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RESEARCH ARTICLE

MODIFIED BINOMIAL FUNCTIONS FOR RINGS THROUGH THE COMBINATION OF ADDITIVE AND MULTIPLICATIVE FUNCTIONS

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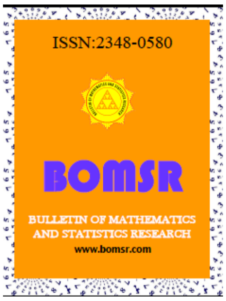

ABSTRACT

Let R be a set of positive integers then we assume that this set has two distinct elements .If the rules for multiplication of elements of R. such that α ∈ F, β ∈ F,we have: • α + β ∈ F • αβ ∈ F. In this note, we characterize that if xy is equal to 0, but both 'a' and 'b' are nonzero elements, then If a = 0 and b = 0, then there exist multiplicative inverses x^{-1} and y^{-1}. Thus, multiplying the equation $xy = 0$ by the product a $^{-1}$ b $^{-1}$ in the set.

Keywords: Additive, Binomial, Multiplicative, NonZero Elements, Rings

1. LITERATURE REVIEW

Elliott, (2006)has suggested A commutative ring which is said to be binomial if a is sans torsion (as a Z-module) and the component a (a − 1) (a − 2) • • • (a − n + 1)/n! of A ⊗Z Q lies in A for each a ∈ An and each positive number n. Binomial rings were initially characterized around 1969 by Philip Hall regarding his earth shattering work in the hypothesis of nilpotent gatherings. They have since had assist applications to the whole number esteemed polynomials, Witt vectors, and λ-rings. For any set X, the ring of number esteemed polynomials in Q[X] is the free binomial ring on the set X. Accordingly, the binomial property gives an all-inclusive property to rings of number esteemed polynomials. We give a few portrayals of binomial rings and their homomorphic pictures. For instance, we demonstrate that a binomial ring is equally a λ-ring A whose Adams operations are all the personality on A. This permits us to build a privilege adjoint BinU for the incorporation from binomial rings to rings which has a few applications in commutative polynomial math and number hypothesis. For instance, there is a characteristic BinU (A) - variable based math structure on the all-inclusive λ-ring Λ (A), and in like manner on the abelian gathering of multiplicative A-number-crunching capacities. So also, there is a characteristic BinU (A) - module structure on the abelian amass 1+a for any perfect an in an as for which a is finished.

Eriksson-Bique, (2012) proposed a hypothesis on complex factors that depends on contemplations of z^m. Imbedding R^{n+1} in the Clifford polynomial math C_n , Leutwiler in Complex Variables 17 (1992) has summed up Cauchy–Riemann conditions to R^{n+1} with x^mas one of the primary arrangements. In any case, since C_n is not commutative, the forces xm are hard to handle. For instance separation recipes are confounded. Th3e author has introduced the binomial hypothesis in R^{n+1} which streamlines the count rules concerning x^m.

Huang Wen-Tao, (1991) has proposed a modified binomial distribution model win the order of K. furthermore, the probability was obtained for the modified binomial theorem and also the Poisson limit was used while modifying the theorem. Analogous arguments were used for concluding the result.

Xantcha, (2012) has suggested that based on the previous study carried out by Ekedahl. He axiomatised and demonstrated indistinguishable Numerical Rings. In this study, the author has developed a Binomial Transfer Principle, which has empowered the combinatorial confirmations of algebraic characters. The final produced binomial rings were totally classified, and like-wise the limitedly created, without torsion modules.

Elliott, (2008) utilized the Witt vectors hypothesis, the author has defined the ring structures based on arithmetic functions and formal Dirichlet series. The arrangement of multiplicative arithmetic number over a commutative ring R is appeared to have an exceptional functional ring structure for which the operation of expansion is Dirichlet convolution and the operation of duplication confined to the totally multiplicative capacities harmonizes with point-wise augmentation. The gathering of added substance number-crunching capacities over R moreover has a functional ring structure. The Similarity with the ghost homomorphism of Witt vectors, there is a functional ring homomorphism from the ring of multiplicative capacities to the ring of added substance works that is an isomorphism if R is a Q-variable based math. The gathering of arithmetic functions that is, the gathering produced by the totally multiplicative capacities, shapes a subring of the ring of multiplicative capacities. The last mentioned ring has the structure of a Bin(R) -polynomial math, where Bin(R) is the general binomial ring outfitted with a ring homomorphism to R. In this study the author has utilized this polynomial math structure to concentrate the request of an objective arithmetic function, also the forces of α for α ∈ Bin(R) of a multiplicative arithmetic function f. For instance, the author has demonstrated comes about the forces of a given multiplicative arithmetic function that were found to be discerning. At last, the author has applied the proposed hypothesis to the investigation of the zeta capacity of a plan of limited type over Z.

2. METHODOLOGY

Two set of binary operations is there in a ring. We will refer to the operations on the set as addition and multiplication, and it will denote as $x+y$ and xy (or $x.y$). A ring must be an abelian group under addition. The multiplication operations must be associative $(x(yz)=((xy)z$ for all $x,y,z\in R$ and multiplication must be distributive over addition, both from the left and from the right $(x(y+z)=xy+yz$ and $(x+y)z=xz+yz$ for all $x,y,z\in R$.

A ring is, in particular, an Abelian group less than addition, so there is an additive uniqueness and each element of the ring has an additive inverse. It will always denote the additive identity of the ring by 0, and the additive inverse of an element a of R will be denoted –a. Similarly, $x+(-y)$ is denoted as $x-y$, and it will refer to the operation $x-y$ as subtraction.

A ring is not a group under multiplication: It does not have to be a multiplicative uniquenessdivision, and multiplicative reverse does not have to exist and also the multiplication is not assumed to be commutative. This will allowto define several special kinds of groups in which multiplication is assumed to have some extra properties.

2.1 Basic Definitions and Properties

Our research is based on the normal additive and multiplicative functions as well as homomorphism between rings. It can be defined similarly to group homeomorphisms: as operation –preserving maps. To avoid some of the difficulties that arise in the more general case, the discussion is controlled to restrained rings. A ring R is an abelian group with a multiplication Operation (x, y) that is associative and satisfies the distributive laws: $(x(y+z)=xy+yz$ and $(x+y)z=xz+yz$ for all $x,y,z\in R$. It assumes that R has at least two elements, including a multiplicative identity 1_R satisfying $a1_R=1_Ra=a$ for all a in R. The multiplicative identity is often written simply as 1, and the additive identity as 0. If x, y and z are arbitrary elements of R, the following properties are derived quickly from the definition of a ring: The general equation of additive and multiplicative functions was given below,

1) $x0=0x=0$ $[x0+x0=x(0+0)=x0;0x+0x=(0+0)x=0x]$
2) $(-x)y=x(-y)=-(xy)$ $[0=0y=(x+(-x))y=xy+(-x)y,$ So $(-x)y=-(xy)$; similarly, $0=x0=x(y+(-y))=xy+x(-y),$ So $x(-y)=-(xy)]$
3) $(-1)(-1)=1$ [take x=1 , y=-1 in (2)]
4) $(-x)(-y)=xy$ [replace b by –b in (2)]
5) $x(y-z)=xy-xz$ $[x(y+(-z))=xy+x(-z)=xy+(-(xz))=xy-xz]$
6) $(x-y)z=xz-yz$ $[(x+(-y))z=xz+(-y)z)=xz-(yz)=xz-yz]$
7) $1\neq 0$ [If 1=0 then for all a we have a=a1=a0=0, So R={0}, contradicting assumption that R has at least two elements]
8) The multiplicative identity is unique [If $1'$ is another multiplicative identity then 1=$11'$=$1'$].

2.2 Definitions and Commands

If x and y are nonzero but xy =0, we say that x and y are zero divisors; if $x\in R$ and for some $y\in R$ we have $xy=yx=1$, we say that x is a unit or that a is invertible.Note that xy need not equal to yx; if this holds for all $a,b\in R$, we say that R is a commutative ring. An integral domain is a commutative ring with no zero divisors. A division ring or skew field is a ring in which every nonzero element a has a multiplicative inverse a^{-1} (i.e. $aa^{-1}=a^{-1}a=1$). Thus the nonzero elements form a group under multiplication.

A field is a communicative division ring. Intuitively, in a ring it is possible to do the both addition, subtraction and multiplication without leaving the set, while in a field (or skew field) we can do division as well. Any finite integral domain is a field. To see this, observe that if $x\neq R$, the map $a\to xa, a\in R$ is injective because R is an integral domain. If R is finite, the map is subjective as well, so that $xa=1$ for some a. The characteristic of a ring R (written Char R) is the lowestoptimistic integer such that n1 = 0, where n1 is anshortening for 1 + 1 + · · ·1 (n times). If n1 is never 0, we say that R has characteristic 0. Note that the characteristic can never be 1, since $1_R=0$ If R is an integral domain and Char $R\to 0$, then Char R must be a prime number. For if Char $R=n=rs$ where r and s are positive integers greater than 1, then (r1) (s1) = n1 = 0, so either r1 or s1 is 0, opposing the similarity of n. A subring of a ring R is a subset S of R that forms a ring under the operations of addition and multiplication defined on R. In other words, S is an additive subsection of R that comprehends 1R and is closed under multiplication. Note that 1R is automatically the multiplicative identity of S, since the multiplicative identity is unique.

2.3 Objectives

- To identify the properties of rings through additive and multiplicative functions.
- To identify the properties of the relationship between binomial properties and rings.
- To derive the properties of rings based on the additive and multiplicative functions.

2.3 Proposed Methodology

Binomial theorem for Lemma,

The normal associative law holds for multiplications in a ring. There is also a generalized distributive law:

1) **Theorem 1:** The additive and multiplicative functions equations based on the Lemma is given by,

$$(x+y)+[(x_1+...+x_m)(y_1+...+y_n)] = x+y+\left(\sum_{i=1}^{m}\sum_{j=1}^{n} x_i y_j\right) \tag{1}$$

$$xy[(x_1+...+x_m)(y_1+...+y_n)] = xy\left(\sum_{i=1}^{m}\sum_{j=1}^{n} x_i y_j\right) \tag{2}$$

Where,

x and y is always 0.

Equation (1) is an additive combination of x and y and (2) is a multiplicative combination of x and y.

Proof: The argument for the generalized associative law is exactly the same as for groups; the generalized distributive law is proved in two stages. First set m=1 and work by induction on n, using the left distributive law $x(y+z) = xy+xz$. Then use induction on m and the right distributive law $(x+y)z = xz+yz$ on ($x_1+...+x_m+x_{m-1})(y_1+...+y_n)$

2) **Theorem 2:**The binomial theorem $(x+y)^n = xy+\sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}$ is valid in any ring, if $x, y = 0$ and $xy = yx$

Proof: The standard proof via elementary combinatorial analysis works. Specifically, $(x+y)^n = (x+y)...(x+y)$ and we can expand this product by multiplying an element (a or b) from,

Objective 1 (the first $(x+y)$) times an element from object 2 times … times an element from objective n, in all possible ways. Since $xy = yx$, these terms are of the form $x^k y^{n-k}, 0 \le k \le n$. The number of terms corresponding to a given k is the number of ways of selecting k objectives from a collection of n, namely $\binom{n}{k}$.

3) **Theorem 3:** Every proper ideal I is the kernel of a ring homomorphism.

Proof:

Define the natural or canonical map $\prod$: $R \to R/Q$ by $\prod(r) = xy+r+Q$. Where, $\prod$ is a homomorphism of abelian groups and its kernel is Q. To verify that $\prod$ preserves multiplication, note that

$\prod(rs) = rs+Q = (r+Q)(s+Q) = \prod(r)\prod(s)$

Since

$\prod(1_R) = 1_R + Q = 1_{RQ}$

Where,

$\prod$ - Ring homomorphism.

x, y - Always 0.

4) **Theorem 4:** If $g \in R[X]$ and $a \in R$, then for some unique polynomial $p(X)$ in $R[X]$

$g(X) = p(X)(X - x) + g(x);$

Hence g (a) =0 if and only if X-a divides g(X).

Proof: By the division algorithm, we may write g(X) = p(X) (X-x) +r(X)

Where,

The degree of r is less than 1.i.e r is a constant. Apply the findings of homomorphism $X \to x$ to show that r=g(x).

3. CONCLUSION

In this paper, we have adopted a modified version of lemma's theorem which is applied forhomomorphism, binomial and ideal functions for rings based on the properties of additive and multiplicative functions. The different attributes in polynomial functions were analyzed to determine the applications of additive and multiplicative functions. In future, this research cab be extended even to improve the real-time network performance.

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