Vol.5.Issue.2.2017 (April-June)
©KY PUBLICATIONS

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com
Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

MHD CONVECTIVE HEAT TRANSFER IN THE BOUNDARY LAYERS ON AN EXPONENTIALLY STRETCHING SURFACE WITH MOMENTUM AND THERMAL SLIP

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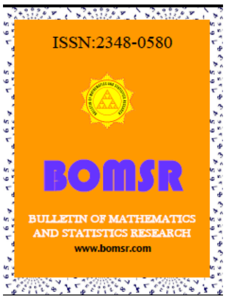

ABSTRACT
The present study analyses the MHD convective flow and heat transfer over an exponentially stretching surface with combined effects of momentum as well as thermal slip.The boundary value problem consisting of nonlinear PDE's are converted into nonlinear ODE's using a suitable similarity transformation. This problem has been solved, using RungeKutta fourth order method with shooting technique.The effects of various physical parameters, such as, Hartmann number Ha, Thermal buoyancy parameter Gr, Prandtl number Pr,Eckert number Ec, heat generation/absorption parameter λ , momentum slip parameter A, and thermal slip parameter, B on flow and heat transfer charecteristics., are computed and represented graphically.

Key Words: Exponentially stretching surface;Hartmannnumber;Momentum and thermal slip;Runge-Kutta shooting method.

1. INTRODUCTION

The problems of heat transfer in the boundary layers of a continuous stretching surface with a prescribed temperature have attracted considerable attention during the last few decades due to their numerous applications in several industrial manufacturing processes. For example, such as the extrusion of plastic sheet, hot rolling wire drawing, glass-fibres and paper production, drawing of plastic films and the cooling of a metallic plate in the cooling bath, materials manufactured by extrusion process, the boundary layer along a liquid film in condensation process and the heat treated materials travelling between a feed roll and the wind-up roll on conveyor belt posesess the features of a moving continuous surface. Annealing and thinning of copper wires is another example in which the final product depends on the rate of heat transfer at the stretching continuous surface.

The flow and heat transfer phenomena over stretching surface have promising applications in a number of technological processes including production of polymer films or thin sheets. The heat transfer rate in the boundary layer stretching sheets is important, because in the mentioned applications the quality of the final product depends on the heat transfer rate between the stretching surface and the fluid during the cooling or heating process. Therefore, the choice of suitable cooling/heating liquid is essential as it has a direct impact on the rate of heat transfer. All the known fluids gaseous or liquid possess the property of viscosity in varying degree. Fluids like water and gaseous (air) viscosity is very small (negligible) but fluid such as oil, glycerine, paints, molecule, and printer ink posses large viscosity, there exists a difference in relative tangential velocity i.e. there is a slip at the boundary. On the other hand the existence of intermolecular attraction makes the fluid to adhere to a solid wall and this gives rise to the shearing stress. The no-slip (without slip or absence of slip) boundary condition (the assumption that a liquid adheres to a solid boundary) is one of the central tenets of the Navier-Stokes [1] and Gadel-Hak [2] theory. However, there are situations wherein this condition does not hold. Partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. Navier [3] proposed a slip boundary condition wherein the slip depends linearly on the shear stress. However, experiments suggest that the slip velocity also depends on the normal stress. H. I. Anderson [4] considered the slip flow of a Newtonian viscous fluid past a linearly stretching sheet. T. Hayat, T. Javed, Z.Abbas [5] studied M.H.D steady flow of second grade fluid with transfer analysis. The flow in a porous space is due to a stretching sheet which also exhibits slip condition. Bikash Sahoo [6] studied the effect of partial slip on the steady flow and heat transfer of an incompressible, thermo-dynamically compatible third grade fluid past a stretching sheet.

A number of models have been developed for describing the slip that occurs at solid boundaries. A brief description of these models can be found in the work of Rao and Rajagopal [7]. Elbashbeshy [8] has added a new dimension in his investigation by considering the flow and heat transfer of a Newtonian fluid over an exponentially stretching continuous surface. He considered an exponential stretching velocity distribution on the coordinate considered in the direction of stretching. P.Donald Ariel [9] studied the steady, laminar, axisymmetric flow of an incompressible viscous Newtonian fluid past a stretching sheet when there is a partial slip at the boundary. BikashSahoo,Younghae [10], investigated the combined effects of the non-Newtonian flow parameters, magnetic field and the partial slip on the flow and heat transfer of an electrically conducting third grade fluid arising due to the linearly stretching sheet. BikashSahoo [11], investigated the combined effects of the non-Newtonian flow parameters, magnetic field and the partial slip on the flow and heat transfer of an electrically conducting third grade fluid arising due to the linearly stretching sheet in presence of heat source (sink). Partha et al. [12] obtained an similarity solution for mixed convection flow past an exponentially stretching surface by taking into account the influence of viscous dissipation on the convective transport. Al-odat et al. [13] explained the effect of magnetic field on thermal boundary layer on an exponentially stretching continuous surface with an exponential temperature distribution.

Recently, Anuar Ishak [14] studied the MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. He solved it numerically by an implicit finite-difference method. V. Singh, Shweta Agarwal [15] explained the effects of heat transfer for two types of viscoelastic fluid over and exponentially stretching sheet with thermal conductivity and radiation in porous medium. He solved it by well known fourth order Runge-Kutta method with shooting technique. Krishnendu Bhattacharyya [16] explained the effect of steady boundary layer flow and reactive mass transfer past an exponentially stretching surface in an exponentially moving free stream. R.N.Jat and Gopi Chand [17] worked on MHD flow and heat transfer over an exponentially stretching sheet with viscous dissipation and radiation effects. BikashSahoo [18] explained the effects of flow and heat transfer of third grade fluid past an exponentially stretching sheet with partial slip boundary condition. Sohail Nadeem and Changhoon Lee [19] studied the boundary layer flow of nanofluid over an exponentially stretching surface. He solved it analytically by HAM method.

Thus motivated by the above mentioned investigations and applications, we investigated the unsteady flow of an electrically conducting fluid with magneto hydrodynamic mixed convection heat transfer on a vertical surface of variable temperature that is stretched exponentially with momentum and thermal slip. It is assumed that the impermeable surface is stretched with exponential velocity in quiescent fluid and the surface is maintained at a constant temperature. The system is controlled by a uniform transverse magnetic field, as well as by internal heat generation, viscous dissipation and the partial slip (slip) effects. The obtained results have promising applications in engineering. The current investigation is not only important because of its technological significance, but also in view of the interesting mathematical features presented by the equations governing the slip flow. The aim of the present paper is to extend the work of Dulal Pal [20] to slip aspects.

2. MATHEMATICAL FORMULATION

Consider a two-dimensional flow of an electrically conducting and incompressible viscous fluid an impermeable plane wall stretching with velocity $u_w(x)$ and a given temperature distribution $T_w(x)$ .The x-axis is directed along the continuous stretching surface and points in the direction of motion. The y-axis is perpendicular to x-axis hence the continuous stretching plane surface issues from a slit (see Fig.1)

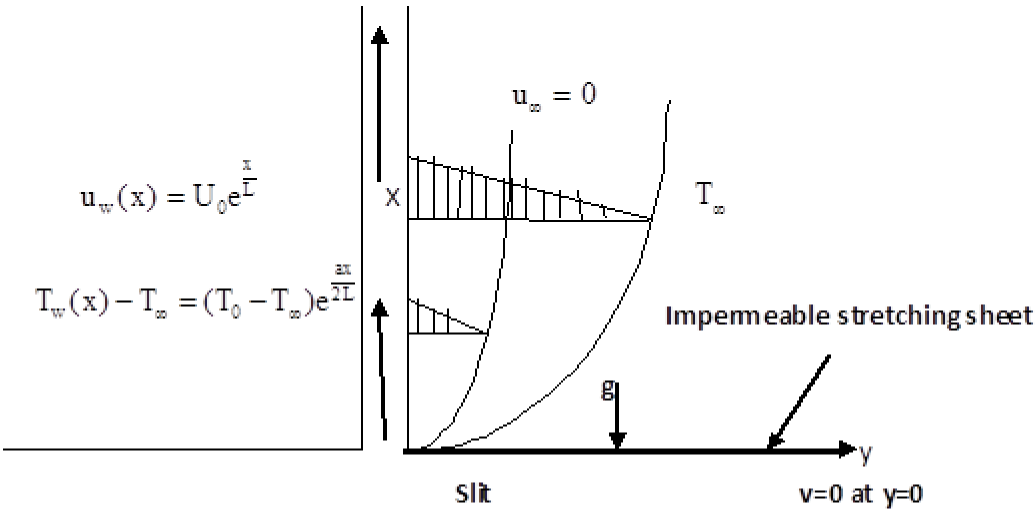

Fig. 1

A uniform magnetic field B_0 is assumed to be applied in the y-direction. It is assumed that the induced magnetic field of the flow is negligible in comparison with applied one which corresponds to a very small magnetic Reynolds number. Under boundary layer as well as with the Boussinesq approximation, the continuity, momentum and energy equations can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\,\beta\,(T-T_\infty)-\frac{\sigma\,\beta_0^{\,2}}{\rho}u \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\sigma}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho c_p}(T-T_\infty) \tag{2.3}$$

The associated boundary conditions to the problem are

$$u-u_w(x)=L_1\frac{\partial u}{\partial y}, v=0,\quad T-T_w(x)=L_2\frac{\partial T}{\partial y}\ \text{at}\ \ y=0 \tag{2.4}$$

$$u\to 0\quad T\to T_\infty\quad \text{as}\ \ y\to\infty \tag{2.5}$$

Where u and v are the x and y component of the velocity field, of the steady plane boundary flow, respectively, υ denotes the kinematic viscosity and α is the thermal diffusivity of the ambient fluid. Both are assumed to be constant. σ is the electrical conductivity and B_0 is the magnetic field flux density. The fluid flow is independent of the temperature field, T_∞ is the temperature of the ambient fluid and Q is the internal heat generation/absorption coefficient.

The stretching velocity $u_w(x)$ and the exponential temperature distribution $T_w(x)$ are defined as :

$$u_w(x)=U_0 e^{x/L}, \tag{2.6}$$

$$T_w(x)=T_\infty\ +\ (T_0-T_\infty)\ e^{\frac{ax}{2L}}, \tag{2.7}$$

Where T_0 is reference temperature and **a** isparameter of temperature distribution on the stretching surface.

Introducing the following non-dimensional parameters [1999, 2006]

$$\eta=\sqrt{\frac{Re}{2L}}\ y\ e^{\frac{x}{2L}},\quad \psi(x,\eta)=\sqrt{2Re}\ \nu e^{\frac{x}{2L}}f(\eta), \tag{2.8}$$

$$T(x,y)\ =\ T_\infty+(T_0-T_\infty)e^{\frac{a\,x}{2L}}\theta(\eta) \tag{2.9}$$

Where ψ is the stream function which is defined in the usual form as

$$u=\frac{\partial\psi}{\partial y}\quad \text{and}\quad v=-\frac{\partial\psi}{\partial x} \tag{2.10}$$

The correct form of η is taken from [1999] where as [2006] has typographical error which leads to incorrect solution. Thus substituting (5.2.8) and (5.2.9) into Eq. (5.2.10).We obtain u and v as follows:

$$u(x,y)=U_0 e^{\frac{x}{L}}f'(\eta),\quad v(x,y)=-\frac{\nu}{L}\sqrt{\frac{Re}{2}}\ e^{\frac{x}{2L}}[f(\eta)+\eta f'(\eta)] \tag{2.11}$$,

Eqs. (5.2.1) to (5.2.5) is transformed into the ordinary differential equation with the aid of Eqns. (5.2.8) to (5.2.11).Thus the governing equations using the dimensionless functions $f(\eta)$ and $\theta(\eta)$ become

$$f''' + ff' - 2f'^2 - 2\frac{Ha^2}{Re}e^{-X}f' + 2Gre^{\frac{X(a-4)}{2}}\theta = 0 \tag{2.12}$$

$$Pr^{-1}\theta'' + f\theta' - \mathbf{a}f'\theta + e^{\frac{X(2-\mathbf{a})}{2}}Ec\left(2\frac{Ha^2}{Re}f'^2 + f''^2e^X\right) + 2\lambda e^{-X}\theta = 0 \tag{2.13}$$

Where $X = \frac{x}{L}$ and a prime denotes the derivative with respect to η.

The boundary conditions (2.4) and (2.5) reduce to

$$f(0) = 0,\quad f'(0) = 1 + Af''(0),\quad \theta(0) = 1 + B\theta'(0), \tag{2.14}$$

$$f'(\infty) \to 0,\qquad \theta(\infty) \to 0 \tag{2.15}$$

where,

$$\left.\begin{array}{ll} Ha = \left(\frac{\sigma B_0^2 L^2}{\rho\nu}\right)^{\frac{1}{2}} \to \text{Hartman number}, & Gr = \frac{Gr_1}{Re^2} \to \text{thermal buoyancy parameter}, \\ Ec = \frac{U_0^2}{c_p(T_0 - T_\infty)} \to \text{Eckert number}, & Pr = \frac{\nu}{\alpha} \to \text{Prandtl number}, \\ Gr_1 = \frac{g\beta(T_0 - T_\infty)L^3}{\nu^2} \to \text{Grashof number}, & A = L_1\sqrt{\frac{Re}{2}}\,e^{\frac{X}{2}} \to \text{velocity or momentum slip parameter and} \\ Re = \frac{U_0 L}{\nu} \to \text{Reynolds number} & B = L_2\sqrt{\frac{Re}{2}}\,e^{\frac{X}{2}} \to \text{thermal slip parameter.} \\ \lambda = \frac{QL^2}{\mu c_p Re} \to \text{dimensionless heat generation /absorption parameter,} & \end{array}\right\} \tag{2.16}$$

In the above system of local similarity equations, the effect of the magnetic field is included as a ratio of the Hartman number to the Reynolds number.

The important physical quantities of interest in this problem are local Skin friction coefficient C_f and the local Nusselt numb er Nu_x are defined as:

$$C_f = -\frac{\tau_w}{\rho u_w^2},\qquad Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \tag{2.17}$$

Where wall shear stress τ_w, wall heat flux q_w are given by:

$$\tau_w = \rho\nu\left(\frac{\partial u}{\partial y}\right)_{y=0},\qquad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{2.18}$$

Where C_f, $Nu_x(Nur)$, $Sh_x(Shr)$, Re_x are the skin friction, local Nusselt number, local Sherwood number and local Reynolds number respectively.

By solving eqs. (2.16) using eqs.(2. 8),(2.9),(2.11) and (2.18).We get

$$C_f\sqrt{2Re_x} = -f''(0),\qquad \sqrt{\frac{2}{X}}\left(\frac{Nu_x}{\sqrt{Re_x}}\right) = -\theta'(0) = Nur \tag{2.19}$$

3. NUMERICAL SOLUTION

The set of coupled non-linear differential equations (2.12) and (2.13) subject to the boundary conditions (2.14) and (2.15) are integrated numerically using a very efficient method known as Runge-Kutta method with Shooting technique. The most important factor of this method is to choose the appropriate finite values of $\eta \to \infty$.In order to determine η_∞ for the boundary value problem started by Eqs. (2.12) and (2.13) we start with some initial guess value for some particular set of physical parameters to obtain $f''(0)$ and $\theta'(0)$.The solution procedure is repeated with another large value of η_∞ until two successive values of $f''(0)$ and $\theta'(0)$ differ only by the specified significant digit. The last value of η_∞ is finally chosen to be the most appropriate value of the limit $\eta \to \infty$ for that particular set of parameters. The last value of η may change for another set of physical parameters. Once the finite value of η is determined then the coupled boundary value problem given by Eqs.(2.12)-(2.15) are solved numerically using the method of superposition.

In this method the third-order non-linear Eq.(2.12) and second order Eq. (2.13) have reduced to five simultaneously ordinary differential equations as follows :

$$f_1' = f_2,$$
$$f_2' = f_3,$$
$$f_3' = -f_1 f_3 + 2f_2^2 + \frac{2Ha^2}{Re} e^{-X} f_2 - \frac{2Gr_1}{Re^2} e^{\frac{X(a-4)}{2}} f_4,$$
$$f_4' = f_5,$$
$$f_5' = -Pr\left[f_1 f_5 - a f_2 f_4 + e^{\frac{X(2-a)}{2}} Ec\left(\frac{2Ha^2}{Re} f_2^2 + f_3^2 e^X\right) + 2\lambda e^{-X} f_4\right], \quad (3.20)$$

Where $f_1 = f,\ f_2 = f',\ f_3 = f'',\ f_4 = \theta,\ f_5 = \theta'$, and a prime denotes differentiation with respect to η.

The boundary conditions now become

$$f_1 = 0,\ f_2 = 1 + A*S_1,\ f_3 = S_1,\ f_4 = 1 + B*S_2,\ f_5 = S_2, \quad \text{at} \quad \eta = 0 \quad (3.21)$$
$$f_2 = 0,\ f_4 = 0 \quad \text{as} \quad \eta \to \infty \quad (3.22)$$

Where S_1 and S_2 are determined such that it satisfied $f_2(\infty) = 0$ and $f_4(\infty) = 0$.Thus , to solve this resultant system, we need five initial conditions, but we have only two initial conditions on f and one initial condition on θ.The third condition on f (i.e. $f''(0)$ and the second condition on θ (i.e. $\theta'(0)$) are not prescribed which are to be determined by shooting method by using the initial guess values S_1 and S_2 until the boundary conditions $f_2(\infty) = 0$ and $f_4(\infty) = 0$ are satisfied. In this way, we employ shooting technique with Runge-Kutta method to determine two more unknowns in order to convert the boundary value problem to initial value problem. Once all the five initial conditions are determined the resulting differential equations can then be easily integrated, without any iteration by initial value solver. For this purpose, the well known fourth-order Runge-Kutta method has been used. In this manner any non-linear equation involved in boundary value problem can easily be solved by this technique. To study the behaviour of the velocity and temperature profiles, curves are drawn for various values of the parameters that describe the flow.

4. RESULTS AND DISCUSSION

Numerical calculations were performed for velocity, temperature distribution, skin-friction coefficient and the wall temperature gradient for various values of physical parameters such as Ha^2/Re, **a**, Ec , Pr ,Gr and X location as can be seen clearly from the governing Eqs.(2.12) and (2.13).The Wall-temperature gradient $\theta'(0)$ values (for both with slip and without slip) computed by the present method for Gr=Ec= λ =Ha=X=0 are compared with that of Magyari and Keller [1999] in the absence of magnetic field and Al-odat et al. [2006], in the absence of buoyancy effects and Dulal pal [2010] are shown in **Table1**.Therefore, we are confident that results obtained by us are very much accurate to analyze the flow problem.

Table 1

a	Pr	(1) Magyari& Keller [1999] A=B=0	(2) AL-odat et al. [2006] A=B=0	(3) Dulal Pal [2010] A=B=0	(4)(i) Present Method Without slip A=B=0	4(ii) Present Method With slip A=B=0.1
		$\theta'(0)$	$\theta'(0)$	$\theta'(0)$	$\theta'(0)$	$\theta'(0)$
-1.5	0.5	0.20405	0.19191	0.20405	0.192611	0.182097
	1	0.37741	0.36152	0.37741	0.376878	0.368736
	3	0.92386	0.90309	0.92386	0.923837	0.955936
	5	1.35324	1.34143	1.35324	1.353219	1.465334
	8	1.88850	1.82858	1.88849	1.888460	2.170729
	10	2.20000	2.13693	2.20003	2.199974	2.633748
-0.5	0.5	-0.17582	-0.18187	-0.17582	-0.181936	-0.170205
	1	-0.29988	-0.32697	-0.29988	-0.300036	-0.275656
	3	-0.63411	-0.67215	-0.63411	-0.634105	-0.565047
	5	-0.87043	-0.84156	-0.87043	-0.870426	-0.759634
	8	-1.15042	-1.08391	-1.15032	-1.150320	-0.979965
	10	-1.30861	-1.25074	-1.30861	-1.308617	-1.099983
0.0	0.5	-0.33049	-0.31006	-0.33049	-0.334807	-0.307616
	1	-0.54964	-0.53104	-0.54964	-0.549735	-0.493640
	3	-1.12219	-1.08522	-1.12209	-1.122082	-0.958212
	5	-1.52124	-1.47558	-1.52124	-1.511331	-1.256255
	8	-1.99185	-1.92633	-1.99184	-1.991836	-1.583351
	10	-2.25743	-2.18847	-2.25742	-2.240004	-1.757370
1.0	0.5	-0.59434	-0.91903	-0.59434	-0.596798	-0.534174
	1	-0.95478	-0.57771	-0.95478	-0.954813	-0.827207
	3	-1.86908	-1.81039	-1.86907	-1.869061	-1.500402
	5	-2.50014	-2.28864	-2.50013	-2.500126	-1.910219
	8	-3.24213	-3.00587	-3.24212	-3.242109	-2.344234

	10	-3.66038	-3.18620	-3.66037	-3.660357	-2.568963
3.0	0.5	-1.00841	-0.97665	-1.00841	-1.009505	-0.870509
	1	-1.56029	-1.46569	-1.56030	-1.560296	-1.284371
	3	-2.93854	-2.89007	-2.93854	-2.927806	-2.172415
	5	-3.88656	-3.78072	-3.88656	-3.878062	-2.685087
	8	-5.00047	-4.86245	-5.00046	-5.000449	-3.207785
	10	-5.62820	-5.58576	-5.62820	-5.628163	-3.470708

Table 1 shows the values of Wall-temperature gradient $\theta^{'}(0)$ for different values of Pr and a .It is observed from the table that for **a**=-0.5,0,1,3 , the values of $\theta^{'}(0)$ are negative which shows that heat is transferred from the stretched surface to the ambient fluid and the reverse effect occurred for $\theta^{'}(0)$ when **a**= -1.5 for different values of Pr. The case of no heat transfer between the stretched surface and the ambient fluid corresponds to $\theta^{'}(0)=0$ .The value of Eckert number Ec is chosen to be positive for all the predicted results shown graphically. Positive value of Ec means that the reference temperatures T_0 must be greater than the free stream temperature T_∞ i.e. $(T_0 - T_\infty) > 0$ .Which indicates that heat is transferred from the wall to the fluid. Further, it is observed that the above effects are true in case of with slip and without slip boundary conditions related to present method in our problem.

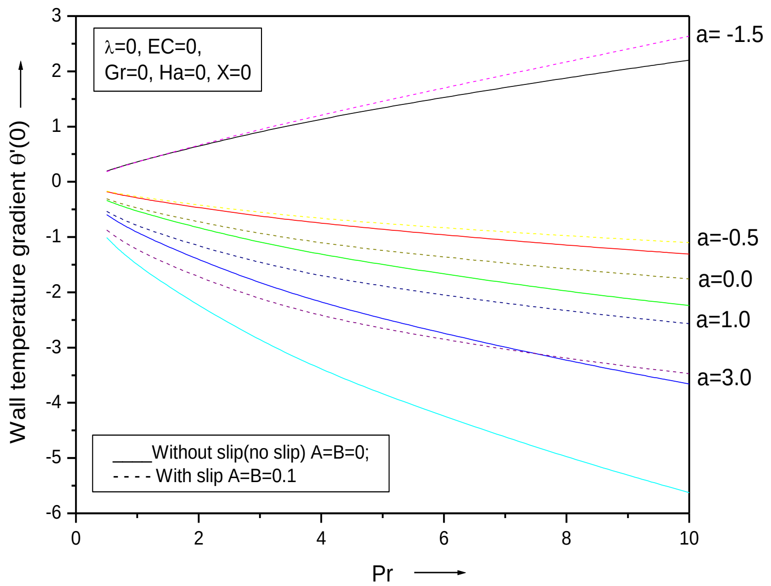

Fig 2. Variation of wall temperature gradient θ'(0) profile vs Prandtl number for a= -1.5, -0.5, 0, 1, 3

Fig 2. has been plotted to depict the variation of wall temperature gradient $\theta^{'}(0)$ vs. Pr for different values of **a** = -1.5,-0.5, 0.0, 1.0, 3.0. Analysis of graph shows that as the values of **a** and Pr increases, the wall temperature gradient $\theta^{'}(0)$ decreases. This is true in case of both with and without slip cases.

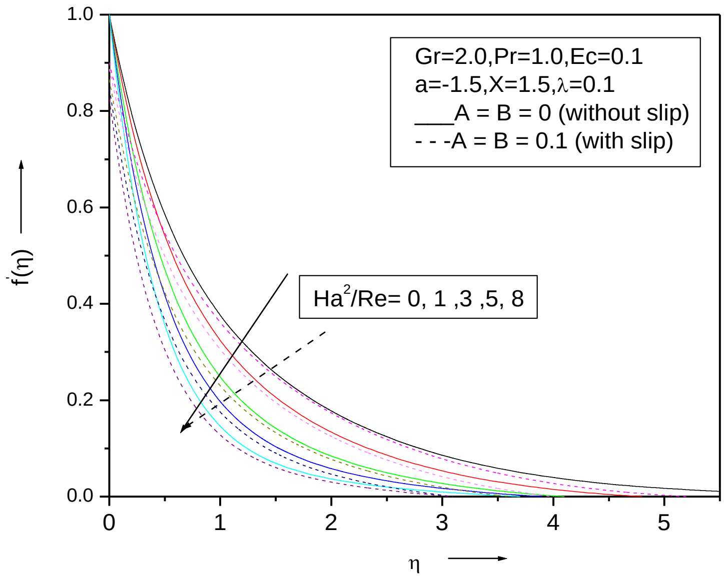

Fig 3.Variation of velocity profiles vs η for different values of magnetic field.

Fig 3.represents the graph of Horizontal velocity profile $f'(\eta)$ vs. η for different values of magnetic field parameter Ha^2/Re, for both with and without slip cases. Analysis of graph for both slip and without slip cases shows that the effect of increasing the values of magnetic field parameter is to decrease the magnitude of horizontal velocity flow. This is because of Lorentz force which increases the frictional drag (resistance), which opposes the motion of the fluid flow in the momentum boundary layer. With slip (A=B=0.1) decreases the momentum boundary layer thickness.

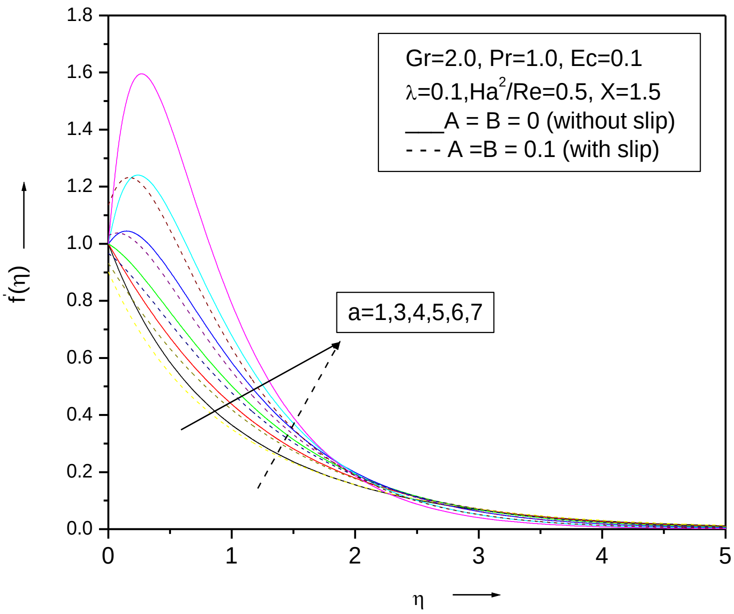

Fig 4. Variations of velocity profiles vs η for various values of a.

Fig 4.represents the graph of horizontal velocity profile $f'(\eta)$ vs. η for various values of (**a**=1,3,4,5,6,7) for both with and without slip cases. From this plot, it is interesting to observe that for both with slip and without slip cases that as the values of **a** increases the dimensionless horizontal velocity profile also increases. The maximum horizontal peak velocity is observed for **a**=7. Further, with slip (A=B=0.1) decreases the momentum boundary layer thickness.

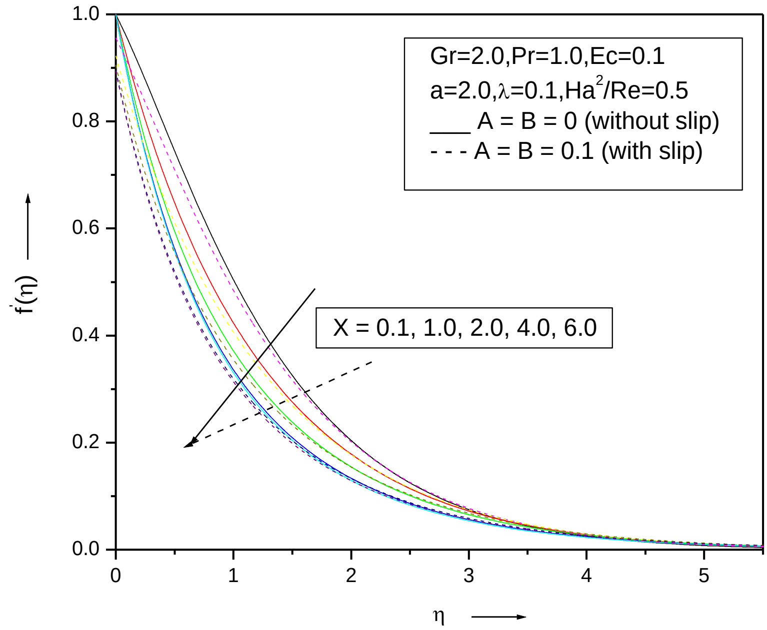

Fig 5. Variation of velocity profiles vs η for various values of X.

Fig 5.shows the variation of velocity profile with η for different values of dimensionless X, for both with slip and without slip case. From this figure, for both with slip and without slip cases, the horizontal velocity profile decreases with increase in the value of X in the momentum boundary layer but the significant effect is noticed for flow adjacent (near) to the stretching sheet. Further, with slip (A=B=0.1) decreases the momentum boundary layer thickness.

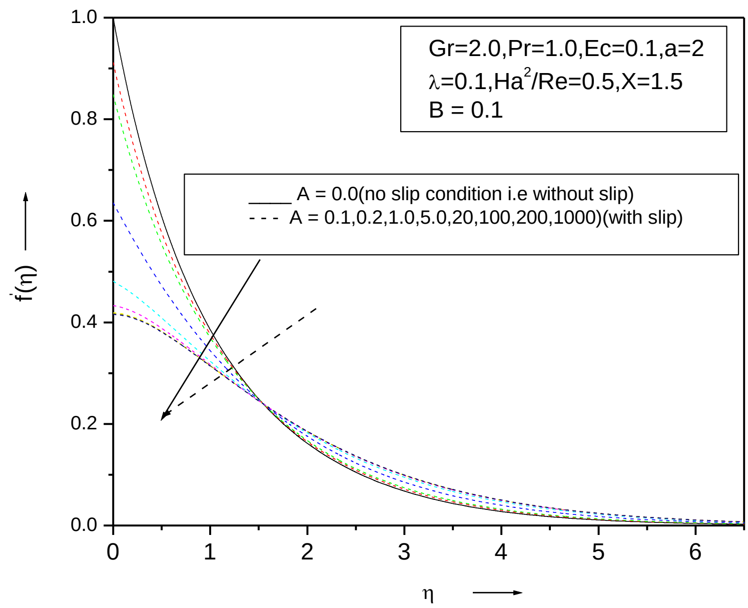

Fig 6. Variations of velocity profiles vs η for different values of momentum slip parameter A.

Fig 6.shows the variations of velocity profiles vs η for some different values of the momentum slip factor A. It is readily seen that A has substantial effect on the velocity profile. It is noticed that as the value of momentum slip parameter A=(0,0.1,0.2,1,5,10,100,200,1000), increases, the velocity profile decreases near the surface of the sheet, and then increases away from it resulting a crossover in the velocity profile .For without slip for $A = 0$ (total adhesion) and towards full slip as A tends to infinity. In the limiting case $A \to \infty$ implies that the frictional resistance between the viscous fluid and the surface is eliminated and the stretching of the sheet does no longer impose any motion on the fluid, i.e. the flow behaves as though it were inviscid(frictionless).

Figs 3,4,5,6 onecommon thing is noticed i.e. with slip decreases the momentum boundary layer thickness and the horizontal fluid velocity is found lower for the case of with slip, than without slip flow.

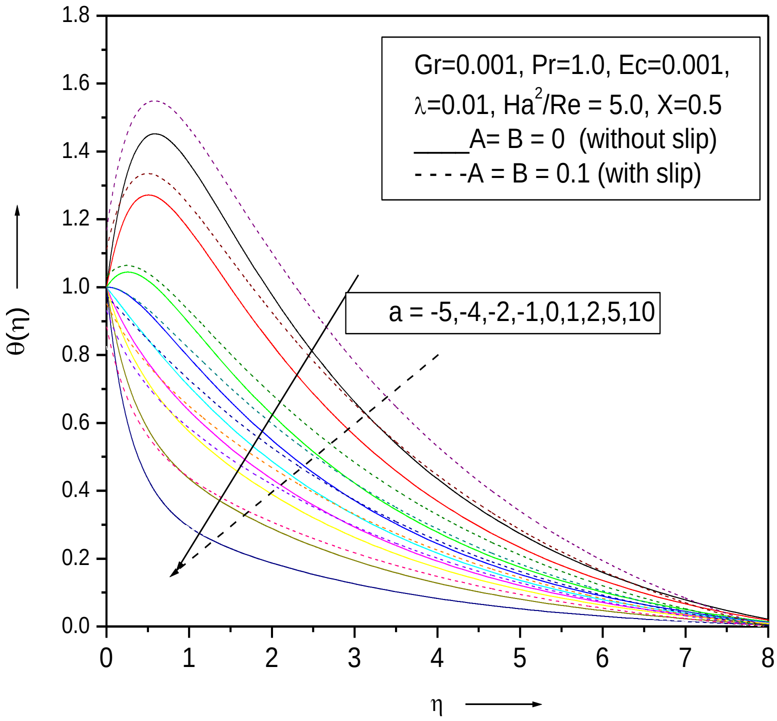

Fig 7. Variation of temperature profile vs η for different values of a.

Fig 7.depicts variation of dimensionless profiles $\theta(\eta)$ vs. η for various values of **a** for both with slip and without slip cases. For both slip and without slip cases, it is observed from this figure that temperature decreases with increase in the value of **a** (**a** = -5,-4,-2.-1, 0, 1, 2, 5, 10) .Further, it can be seen that the thermal boundary layer thickness decreases with increase in **a**. It is observed that thickness of thermal boundary layer is higher for the with slip case than the without slip case.

Fig 8.shows the temperature Profile $\theta(\eta)$ vs. η for various values of X= 1.5,2,2.5. It is noticed that temperature increases with the increasing value of X, Which results in increase of boundary layer thickness of the fluid. These results are true for both with slip and without slip boundary condition. Further, the temperature increases more in case of without slip (A=B=0) compared to with slip (A=B=0.1) boundary condition i.e. the effect of parameter X is more prominent(effective) in case of without slip compared to with slip.

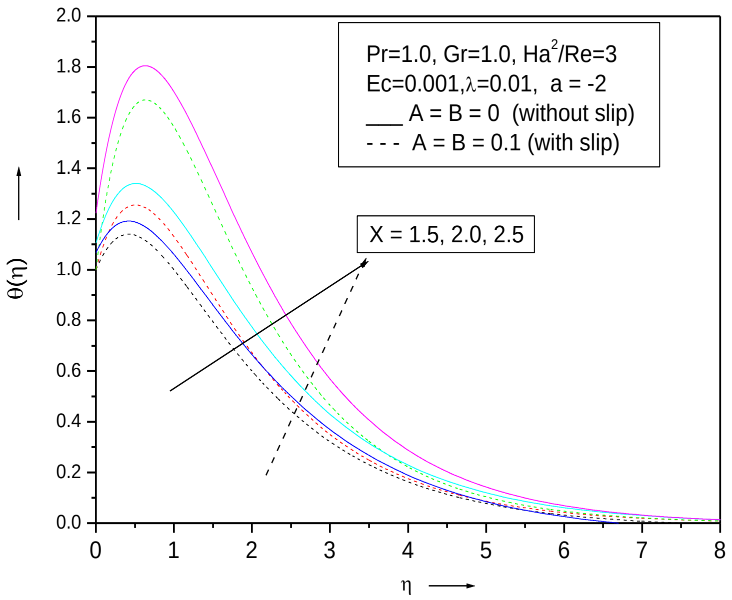

Fig 8. Temperature profiles vs η for various values of x.

Fig 9.depicts the variation of the temperature profiles $\theta(\eta)$ for various values of magnetic field parameter $(Ha^2/Re = 0,6,8)$.For both slip and without slip, it can be seen that temperature profile increases as magnetic field parameter $(Ha^2/Re = 0,6,8)$ increases. This results in increase of thermal boundary layer thickness. This is due to the fact that magnetic field produces a Lorentz force which results in retarding force on the velocity which increases the temperature. Further, thermal boundary layer thickness increases more in case of without slip compared to with slip i.e. the effect of magnetic field is more prominent in case of without slip.

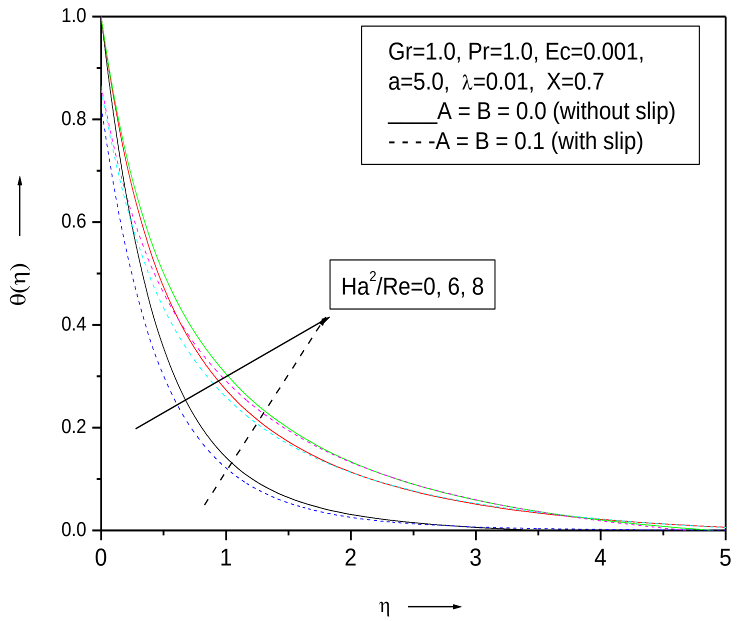

Fig 9. Temperature profile vs η for various values of magnetic field Ha^2/Re.

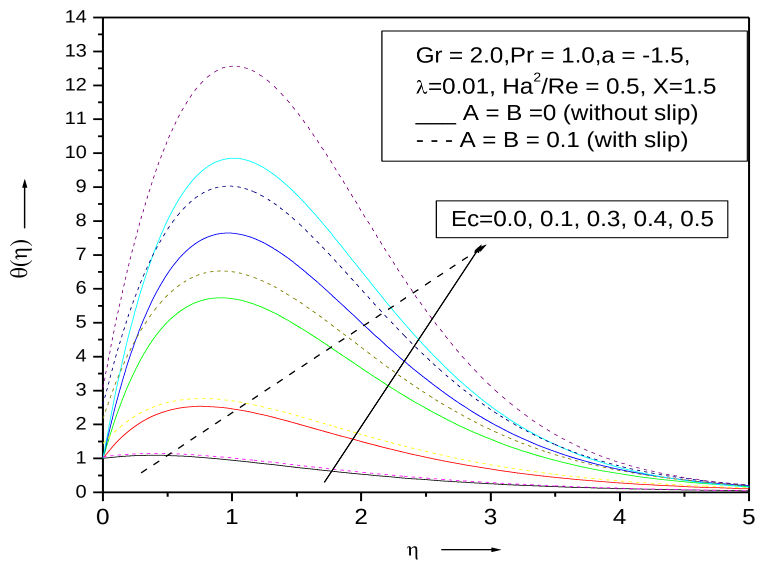

Fig 10. Variations of temperature vs η for different values of Eckert number Ec.

Fig 10.shows the temperature distribution $\theta(\eta)$ vs. η for different values of Eckert number Ec. By analyzing the graph for both slip and without slip, it is noticed that temperature increases with the increasing value of Eckert number Ec = (0.0, 0.1, 0.3, 0.4, 0.5) which results in increase of thermal boundary layer thickness of the fluid. This is due to the fact that the heat energy is stored in the fluid due to the frictional heating. Thus the effect of increasing the Eckert number Ec is to enhance the temperature at any point in the fluid. It is interesting to note that the temperature overshoot near the stretching surface, there is significant heat generation due to fluid friction near the sheet. Further, thermal boundary layer thickness increases more in case of with slip compared to without slip i.e. the effect of Ec is more prominent in case of with slip.

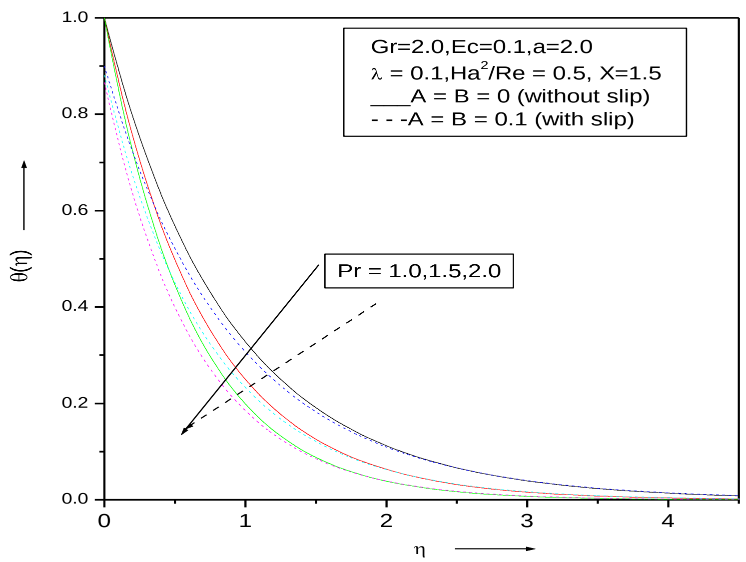

Fig 11. Variations of temperature vs η for different values of Prandtl number Pr

Fig 11.shows the effect of Prandtl number on the temperature profile for both with slip and without slip cases respectively. It is noticed that temperature profile decreases with the increasing value of

Prandtl number (Pr= 1,1.5,2) where as thermal boundary layer thickness decreases due to increase in Pr. From these plots it is evident that large values of Prandtl number results in thinning of thermal boundary layer. Further, thermal boundary layer thickness decreases more in case of with slip compared to without slip i.e. the effect of Pr is more prominent in case of with slip.

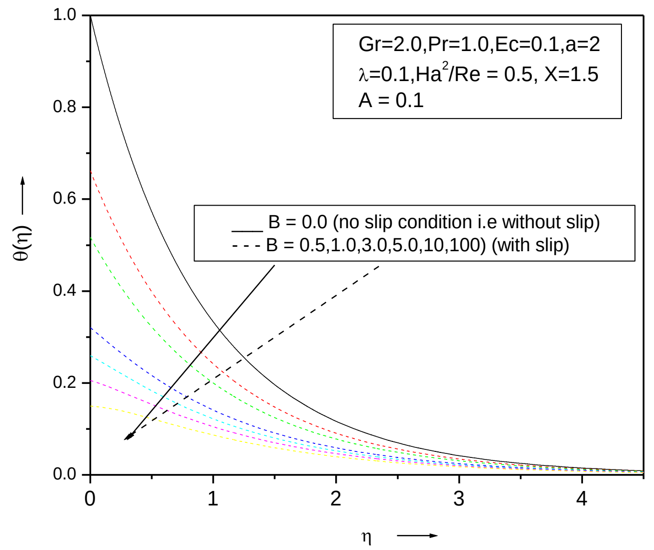

Fig 12. Variations of temperature profiles vs η for different values of thermal slip parameter B.

Fig 12.shows the variations of temperature velocity profiles vs. η for some different values of the thermal slip factor B = (0,0.1,0.2,1,5,20,200,100). For B=0(no-slip) the behaviour of graph is according to equation (5.2.14) i.e. it satisfies the usual boundary condition $\theta(0)$ =1 From the plot it is noticed that the temperature profile decreases with increasing value of thermal slip parameter B, which results in thinning of the thermal boundary layer. In fact, the amount of with slip $1-\theta(0)$ increases monotonically with B from without slip for $B=0$ (total adhesion) and towards full slip as B tends to infinity. In the limiting case $B \to \infty$ implies that the frictional resistance between the viscous fluid and the surface is eliminated and the stretching of the sheet does no longer impose any motion on the fluid i.e. the flow behaves as though it were inviscid(frictionless).

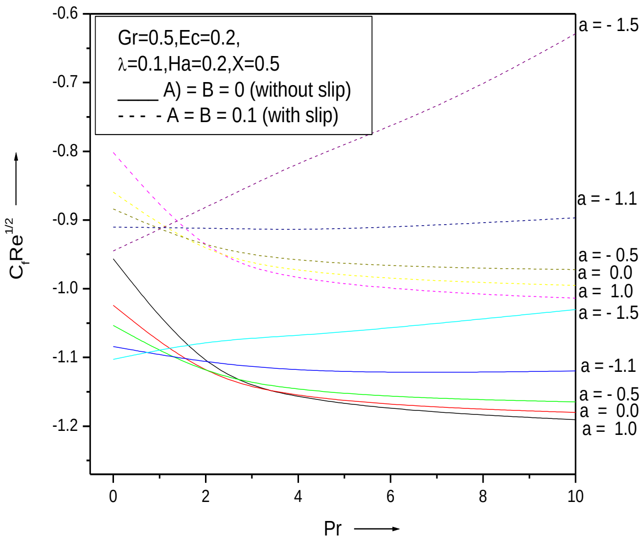

Fig 13. Variation of Skin-friction coefficient vs Pr for differnt value of a.

Fig 13.has been plotted to depict the variation of skin-friction coefficient vs. Prfor different values of **a** = (-1.5,-1.1,-0.5,0,1). Analysis of the graph shows that the effect of **a** is to decrease skin-friction coefficient. Similar effect is observed by increasing the value of the Prandtl number on skin-friction coefficient. The Skin-friction coefficient is found higher for the case of with slip than without slip flow.

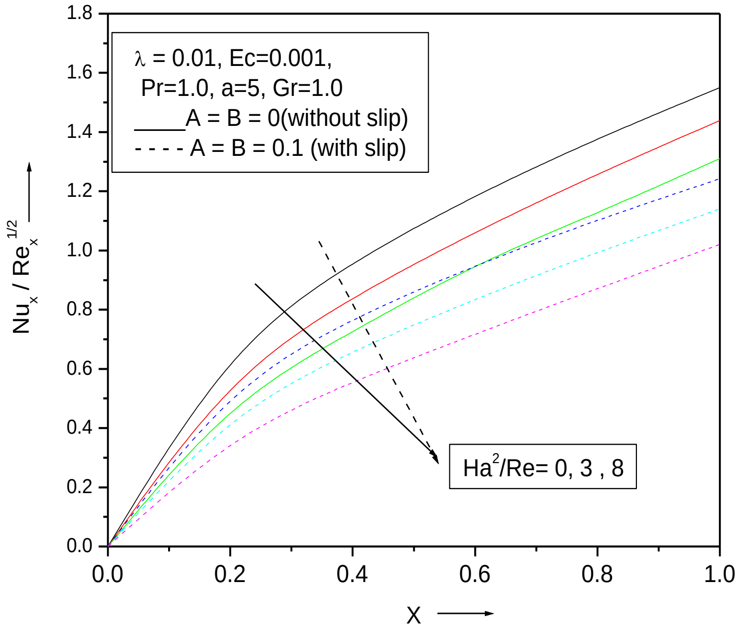

Fig 14. Effect of magnetic field on local Nusselt number with X.

Fig 14.presents the effect of magnetic field (represented by the Hartman number) for local Nusselt number. It can be seen that an increase in the strength of the magnetic field leads to decrease in local Nusselt number. After observing the graph it is interesting to note that the behaviour of graph (i.e. decreasing) is true for both with slip and without slip. But local Nusselt number decreases more in case of with slip compared to without slip.

5. CONCLUSIONS.

A numerical study corresponding to the convection heat transfer in steady flow region in exponentially continuous stretching surface with exponential temperature variations at the wall in the presence of the magnetic field, viscous dissipation, buoyancy force and internal heat generation has been presented.

The following conclusions have been drawn from the present study:

1. As the value of Pr and **a** increases wall temperature gradient $\theta'(0)$ decreases more in case of without slip than with slip

2. With slip decreases the momentum boundary layer thickness.

3. The horizontal fluid velocity is found lower for the case of with slip compared to without slip flow.

4. For both with slip and without slip cases, the increase of magnetic field parameter ($\mathrm{Ha}^2/\mathrm{Re}$) decreases the momentum boundary layer thickness and increases the thermal boundary layer thickness.

5. For both with slip and without slip cases, the increase of temperature distribution parameter **a** increases the momentum boundary layer (velocity profile) thickness and decreases the thermal boundary layer thickness.(thermal profile).

6. For both with slip and without slip cases, the increase of X decreases the momentum boundary layer thickness and increases the thermal boundary layer thickness.

7. For both with slip and without slip cases, the increase of Ec increases the thermal boundary layer thickness whereas increase of Pr decreases the thermal boundary layer thickness.

8. For with slip case, the increase of momentum slip parameter A decreases the momentum boundary layer thickness. For A = 0(i.e. without slip), $f^{'}(0)=1$ the solution derived by Crane [1970] is recovered in the case of velocity profile.

9. For with slip case, the increase of thermal slip parameter B decreases the thermal boundary layer thickness. For B =0(i.e. without slip), $\theta(0)=1$ in the case of temperature profile.

10. For both with slip and without slip cases, the skin-friction coefficient decreases with increasing the value of Pr and **a .** Skin- friction coefficient is found higher for the case of with slip than without slip flow.

11. For both with slip and without slip cases, as the value of magnetic field parameter (Ha^2/Re) and X increases, the local nusselt number decreases. Further, local nusselt number decreases more in case of with slip than compared to without slip flow.

Comparison of Wall- temperature gradient $\theta'(0)$ calculated by

(1) Magyari and Keller [1999] for Ha=Gr=Ec= λ =0

(2) Al-odat et al. [2006] for Gr=Ec= λ =0

(3) Dulal Pal [2010] for Ha=Gr=EC= λ =0

(4) (i) Present method without slip A = B =0, Ha=Gr=Ec= λ =X=0

(ii) Present method with slip A = B = 0.1, Ha=Gr=Ec= λ =X=0

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