Vol.5.Issue.4.2017 (Oct-Dec)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

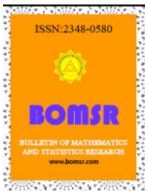
http://www.bomsr.com
Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

CONVOLUTION FOR KAMAL AND MAHGOUB TRANSFORMS

ROAA AZIZ FADHIL
General Directorate of Education Qadsiyah, Intermediate Alnwaris for Girls ,Iraq
roaaazizfadhil@gmail.com



ABSTRACT

We have to get the convolution for Kamal transform and Mahgoub transform and to be used to solve differential and integral equations.

Keywords : Kamal transform, Mahgoub transform, convolution

1. INTRODUCTION

there are integral transforms found convolution base for them mentioned some as Laplace [3] and Elzakytransforms [6,8] this paper to find the convolution for a transforms new " Kamal transform" and "Mahgoub transform" and solve integral equation and some ordinary equation by use of convolution type for both transforms.

1.1 KAMAL TRANSFORM [1]

In set A the function is defined in the form

$$A = \left\{ f: |f(t)| < pe^{\frac{|t|}{k_i}} \text{ if } t \in (-1)^i \times [0, \infty) \,, i = 1,2 \,; k_i > 0 \right\} (1)$$

For a given function in the set A the constant p must be finite number k_1, k_2 may by finite or infinite. Then the kamal transform denoted by the operator K(.) defined by the integral equation :

$$K\big(f(t)\big) = G(v) = \int_0^\infty f(t) e^{\frac{-t}{v}} dt \quad , t \geq 0 \ , \qquad k_1 \leq v \leq k_2 \qquad (2)$$

1.2 MAHGOUB TRANSFORM[5]

Mahgoub transform has same background with Kamal transform thusfrom equation (1), mahgoub integral transform denoted by the operator M(.) is defind by

$$M\big(f(t)\big) = H(v) = v \int_0^\infty f(t) e^{-vt} dt \quad , \qquad t \geq 0 \ , \quad k_1 \leq v \leq k_2 \ (3)$$

1.3 KAMAL TRANSFORM AND MAHGOUB TRANSFORM OF SOME FUNCTIONS [1,5]

S.NO.	f(t)	K[f(t)]	M[f(t)]
1	1	v	1
2	t	v^2	$\frac{1}{v}$
3	t^n	$n!\,v^{n+1}$, $n \geq 0$	$\frac{n!}{v^n}$, $n > 0$
4	e^{at}	$\frac{v}{1-av}$	$\frac{v}{v-a}$
5	Sinat	$\frac{av^2}{1+av^2}$	$\frac{av}{v^2+a^2}$
6	Cosat	$\frac{v}{1+av^2}$	$\frac{v^2}{v^2+a^2}$
7	Sinhat	$\frac{av^2}{1-av^2}$	$\frac{av}{v^2-a^2}$
8	Coshat	$\frac{v}{1-av^2}$	$\frac{v^2}{v^2-a^2}$

1.4 KAMAL TRANSFORM AND MAHGOUB TRANSFORM FOR DERIVATIVES[1,6]

Let $K[f(t)] = G(v)$ and $M[f(t)] = H(v)$ then

Kamal transform	MAhgoub transform
$K[f'(t)] = \frac{1}{v}G(v) - f(o)$	$M[f'(t)] = vH(v) - vf(o)$
$K[f''(t)] = \frac{1}{v^2}G(v) - \frac{1}{v}f(o) - f'(0)$	$M[f''(t)] = v^2H(v) - vf'(0) - v^2f(o)$
$K[f^n(t)] = \frac{1}{v^n}G(v) - \sum_{k=0}^{n-1} v^{k-n+1}f^k(0)$	$M[f^n(t)] = v^nH(v) - \sum_{k=0}^{n-1} v^{n-k}f^k(0)$

2. CONVOLUTION

Definition(1) :[7]the convolution $f * g$ of two functions f and g defined by

$$(f * g)_{(t)} = \int_0^t f(T)g(t-T)dT$$

Theorem(1): Convolution theorem for Kamal Transform

If $K(f)$ and $K(g)$ are the Kamal transform of f andgrespectively then

$$K(f * g) = K(f)K(g)$$

Proof : let

$$K(f) = \int_0^\infty e^{\frac{-q}{v}} f(q)dq \qquad \text{and} \qquad K(g) = \int_0^\infty e^{\frac{-T}{v}} g(T)dT$$

$$K(f)K(g) = \int_0^\infty e^{\frac{-q}{v}} f(q)\, dq \int_0^\infty e^{\frac{-T}{v}} g(T)dT$$

Let us put $t = q + T$, $T = t - q$ we get

$$K(f)K(g) = \int_0^\infty e^{\frac{-q}{v}} f(q) \int_T^\infty e^{\frac{-(t-q)}{v}} g(t-q)dqdt = \int_0^\infty e^{\frac{-q}{v}} f(q)e^{\frac{q}{v}} \int_T^\infty e^{\frac{-t}{v}} g(t-q)dqdt$$

$$= \int_0^\infty f(q) \int_T^\infty e^{\frac{-t}{v}} g(t-q)dqdt = \int_0^\infty e^{\frac{-t}{v}} \int_0^t f(q)g(t-q)dqdt =$$

$$\int_0^\infty e^{\frac{-t}{v}} (f * g)dt = K(f * g)$$

Theorem(2): Convolution theorem for Mahgoub Transform

If $M(f)$ and $M(g)$ are theMahgoub transform of f and grespectively then

$$M(f * g) = \frac{1}{v} M(f)M(g)$$

Proof:

Let

$$M(f) = v\int_0^\infty e^{-vp} f(p)dp \quad \text{and} \quad M(g) = v\int_0^\infty e^{-vT} g(T)dT$$

$$M(f)M(g) = v^2 \int_0^\infty e^{-vp} f(p)dp \int_0^\infty e^{-vT} g(T)dT$$

Let us put $t = p + T$, $T = t - p$ we get

$$M(f)M(g) = v^2 \int_0^\infty e^{-vp} f(p) \int_T^\infty e^{-v(t-p)} g(t-p)dpdt$$

$$= v^2 \int_0^\infty e^{-vp} f(p)e^{vp} \int_T^\infty e^{-vt} g(t-p)dpdt = v^2 \int_0^\infty f(p) \int_T^\infty e^{-vt} g(t-p)dpdt$$

$$= v^2 \int_0^\infty e^{-vt} \int_0^t f(p)g(t-p)dpdt = v\int_0^\infty e^{-vt} (f * g)dt = vM(f * g)$$

Example 1 : $f(t) = e^{nt}$, $g(t) = e^{mt}$ and $n \neq m$

verify that 1) $K(f * g) = K(f)K(g)$,2) $M(f * g) = 1/v\, M(f)M(g)$

solution : 1) the first find

$$(f * g)_{(t)} = \int_0^t e^{nT} e^{m(t-T)} dT = e^{mt} \int_0^t e^{(n-m)T} dT = \frac{e^n - e^m}{(n-m)} \quad (4)$$

And

$$K(e^{nt}) = \frac{v}{1-nv} \quad , \quad K(e^{mt}) = \frac{v}{1-mv}$$

$$K(f * g) = \frac{1}{(n-m)}\left[\frac{v}{1-nv} - \frac{v}{1-mv}\right] = \frac{v}{(n-m)}\left[\frac{nv-mv}{(1-nv)(1-mv)}\right]$$

$$= \frac{v}{(1-nv)} \frac{v}{(1-mv)} = K(e^{nt})K(e^{mt})$$

2) from equation (4) and

$$M(e^{nt}) = \frac{v}{v-n}, \qquad M(e^{mt}) = \frac{v}{v-m}$$

$$vM(f*g) = \frac{v}{(n-m)}\left[\frac{v}{v-n} - \frac{v}{v-m}\right] = \frac{v^2}{(n-m)}\left[\frac{v-m-v+n}{(v-n)(v-m)}\right]$$

$$= \frac{v^2}{(n-m)}\left[\frac{(n-m)}{(v-n)(v-m)}\right] = \frac{v}{(v-n)}\frac{v}{(v-m)} = M(e^{nt})M(e^{mt})$$

Example 2:To find the solution of the differential equation

$$y'' + y' - 2y = e^{2t} \quad , \quad y(0) = 0 \quad , \quad y'(0) = 0 \qquad (5)$$

by a) the convolution theorem Kamal transform b) the convolution theorem Mahgoub transform solution : a) Take Kamal transform of equation (5) we have

$$v^{-2}K(y) - v^{-1}y(0) - y'(0) + v^{-1}K(y) - y(0) - 2K(y) = \frac{v}{1-2v}$$

$$(v^{-2} + v^{-1} - 2)K(y) = \frac{v}{1-2v}$$

$$K(y) = \frac{v}{1-2v}\cdot\frac{v^2}{(1+2v)(1-v)}$$

Take invers to this Equation and the convolution theorem Kamal transform we get the solution of equation (5)

$$y = K^{-1}\left(\frac{v}{1-2v}\right)K^{-1}\left(\frac{v}{1+2v}\right)K^{-1}\left(\frac{v}{1-v}\right) = e^{2t}e^{-2t}e^{t} = e^{t}$$

b) Take Mahgoub transform of equation (5) we have

$$v^2H(v) - vy'(0) - v^2y(0) + vH(v) - vy(0) - 2H(v) = \frac{v}{v-2}$$

$$(v^2 + v - 2)H(v) = \frac{v}{v-2}$$

$$H(v) = \frac{v}{v-2}\cdot\frac{v^2}{(v+2)(v-1)}$$

Take invers and the convolution theorem Mahgoub transform we get

$$y = H^{-1}\left(\frac{v}{v-2}\right)H^{-1}\left(\frac{v}{v+2}\right)H^{-1}\left(\frac{v}{v-1}\right) = e^{2t}e^{-2t}e^{t} = e^{t}$$

this solution of equation (5).

Example 3 : Solve the integral equation

$$g(t) = \int_0^t \cos mT \sin n(t-T)\,dT \qquad |n| \neq |m| \qquad (6)$$

by implied a) theorem (1) b) theorem (2)

Solution : a) At first write the equation (6) in the form

$$g = \cos mt \ \sin nt \qquad (7)$$

And applying theorem convolution Kamal transform we get

$$y = \frac{v}{1+m^2v^2}\cdot\frac{nv^2}{1+n^2v^2}$$

$y = K(g)$, now by a partial fraction method we get

$$y = \frac{n}{n^2-m^2}\left[\frac{v}{1+m^2v^2} - \frac{v}{1+n^2v^2}\right]$$

And take invers Kamal transform

$$g(t) = \frac{n}{n^2-m^2}\left[\cos(mv) - \cos(nv)\right]$$

b) from equation (7) and applying theorem (2) we have

$$\frac{1}{v}J = \frac{av}{v^2+m^2}\cdot\frac{v^2}{v^2+n^2}$$

$J = M(g)$, now by a partial fraction method we get

$$J = \frac{n}{n^2-m^2}\left[\frac{v^2}{v^2+m^2}-\frac{v^2}{v^2+n^2}\right]$$

And take invers Mahgoub transformwe have solution of equation (6)

$$g(t) = \frac{n}{n^2-m^2}[\cos(mv) - \sin(nv)]$$

Example 4: SolveVolterraintegral equation of the second kindby applied a) theorem (1) b) theorem (2)

$$f(t) - \int_0^t f(T)\sin(t-T)dT = t \tag{8}$$

a) nowthis equation can be written

$$q - q\sin(t) = t \tag{9}$$

by theorem (1) and writing $G(v) = K(q)$and

$$K(\sin(t)) = \frac{v^2}{1+v^2} \quad , \quad k(t) = v^2$$

we get

$$G(v) - G(v)\frac{v^2}{1+v^2} = v^2$$

thus

$$G(v) = v^2(1+v^2) = v^2 + v^4$$

And take invers transform we get solution of equation (8).

$$q(t) = t + \frac{t^3}{6}$$

b) from equation (9) and by convolution theorem Mahgoub transform and writing $H(v) = M(q)$and

$$M(\sin(t)) = \frac{v}{v^2+1} \quad , \quad M(t) = \frac{1}{v}$$

we havet

$$H(v) - \frac{1}{v}H(v)\frac{v}{v^2+1} = \frac{1}{v}$$

thus

$$H(v) = v^{-3}(v^2+1) = v^{-1} + v^{-3}$$

And take invers transform we have

$$q(t) = t + \frac{t^3}{6}$$

3. Conclusion

Convolution for Kamal transform and Mahgoub transform can solve some of the differential equations and solve the integrative equations without complexity, suggested proving shifting theorem to both transforms to solve equations of other formulas.

Reference

[1]. Abdelilah.K. Hassan Sedeeg , The New Integral" Kamal Transform", Advances in Theoretical and Applied Mathematics,Vol. 11, No. 4, 2016 , pp451-458.

[2]. Herbert Kreyszig, Edward J. Norminton , “ Advanced Engineering Mathematics” Ohio State University Columbus, Ohio,Tenth edition, 2011.

[3]. Joel L.Sciff , The Laplace Transform theory and Application , springer,2010.
[4]. KimHwaJoon, The time shifting for Elzaki transform , International Journal of Pure and Applied Mathematics, 2013, Vol,87 ,No.2 ,261-271.
[5]. MohnadMahgoub AM .the new integral transform "Mahgoub transform " Advances in theoretical and Applied Mathematics. 2016,vol.11.No.4, 391-398.
[6]. S.S.Ali and M.S. Chaudhary, On a new integral transform and solution of some integral equations , International Journal of Pure and Applied Mathematics , 2011Vol.73, No.3, 299-308.
[7]. William F.Trench, "Elementary Differential Equations" Trinity University, 2013.
[8]. Yechan Song , The solution of volterra integral equation of the second kind by using the Elzakitransform,Applied Mathematical science , 2014,Vol.8 ,No,11,525-530.