Vol.5.Issue.2.2017 (April-June)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com
Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

STABILITY OF CONNECTED REGULAR DOMINATION OF SOME GRAPHS

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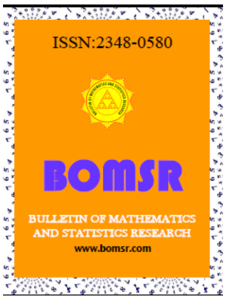

ABSTRACT

The connected dominating set was first suggested by S. T. Hedetniemi and elaborate treatment of this parameter appears in E. Sampathkumar and H. B. Walikar[5]. The concept of regular domination was first introduced by Prof. E. Sampathkumar. On combining these two parameters we obtain a new parameter namely connected regular domination. In networks, the radius of the graph model represents a measure of the shortest possible time required to broadcast a message from a single vertex to all the other vertices. Changing and unchanging studies then reflect how this parameter can vary as a result of vertex or edge removal or edge addition. We will consider the effects which can occur by removing a single vertex or edge and by adding a single edge responding in a fixed way for all vertices or edge. In this paper, we evaluate the stability of connected regular domination number for complete graph and for some standard graphs.

Subject Classification: 05C25, 05C69

Keywords: Connected domination, Regular domination

1. INTRODUCTION

Domination in graphs has been an extensively researched branch of graph theory. The topic of domination was given a formal mathematical definition, first by C. Berge in [4] and later by Ore [6] in 1962. Berge called the domination as external stability and domination number as coefficient of external stability. Ore introduced the word domination in his famous book Theory of Graphs published in 1962. A recent book on the topic of domination [5] lists over 1,200 papers related to domination in graphs, and several hundred papers on the topic have been written[3,7,8,9] since the publication of the book few years ago. The first simplification of Beck's zero divisor graph was introduced by D. F. Anderson and P. S. Livingston [2]. The connected dominating set was first suggested by S. T. Hedetniemi and elabourate treatment of this parameter appears in E. Sampathkumar and H. B. Walikar[10]. The concept of regular domination was first introduced by prof. E. Sampathkumar. On combining we obtain a new parameter called connected regular domination.

Definition 1.1. Dominating set

A dominating set is a set of vertices such that each vertex of V is either in D or has atleast one neighbour in D. The minimum cardinality of such a set is called the domination number of *G* denoted by $\gamma(G)$.

Definition 1.2. Connected dominating set

A connected dominating set D is a set of vertices of a graph G such that every vertex in V- D is adjacent to atleast one vertex in D and the subgraph ⟨D⟩ induced by the set D is connected. The connected domination number $\gamma_c(G)$ is the minimum cardinality of the connected dominating sets of *G.*

Definition 1.3. Regular dominating set

A regular dominating set is a dominating set D of V (G) if ⟨D⟩ is regular. The minimum cardinality of a regular dominating set is called regular domination number of G and is denoted by $\gamma_r(G)$.

On combining the two parameters we obtain a new parameter namely connected regular domination.

Definition 1.4. Connected Regular dominating set

Let *G* be a graph, V is a vertex set of *G* and D ⊆ V , then D is said to be Connected Regular dominating set, if it satisfies the following conditions, (i) D is dominating set (ii) D is regular (iii) D is connected. The minimum cardinality of connected regular domination set is denoted by $\gamma_{cr}(G)$.

Definition 1.5. Stability of Connected Regular Domination

Let G - v (respectively, G - e) denote the graph formed by removing vertex v (respectively edge e) from G. We use acronyms to denote the following classes of graphs (C represents changing; U-unchanging; V-vertex; E-edge; R-removal; A-addition). The stability of Connected Regular domination of G is defined as,

$$(CVR) \Rightarrow \gamma_{cr}(G-v) \neq \gamma_{cr}(G), \forall v \in V$$
$$(UVR) \Rightarrow \gamma_{cr}(G-v) = \gamma_{cr}(G), \forall v \in V$$
$$(CER) \Rightarrow \gamma_{cr}(G-e) \neq \gamma_{cr}(G), \forall e \in E$$
$$(UER) \Rightarrow \gamma_{cr}(G-e) = \gamma_{cr}(G), \forall e \in E$$
$$(CEA) \Rightarrow \gamma_{cr}(G+e) \neq \gamma_{cr}(G), \forall e \in E(\bar{G})$$
$$(UEA) \Rightarrow \gamma_{cr}(G+e) = \gamma_{cr}(G), \forall e \in E(\bar{G})$$

Let $V = V^0 \cup V^+ \cup V^-$ for,

$$V^0 = \{v \in V: \gamma_{cr}(G-v) = \gamma_{cr}(G)\}$$
$$V^+ = \{v \in V: \gamma_{cr}(G-v) > \gamma_{cr}(G)\}$$
$$V^- = \{v \in V: \gamma_{cr}(G-v) < \gamma_{cr}(G)\}$$

similarly, the edge set can be partitioned into,

$$E^0 = \{uv \in E: \gamma_{cr}(G-uv) = \gamma_{cr}(G)\}$$
$$E^+ = \{uv \in E: \gamma_{cr}(G-uv) > \gamma_{cr}(G)\}$$

2. Stability of Connected Regular Domination of Some Standard graphs

Definition 2.1. Wheel graph

A Wheel graph, W_n of order n is a graph that contains an outer cycle of order n -1 and for which every vertex in the cycle is connected to one other vertex(which is known as the hub). The number of vertices in W_n is n and the number of edges is 2(n-1).

Theorem 2.2

For a Wheel graph W_n for n ≥5, we have the following

(i) there exist a vertex *v* with maximum degree such that $\gamma_{cr}(W_n - v) > \gamma_{cr}(W_n)$, which implies that $W_n \in CVR$.

(ii) there exist a vertex *v* with minimum degree such that $\gamma_{cr}(W_n - v) = \gamma_{cr}(W_n)$, which implies that $W_n \in UVR$.

(iii) there exist an edge e ∈ E(W_n), such that $\gamma_{cr}(W_n - e) > \gamma_{cr}(W_n)$, which implies that $W_n \in CER$.

(iv) there exist an edge e ∈ E(W_n), such that $\gamma_{cr}(W_n - e) = \gamma_{cr}(W_n)$, which implies that $W_n \in UER$.

(v) there exist an edge e ∈ E($\overline{W_n}$), such that $\gamma_{cr}(W_n \cup e) = \gamma_{cr}(W_n)$, which implies that $W_n \in UEA$.

(vi) for any Wheel graph W_n is in V^0 and V^+, that is V(W_n) = $V^0 \cup V^+$.

(vii) for any Wheel graph W_n is in E^0 and E^+, that is E(W_n) = $E^0 \cup E^+$.

Proof:

Let *G* = (V, E) be the wheel graph W_n with the vertex set V ={$v_1, v_2, v_3, \ldots, v_{n-1}, v_n$ } and the edge set as E = {v_1v_i : $2 \leq i \leq n$} ∪ {v_iv_{i+1}: $2 \leq i \leq (n-1)$} ∪ {v_nv_2}. The total number of vertices in W_n is n and the number of edges is 2(n-1). We know that the connected regular domination of W_n is 1.

(i) Let v be a vertex in W_n with maximum degree n-1. In such a way that v is adjacent to all the remaining vertices in W_n. Clearly $\gamma_{cr}(W_n)$ = 1 which is the maximum degree vertex. Thus by removing the vertex v from W_n, the connected regular domination number will increase by n-2. That is $\gamma_{cr}(W_n - v) = n - 1 > \gamma_{cr}(W_n)$. Thus $W_n \in CVR$.

(ii) Let M = {$u_1, u_2, \ldots, u_{(n-)}$ } be the set of all minimum degree vertices. Let us consider, any three consecutive vertices u_i , u_j , u_k in M, for $1 \leq i < j < k \leq n-1$. Clearly the vertices u_j is adjacent with the vertices u_i and u_k. Using case (i), there exist a maximum degree vertex v which is adjacent to all the vertices in the vertex set M. After the removal of the vertex u_j the vertices u_i and u_k are again adjacent to the maximum degree v. Thus by the removal of the vertex u_j, the connected regular domination number never changed. Therefore $W_n \in UVR$.

(iii) There exists an edge e which is adjacent to the maximum degree vertex v to any one of the vertices from the vertex set M. The removal of an edge e from W_n will increase the connected regular domination number by 1. Thus $W_n \in CER$.

(iv) Let e be an edge, which is incident to any adjacent vertices in M. The removal of such an edge does not affects the connected regular domination number of W_n . Thus $W_n \in UER$.

(v) Let e be an edge in E($\overline{W_n}$). We added the edge e between any two non- adjacent vertices in M. Clearly the addition of new edge does not affects the connected regular domination number because by using case (i) there exists an maximum degree vertex v which is adjacent to all the vertices in M. Thus $W_n \in UEA$

(vi) Using case (i) and (ii), we have $W_n \in CVR$ and $W_n \in UVR$. Thus the removal of the vertex v from *V* (W_n) will either increase the connected regular domination number by n-2 or it will not affect the connected regular domination number. Clearly all the vertices in V (W_n) lies on either v^+, if the connected regular domination number increases or V^0, if the connected regular domination number has no change. Therefore V(W_n) = $V^0 \cup V^+$.

(vii) Using case (iii) and (iv), we have $W_n \in CER$ and $W_n \in UER$. Thus the removal of an edge from E (W_n) will either increase the connected regular domination number by 1 or it will not affect the connected regular domination number. Clearly all the edges in E (W_n) lies either on E^+, if the connected regular domination increases or E^0, if the connected regular domination has no changes. Thus E(W_n) = $E^0 \cup E^+$.

Definition 2.3. Helm Graph

A **Helm graph**, H_n of order n is a graph that obtained from a wheel graph W_n by attaching a pendent edge at each vertex of the n cycle of the wheel. The number of vertices in H_n is 2n -1 and the number of edges is 3(n- 1).

Theorem 2.4

For any Helm graph H_n, we have the following

(i) there exist a vertex $v \in V(H_n)$ such that $\gamma_{cr}(H_n - v) = \gamma_{cr}(H_n)$, which implies that $H_n \in UVR$.

(ii) there exist an edge e ∈ E(H_n), such that $\gamma_{cr}(H_n - e) > \gamma_{cr}(H_n)$, which implies that $H_n \in CER$.

(iii) there exist an edge e ∈ E(H_n), such that $\gamma_{cr}(H_n - e) = \gamma_{cr}(H_n)$, which implies that $H_n \in UER$.

(iv) there exist an edge e ∈ E($\overline{H_n}$), such that $\gamma_{cr}(H_n \cup e) = \gamma_{cr}(H_n)$, which implies that $H_n \in UEA$.

(v) for any Helm graph H_n is in V^0, that is V^0 is a proper subset of V.

(vi) for any Helm graph H_n is in E^0 and E^+, that is E^0 and E^+ are the proper subsets of E.

Proof:

Let H_n be the Helm graph with the vertex set V ={$v_1,v_2,v_3, \ldots, v_{n-1},v_n,u_1,u_2,u_3, \ldots, u_{n-1}$} and the edge set as E = {v_1v_i: $2 \le i \le n$} ∪ {v_iv_{i+1},v_iu_{i-1}: $2 \le i \le (n-1)$} ∪ {v_nv_2}. The number of vertices of H_n is 2n-1 and the number of edges are 3(n-1).

(i) By the definition of Helm graph, the inner circuit has n-1 vertices, which must contain (n-1)pendent vertices. Thus the inner circuit dominates all thevertices in Helm graph. Also these vertices are connected and regular.

Sub case(a) Let v be any vertex from the vertex subsets {$v_2,v_3,\ldots,v_n$} which is located in the inner circuits of the Helm graph. The removal of this vertex v makes the Helm graph disconnected. So the connected regular domination does not exist.

Sub case(b) The removal of the vertex v = v_1, which is in the centre of the Helm graph and adjacent to the inner circuit, the connected regular domination number does not change. That is$\gamma_{cr}(H_n - v) = \gamma_{cr}(H_n)$.

Sub case(c) The removal of any vertices from the n-1 pendent vertices v = {$u_1,u_2,\ldots,u_{n-1}$} which are located on the outer circuits of the Helm graph, the connected regular domination number does not change from the original one.That is $\gamma_{cr}(H_n - v) = \gamma_{cr}(H_n)$.

From (b) and (c), we have $H_n \in UVR$.

(ii) From the definition of the Helm graph, it contains 3(n-1) edges. As the Helm graph has three layers, the removal of an edge must produce the following changes.

Sub case (a) The removal of any edge between the inner circuit of the Helm graph that is between the vertices {$v_2,v_3,\ldots,v_n$}, the connected regular domination number becomes n.

That is $\gamma_{cr}(H_n - e) = n > n-1 = \gamma_{cr}(H_n)$. Thus the connected regular domination number increased by 1, which implies $H_n \in CER$.
Sub case (b) The removal of any edge from the n-1 pendent vertices, the Helm graph must contain isolated vertex. Thus the connected regular domination does not exist for (H_n - e), where e is any edge from any pendent vertices.

(iii) The removal of the edge from the maximum degree vertex, which is from v1 to any other vertices in the inner circuit of the Helm graph, the connected regular domination number never changes. That is $\gamma_{cr}(H_n - e) = \gamma_{cr}(H_n)$. Thus $H_n \in UER$.

(iv) From the definition of the Helm graph, it contains n-1 pendent vertices. Let e be an edge in E($\overline{H_n}$). Add a new edge in H_n which are not incident in H_n . The addition of such an edge does not change the connected regular domination number of the Helm graph. That is $\gamma_{cr}(H_n \cup e) = \gamma_{cr}(H_n)$. Thus $H_n \in UEA$.

(v) By using (i), we obtain that there exists some vertex such that the removal of such vertex does not change the connected regular domination number of the Helm graph, that is $\gamma_{cr}(H_n - v) = \gamma_{cr}(H_n)$ implies that v ∈ V^0. Also by using the Sub case (a) of (i) there exist some vertices, such that by the removal of the vertex, the connected regular domination number does not occur. Thus $V^0$is the proper subset of V (H_n).

(vi) By using Sub case (a) and (b) of (i), we have $H_n \in UVR$ and $H_n \in CVR$, that is the removal of an edge does not change the connected regular domination number of a Helm graph. Thus e ∈ E^0. Also the removal of a certain edges must increase the connected regular domination number by 1.Thus e ∈ E^+. Also there exists some edges, the removal of such edges must affect the existence of the connected regular domination number. Thus we have
E^0 and E^+ as the proper subset of E(H_n).

Definition 2.5. Flower graph

A Flower graph, Fn is a graph that obtained from a helm graph H_n by joining each pendent vertex to the central vertex of the helm. The number of vertices in F_n is 2n-1 and the number of edges is 4(n-1).

Theorem 2.6.

For any Flower graph F_n , we have the following
(i) there exist a vertex $v \in V(F_n)$ such that $\gamma_{cr}(F_n - v) > \gamma_{cr}(F_n)$, which implies that
$F_n \in CVR$.
(ii) there exist a vertex $v \in V(F_n)$ such that $\gamma_{cr}(F_n - v) = \gamma_{cr}(F_n)$, which implies that
$F_n \in UVR$.
(iii) there exist an edge e ∈ E(F_n), such that $\gamma_{cr}(F_n - e) > \gamma_{cr}(F_n)$, which implies that
$F_n \in CER$.
(iv) there exist an edge e ∈ E(F_n), such that $\gamma_{cr}(F_n - e) = \gamma_{cr}(F_n)$, which implies that
$F_n \in UER$.
(v) there exist an edge e ∈ E($\overline{F_n}$), such that $\gamma_{cr}(F_n \cup e) = \gamma_{cr}(F_n)$, which implies that
$F_n \in UEA$.
(vi) for any Flower graph F_n is in V^0 and V^+, that is V(F_n) = $V^0 \cup V^+$.
(vii) for any Flower graph F_n is in E^0 and E^+, that is E(F_n) = $E^0 \cup E^+$.

Proof:

Let F_n be the Flower graph with the vertex set V ={$v_1, v_2, v_3,\ldots, v_{n-1}, v_n, u_1, u_2,\ldots, u_{n-1}$} and the edge set as E = {v_1v_i : $2 \le i \le n$} ∪ {v_iv_{i+1}, v_iu_{i-1}: $2 \le i \le (n-1)$} ∪ {v_nv_2} ∪ {v_1u_i : $1 \le i \le (n-1)$}. The total number of vertices of F_n is 2n-1. The vertex set can be partitioned into three vertex subsets $V_1=v_1$,

$V_2=\{v_2,v_3,\ldots,v_n\}$ and $V_3=\{u_1,u_2,\ldots,u_{n-1}\}$. Let the maximum degree vertex be $v_1$lies in the centre,V_2 lies in the inner circuit and the vertex subset V_3 lies in the outer circuit.

i. Let v = v_1 be the maximum degree vertex, which is adjacent to all the remaining vertices of F_n. Thus v acts as a connected regular domination for F_n. By removing the vertex v from F_n , the connected regular domination number increases by n. That is$\gamma_{cr}(F_n - v) = n - 1 > 1 = \gamma_{cr}(F_n)$. Thus by removing the maximum degree vertex v from F_n, $\gamma_{cr}(F_n - v) > \gamma_{cr}(F_n)$, which implies that $F_n \in CVR$.

ii. Let v be any one of the vertex from V_2 or V_3. As the maximum degree vertex v_1 is adjacent to all the vertices in F_n, the removal of any other vertex from F_n does not change the connected regular domination number of F_n. That is $\gamma_{cr}(F_n - v) = \gamma_{cr}(F_n)$, which implies that $F_n \in UVR$.

iii. Let e be any edge from the vertex subset V_1 to any one of the vertex subsets V_2 or V_3. As v_1 is adjacent to all the vertices of both V_2 and V_3, the removal of any edges from V_1 must increase the connected regular domination number by 1. That is $\gamma_{cr}(F_n - e) > \gamma_{cr}(F_n)$. Thus $F_n \in CER$.

iv. Let e be any edge in the vertex subset V_2 or any edge connecting the vertices of V_2 and V_3. The removal of edge e does not change the connected regular domination of F_n. That is $\gamma_{cr}(F_n - e) = \gamma_{cr}(F_n)$. Thus $F_n \in UER$.

v. Let e be an edge in E($\overline{F_n}$). By adding a new edge between any non -incident vertices does not change the connected regular domination number. That is $\gamma_{cr}(F_n \cup e) = \gamma_{cr}(F_n)$. Thus $F_n \in UEA$.

vi. rom (i) $F_n \in CVR$, that is the removal of the maximum degree vertex v from V (F_n), increase the connected regular domination. That is $\gamma_{cr}(F_n - v) > \gamma_{cr}(F_n)$, implies that v ∈ V^+. From (ii) $F_n \in UVR$, that is the removal of a vertex v does not change the connected regular domination number of F_n. That is $\gamma_{cr}(F_n - v) = \gamma_{cr}(F_n)$, implies that v ∈ V^0. Thus we have $V(F_n) = V^0 \cup V^+$.

vii. From (iii) $F_n \in CER$, that is $\gamma_{cr}(F_n - e) > \gamma_{cr}(F_n)$, implies that e ∈ E^+. From (iv) $F_n \in UER$, that is $\gamma_{cr}(F_n - e) = \gamma_{cr}(F_n)$, implies that e ∈ E^0. Thus we have $E(F_n) = E^0 \cup E^+$.

Definition 2.7. Friendship graph

A Friendship graph, is the planar undirected graph with (2n + 1) vertices and 3n edges. It can be constructed by joining n copies of the cycle graph C_3 with a common vertex. Let us denote it by Fr_n. The number of vertices in Fr_n is 2n + 1 and the number of edges is 3n.

Theorem 2.8

For any Friendship graph Fr_n , we have the following

(i) there exist a vertex $v \in V(Fr_n)$ such that $\gamma_{cr}(Fr_n - v) = \gamma_{cr}(Fr_n)$, which implies that $Fr_n \in UVR$.

(ii) there exist an edge e ∈ E(Fr_n), such that $\gamma_{cr}(Fr_n - e) > \gamma_{cr}(Fr_n)$, which implies that $Fr_n \in CER$.

(iii) there exist an edge e ∈ E(Fr_n), such that $\gamma_{cr}(Fr_n - e) = \gamma_{cr}(Fr_n)$, which implies that $Fr_n \in UER$.

(iv) there exist an edge e ∈ E($\overline{Fr_n}$), such that $\gamma_{cr}(Fr_n \cup e) = \gamma_{cr}(Fr_n)$, which implies that $Fr_n \in UEA$.

(v) for any Friendship graph Fr_n is in V^0, that is V^0 is a proper subset of V.

(vi) for any Friendship graph Fr_n is in E^0 and E^+, such that $E(Fr_n)= E^0 \cup E^+$.

Definition 2.9. Lollipop graph

The Lollipop graph $L_{m,n}$, is the graph obtained by joining a complete graph K_m to a path P_n with a bridge.

Theorem 2.10

For any Lollipop graph $L_{m,2}$, we have the following

(i) there exist a vertex $v \in V(L_{m,2})$ such that $\gamma_{cr}(L_{m,2} - v) < \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in CVR$.

(ii) there exist a vertex $v \in V(L_{m,2})$ such that $\gamma_{cr}(L_{m,2} - v) = \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in UVR$.

(iii) there exist an edge $e \in E(L_{m,2})$, such that $\gamma_{cr}(L_{m,2} - e) > \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in CER$.

(iv) there exist an edge $e \in E(L_{m,2})$, such that $\gamma_{cr}(L_{m,2} - e) = \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in UER$.

(v) there exist an edge $e \in E(\overline{L_{m,2}})$, such that $\gamma_{cr}(L_{m,2} \cup e) < \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in CEA$.

(vi) there exist an edge $e \in E(\overline{L_{m,2}})$, such that $\gamma_{cr}(L_{m,2} \cup e) = \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in UEA$.

(vi) for any Lollipop graph $L_{m,2}$is in V^0 and V^+, that is $V(L_{m,2}) = V^0 \cup V^+$.

(vii) for any Lollipop graph $L_{m,2}$ is in E^0, such that E^0 is the proper subset of $E(L_{m,2})$.

Proof:

Let $L_{m,2}$ be the lollipop graph containing K_m complete graph connected with the path of length 2. Thus the number of vertices in $L_{m,2}$ is m+2. Let us consider the vertex from the complete graph K_m adjacent to the path vertex as v_0,the support vertex of the path as v_1 and the end vertex of the path as v_2. Clearly v_0 is the maximum degree vertex.

(i) By removing the vertex from the longest path of $L_{m,2}$ we have the following cases.

Case(a): Let v be the vertex v_0, that is the maximum degree vertex. Then the removal of this vertex v from the $L_{m,2}$ disconnect the Lollipop graph. Thus the connected regular domination does not exist.

Case(b): Let v be the vertex v_1 then the removal of this support vertex v from $V(L_{m,2})$ disconnect the longest path of $L_{m,2}$ and form an isolated vertex. Thus the connected regular domination does not exist.

Case(c): Let v be the vertex v_2 then the removal of this end vertex v from $V(L_{m,2})$ reduce the length of the longest path by 1. Thus the connected regular domination number decreased by 1. That is $\gamma_{cr}(L_{m,2} - v) < \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in CVR$.

(ii) Let v be any vertex from the complete graph K_m other than v_0 then the removal of the v does not change the length of the longest path in $L_{m,2}$. Thus the connected regular domination number does not change. That is $\gamma_{cr}(L_{m,2} - v) = \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in UVR$.

(iii) The removal of an edge produce the following changes.

Case(a): Let e be any edge in the complete graph K_m , which is incident in v_0. By the removal of an edge e from $L_{m,2}$, the connected regular domination does not exist.

Case(b): Let e be any edge on the path. The removal of an edge e, disconnects the graph $L_{m,2}$. Thus the connected regular domination does not exist.

Case(c): Let e be any edge in the complete graph K_m, which is non incident to v_0. The removal of an edge does not changes the connected regular domination number of $L_{m,2}$. That is $\gamma_{cr}(L_{m,2} - e) = \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in UER$.

(iv) Let e be an edge in E($\overline{L_{m,2}}$). By adding an edge from v_2 to v_0, the vertex v_0 becomes the maximum degree vertex, which is adjacent to all the remaining vertices in $L_{m,2}$. Thus v_0 dominates $L_{m,2}$ and it is connected and regular. Thus we have $\gamma_{cr}(L_{m,2} \cup e) < \gamma_{cr}(L_{m,2})$, where e is an edge between v_2 to v_0. Thus we have $L_{m,2} \in$ CEA.

(v) Let e be an edge in E($\overline{L_{m,2}}$). By adding an edge from any vertices of the complete graph K_m other than v_0 to any one of the vertex v1 or v_2. The connected regular domination number does not change. That is $\gamma_{cr}(L_{m,2} \cup e) < \gamma_{cr}(L_{m,2})$, which implies that $L_{m,2} \in$ UEA.

(vi) By using (ii) $L_{m,2} \in$ UVR, that is the removal of the vertex v does not change the connected regular domination number of $L_{m,2}$. That is $\gamma_{cr}(L_{m,2} - v) = \gamma_{cr}(L_{m,2})$, thus $v \in V^0$.
By using (i) $L_{m,2} \in$ CVR, that is the removal of the vertex v increase the connected regular domination number of $L_{m,2}$ to1. That is $\gamma_{cr}(L_{m,2} - v) = \gamma_{cr}(L_{m,2})$, thus $v \in V^-$.
Also from case(a) and case(b) of (i), there exist some vertices, whose removal, affects the existence of the connected regular domination of $L_{m,2}$. Thus we have $V^0 \cup V^-$ is the proper subset of V($L_{m,2}$) .

(vii) By using case(c) of (iii) $L_{m,2} \in$ UER , that is the removal of an edge from $L_{m,2}$ does not change the connected regular domination number of $L_{m,2}$. That is $\gamma_{cr}(L_{m,2} - e) = \gamma_{cr}(L_{m,2})$, thus $e \in E^0$. By using case (a) and case(b) of (iii) there are some edges, whose removal affects the existence of the connected regular domination of $L_{m,2}$. Thus E^0 is the proper subset of E($L_{m,2}$).

Definition 2.11. Prism graph

In the mathematical field of graph theory a Prism graph, is a graph that has one of the prisms as its skeleton.

Prism graphs are examples of generalized Petersen graphs with parameters GP (n,1).

Theorem 2.12

For any Prism graph of *GP*(n,1), we have the following

(i) there exist a vertex $v \in$ V($GP(n,1)$), such that $\gamma_{cr}(GP(n,1) - v) = \gamma_{cr}(GP(n,1))$, which implies that $GP(n,1) \in UVR$

(ii) there exist an edge e ∈ E($GP(n,1)$), such that $\gamma_{cr}(GP(n,1) - e) > \gamma_{cr}(GP(n,1))$, which implies that $GP(n,1) \in CER$.

(iii) there exist an edge e ∈ E($GP(n,1)$), such that $\gamma_{cr}(GP(n,1) - e) = \gamma_{cr}(GP(n,1))$, which implies that $GP(n,1) \in UER$.

(iv) there exist an edge e ∈ E($\overline{GP(n,1)}$), such that $\gamma_{cr}(GP(n,1) \cup e) = \gamma_{cr}(GP(n,1))$, which implies that $GP(n,1) \in UEA$.

(v) for any Prism graph $GP(n,1)$ is in V^0, that is V^0 is the proper subset of V($GP(n,1)$).

(vi) for any Prism graph $GP(n,1)$ is in E^0 and E^+, such that E($GP(n,1)$) = $E^0 \cup E^+$.

Proof:

For any Prism graph GP(n,1) containing two layers namely outer and inner connected by an edge. Inner layer contains n vertices and the outer layer contains n vertices. Thus the total number of GP(n,1) is 2n. The vertex set of GP(n,1) can be partitioned into two vertex subsets V_1 and V_2, where V_1 containing vertices in the outer layer and V_2 containing vertices in the inner layer.

(i) Let v be any vertex from the vertex subsets V_1 and V_2. The removal of the vertex v from GP (n,1) does not change the connected regular domination number of GP(n,1). That is $\gamma_{cr}(GP(n,1) - v) = \gamma_{cr}(GP(n,1))$. Thus we have GP(n,1) ∈ UVR.

(ii) Let e be any edge from E(GP(n,1)), which is incident between$V_1$and V_2. The removal of an edge e from GP(n,1), increase the connected regular domination number by 2. That is $\gamma_{cr}(GP(n,1) - e) > \gamma_{cr}(GP(n,1))$. Thus we have GP(n,1) ∈ CER .

(iii) Let e be any edge from E(GP (n,1)) , in such a way that both the ends are either in V_1 or in V_2. The removal of an edge e from GP(n,1) does not change the connected regular domination number of GP(n,1). That is $\gamma_{cr}(GP(n,1)-e) > \gamma_{cr}(GP(n,1))$. Thus we have GP(n,1) ∈ UER .

(iv) Let e be any edge from E(GP(n,1)). By adding any non incident edges does not change the connected regular domination number of GP(n,1). That is $\gamma_{cr}(GP(n,1) \cup e) = \gamma_{cr}(GP(n,1))$. Thus we have GP(n,1) ∈ UEA .

(v) By using (i), GP(n,1) ∈ UVR, that is the removal of any vertex v does not change the connected regular domination of GP(n,1). That is $\gamma_{cr}(GP(n,1)-v) = \gamma_{cr}(GP(n,1))$. Thus we have v ∈ V^0. Therefore we have V (GP(n,1)) = V^0.

(vi) By using (ii) GP(n,1) ∈ CER that is the removal of an edge will increase the connected regular domination of GP(n,1) by 2. That is $\gamma_{cr}(GP(n,1)-e) > \gamma_{cr}(GP(n,1))$. Thus we have e ∈ E^+. By using (iii) GP(n,1) ∈ UER that is the removal of an edge does not change the connected regular domination of GP(n,1). That is $\gamma_{cr}(GP(n,1)-e) = \gamma_{cr}(GP(n,1))$. Thus we have e ∈ E^0. Therefore we have E(GP(n,1)) = $E^0 \cup E^+$.

3. CONCLUSION

From our calculation, we conclude that for our standard graphs there exist atleast one vertex and atleast one edge whose removal does not affect the connected regular domination number. That is UVR and UER exists for all standard graphs of connected regular domination. Also there exist an possibility to add an new edge whose addition does not affect the connected regular domination number for our standard graphs. That is UEA exist for all standard graphs of connected regular domination.

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