}, Sx_n) \\ &\le k\omega_\lambda(x_{n-1}, x_n) \\ &\le k\omega_\lambda (Tx_{n-2}, Tx_{n-1}) \\ &\le k^2\omega_\lambda(Sx_{n-2}, Sx_{n-1}) \\ &\le k^2\omega_\lambda(x_{n-2}, x_{n-1}) \ \dots \ \le k^n\omega_\lambda(x_1, x_0) \end{aligned}$$

i.e. $\omega_\lambda (x_n, x_{n+1}) \le k^n\omega_\lambda(x_1, x_0)$ for all $\lambda > 0$ and for each n ϵN

Therefore, $\lim_{n\to\infty} \omega_\lambda (x_n, x_{n+1}) = 0$, for all $\lambda > 0$. So for each $\lambda > 0$, we have for all $\varepsilon > 0$ there exist

$n_0 \epsilon N$ such that $\omega_\lambda (x_n, x_{n+1}) < \in$ for all n ϵN with n ≥ n_0 . Without loss of generality, suppose m,nϵ N and $m > n$. Observe that, for $\frac{\lambda}{m-n} > 0$, there exists $n_{\frac{\lambda}{(m-n)}} \epsilon N$ such that

$$\omega_{\frac{\lambda}{m-n}} (x_n, x_{n+1}) < \frac{\varepsilon}{m-n} \text{ for all n} \ge n_{\frac{\lambda}{m-n}}.$$

Now, we have

$$\omega_\lambda (x_n, x_m) \le \omega_{\frac{\lambda}{m-n}}(x_n, x_{n+1}) + \omega_{\frac{\lambda}{m-n}}(x_{n+1}, x_{n+2}) + \dots + \omega_{\frac{\lambda}{m-n}}(x_{m-1}, x_m)$$

$$< \frac{\varepsilon}{m-n} + \frac{\varepsilon}{m-n} + \dots + \frac{\varepsilon}{m-n} = \varepsilon, \text{ for all m, n} \ge n_{\frac{\lambda}{(m-n)}}.$$

This implies $\{x_n\}$n ϵN is a ω − cauchy sequence. By the completeness of (x_ω), there exist a point x ∈ X_ω such that $x_n \to x$ as n→∞ and subsequently, $\omega_\lambda(Tx_n,x) \to 0$ and $\omega_\lambda(Sx_n,x) \to 0$ as $n\to\infty$ for all $\lambda > o$. By the notion of metric modular ω and by the inequality (ii), we get

$$\omega_\lambda (Tx, x) \leq \omega_{\frac{\lambda}{2}}(Tx , Tx_n) + \omega_{\frac{\lambda}{2}}(Tx_n, x)$$

$$\leq k[\omega_{\frac{\lambda}{2}}(Sx , Sx_n)] + k[\omega_{\frac{\lambda}{2}}(Tx_n, x)] \quad (3.1)$$

Taking limit as n→∞ in (3.1), we get

$$\omega_\lambda (Tx, x) \leq k[\omega_{\frac{\lambda}{2}}(Sx , x) + \omega_{\frac{\lambda}{2}}(x, x)],$$

since $\omega_\lambda (x, Sx) < \infty$, and by the strictness of ω, we have $\omega_\lambda (Tx,x)$=0 for all $\lambda > 0$ and thus $Tx = x$. Hence x is a fixed point of T.

Now we will show that x is a common fixed point S and T. Suppose on the contrary that $S_x \neq x$ or $T_x \neq x$ and put $x = x_n$ and $y= x$ *then by* (ii), we have

$\omega_\lambda (Tx_n, Tx) \leq k\omega_\lambda(Sx_n,Sx)$, taking limit $n \to \infty$ on both sides we get

$\omega_\lambda (x, Tx) \leq k\omega_\lambda(x, Sx)$

Since $\omega_\lambda (x, Sx)<\infty$, therefore by the strictness of ω, and $k \in [0,1/2)$, we get $\omega_\lambda(x, Tx)<\infty$, a contradiction. Hence $Tx = x = Sx$.

Uniqueness. Suppose z is another fixed point of S and T, we have by (ii)

$$\omega_\lambda(x,z)=\omega_\lambda(Tx , Tz)$$
$$\leq k\, \omega_\lambda(Sx, Sz)$$
$$\leq k\, \omega_\lambda (x , z)$$

for all $\lambda > 0$. Since $0 \leq k \leq 1$, and by the strictness of ω, we get $\omega_\lambda (x, z) = 0$, hence for all $\lambda > 0$ implies that $x = z$. Therefore, x is a unique common fixed point of S and T.

Theorem 3.2. Let ω be a metric modular on X, X_ω be a complete modular metric space induced by ω and S, T: $X_\omega \to X_\omega$ satisfying T($X\omega$) ⊂ S(X_ω), and

$$\omega_\lambda(Tx, Ty) \leq k[\omega_{2\lambda}(Tx, Sx) + \omega_{2\lambda}(Ty, Sy)] \quad (3.2)$$

For all x,y ϵX_ω and for all $\lambda > 0$ where k ∈[0,1/2), suppose that $for\ x,y \epsilon X_\omega$ such that $\omega_\lambda(Sx, Ty) < \infty$ for all $\lambda > 0$. Then S, T have a unique common fixed point in X_ω.

Moreover for any $x \epsilon X_\omega$ iterative sequence $\{S^n x\}$ and $\{T^n x\}$ converges to the fixed point.

Proof. Let x_0 be an arbitrary point in X_ω and since T(X_ω) ⊂ S(X_ω), there exists a point $x_1 \epsilon X_\omega$ such that $x_1=Tx_0=Sx_1$, $x_2=Tx_1=Sx_2=T^2x_0$ in general $x_n=Tx_{n-1}=Sx_n=T^n x_0$ for all $n \in N$.We have consider for x $=x_n$ *and* $y=x_{n-1}$, then by (3.2), we get

$$\omega_\lambda (x_{n+1}, x_n) = \omega_\lambda (Tx_n, Tx_{n-1})$$
$$\leq[\omega_{2\lambda} (Tx_n,Sx_n)+ \omega_{2\lambda} (Tx_{n-1},Sx_{n-1})]$$
$$\leq[\omega_{2\lambda} (x_{n+1},x_n)+ \omega_{2\lambda} (x_n,x_{n-1})]$$
$$\leq k\omega_{2\lambda} (x_{n+1},x_n)+k\omega_{2\lambda} (x_n,x_{n-1})$$

i.e. $(1-k)\, \omega_\lambda (x_{n+1},x_n) \leq k\omega_{2\lambda} (x_n,x_{n-1})$ for all $\lambda > 0$ and for all $n \in N$.

Hence $\omega_\lambda (x_{n+1}, x_n) \leq \frac{k}{1-k}\omega_{2\lambda} (x_n, x_{n-1})$ for all $\lambda > 0$ and for all $n \in N$.

Put $\alpha = \frac{k}{1-k}$, since $k \in [0,1/2)$, we get $\alpha \in [0,1)$

and thus $\omega_\lambda (x_{n+1},x_n) \leq \alpha\omega_{2\lambda} (x_n,x_{n-1},)$

$$\leq \alpha^2\omega_\lambda (x_{n-1}, x_{n-2}) \ldots \leq \alpha^n\omega_\lambda (x_1, x_0)$$

for all $\lambda > 0$ and for all $n \in N$.We conclude that $\{x_n\}$ is a Cauchy sequence and by the completeness of X_ω, there exist a point x ∈ X_ω such that $x_n \to x$ as n→∞ and subsequently, $\omega_\lambda(Tx_n,x) \to 0$ and $\omega_\lambda(Sx_n,x) \to 0$ as $n \to \infty$ for all $\lambda > o$.

By the notion of metric modular ω and by the inequality (3.2), we get

$$\omega_\lambda (Tx, x) \leq \omega_{\frac{\lambda}{2}}(Tx\ , Tx_n) + \omega_{\frac{\lambda}{2}}(Tx_n, x)$$

$$\leq k[\omega_\lambda(Tx\ , Sx_n) + \omega_\lambda(Tx_n\ , Sx_n)] + \omega_{\frac{\lambda}{2}}(Tx_n, x) \qquad (3.2.1)$$

Taking limit as n→∞ in (3.2.1), we get $\omega_\lambda (Tx, x) \leq k[\omega_\lambda(Tx\ , x)]$

Since $k \in [0,\frac{1}{2})$, by the strictness of ω, we have $\omega_\lambda (Tx, x) = 0$ for all $\lambda > 0$ and thus $Tx = x$.

Now we will show that x is a common fixed point S and T. Suppose on the contrary that Sx ≠ x or Tx ≠ x and put $x = x_n$ and $y = x$ *then by* (3.2), we have

$$\omega_\lambda(Tx_n,Tx) \leq k\ [\omega_{2\lambda}(Tx_nSx_n) + \omega_{2\lambda}(Tx, Sx)],$$

taking limit n→∞ on both sides we get

$$\omega_\lambda (x,T) \leq k[\omega_{2\lambda} (x,x) + \omega_{2\lambda} (T_x,S_x)]$$

Since $\omega_\lambda(Tx,\ Sx)<\infty$, therefore by the strictness of ω, and $k \in [0,1/2)$, we get $\omega_\lambda (x, Tx) < \infty$, a contradiction.

Hence $Tx = x = Sx$.

Uniqueness. Suppose that z be another common fixed point of S and T. Thus by (3.1) we get

$$\omega_\lambda (x,z) = \omega_\lambda\ (Tx, Tz) \leq k[\omega_{\frac{\lambda}{2}}(Tx\ , Sx) + \omega_{\frac{\lambda}{2}}(Tz, Sz)] = 0$$

for all $\lambda > 0$. Hence $\omega_\lambda (x, z) = 0$. Thus $x = z$ is a unique common fixed point of S and T.

Remark (3.1). By taking the mapping S in Theorem 3.1 as Ix_ω where Ix_ω is an identity mapping on X_ω, we have the following corollary 3.1, which is the main result Theorem 2.1, Theorem 3.2 of Mongkolkeha et al [10,11].

Corollary 3.1. Let ω be a metric modular on X and X_ω be a modular metric space induced by ω. If X_ω is complete modular metric space and $T : X_\omega \to X_\omega$ be a self mapping satisfying the inequality $\omega_\lambda(Tx, Ty) \leq k\omega_\lambda (x,y)$, for all $x,y \epsilon X\omega$, where k ϵ [0, 1). Suppose that there exist x ϵ X_ω such that $\omega_\lambda (x, Tx) < \infty$ for all $\lambda > 0$. Then T has a unique fixed point in X_ω. Moreover, for any $x \epsilon X_\omega$, sequence $\{T^n x\}$ convergses to x .

Remark (3.2). By taking the mapping S in Theorem 3.2 as Ix_ω where Ix_ω is an identity mapping on X_ω, we have the following corollary 3.2, which is the main result Theorem 2.2, Theorem 3.6 of Mongkolkeha et al [10,11].

Corollary 3.2. Let ω be a metric modular on X and $X\omega$ be a modular metric space induced by ω. If X_ω is complete modular metric space and T: $X_\omega \to X_\omega$ be a self mapping satisfying the inequality $\omega_\lambda(Tx, Ty) \leq k[\omega_{2\lambda}(Tx, x) + \omega_{2\lambda}(Ty, y)\ ]$ for all x , $y \epsilon X\omega$, where k ϵ [0,1). Suppose that there exist x ϵ X such that $\omega(x,Tx)<\infty$ for all $\lambda > 0$. Then T has a unique fixed point in X_ω. Moreover, for any $x \epsilon X_\omega$, sequence $\{T^n x\}$converges to x.

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