Vol.6.Issue.3.2018 (July-Sept.)
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BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com
Email:editorbomsr@gmail.com

RESEARCH ARTICLE

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

DETERMINING THE TIME TO RECRUITMENT IN AN ORGANIZATION THROUGH EXPONENTIATED EXPONENTIAL BINOMIAL DISTRIBUTION

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ABSTRACT

Manpower planning plays a vital role in human resource activity of an organization. Human resource is an asset to every organization. Manpower planning mainly involves determining the needs and supply of human resource and the available sources. The organization should make Manpower Planning in such a way that it should satisfy both organization and employer at a higher level. The expected time and variance of the time to recruitment are derived from the model. The analytical results are substantiated with numerical illustrations.

KEYWARDS: Manpower planning, Expected time, Variance, Organization, Recruitment, Exponentiated exponential and binomial distribution

INTRODUCTION

Every organization operates for the purpose of achieving some goals and objective. To facilitate that, firms employ and utilize resources such as man, money, machine, and material. Manpower planning is the commencement stage in manpower management, which is made up of planning for future needs for the workers, planning for the availability of such workers, before taking steps to compare supply and demand. Inyang, (2000). By manpower planning, it first of all becomes necessary to identify specific jobs designed during the organizing process. These specific jobs have to be analyzed in order to be able to determine the right caliber of staff that can best perform the assigned duties. Taking decisions regarding manpower requirements is not restricted to the personnel department.

A new continuous distribution is introduced by compounding exponentiated exponential and binomial distributions, named as exponentiated exponential binomial (EEB) distribution. This distribution has the ability to model lifetime data with increasing, decreasing and upside-down bathtub shaped failure rates. Moreover, the zero-truncated binomial distribution used in compounding is over dispersed. The gamma distributions also contain the exponential distributions, and are constructed by taking powers of the Laplace transform. It is known that the distribution of a sum of a fixed number of independent exponentially distributed random variables is a gamma and the distribution of a minimum number of these variables is again an exponential distribution. Gupta and Kundu (1999) studied an exponentiated exponential model that was constructed by the distribution of a maximum of independent exponential random variables with a fixed sample size. Recently, Karlis (2009) and Kus (2007) considered the exponential-Poisson distribution.

This distribution offers a more flexible distribution for modeling lifetime data, namely in reliability, in terms of its hazard rate shapes, that are decreasing, increasing and upside down bathtub shaped. Mathematical model is obtained for the expected time of breakdown point to reach the threshold level through exponentiated exponential binomial distribution. One can see for more detail in Esary *et.al.,* (1973), Sathiyamoorthi (1980), Pandiyan *et.al.,* (2010), Kannadasan, *and* Vinoth (2015) about the expected time to recruit the manpower in an organization.

ASSUMPTIONS OF THE MODELS

- Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives and promotions are made.
- The exit of every person from the organization results in a random amount of depletion of manpower (in man hours).
- The process of depletion is linear and cumulative.
- The inter arrival times between successive occasions of wastage are i.i.d. random variables.
- If the total depletion exceeds a threshold level Y which is itself a random variable, the breakdown of the organization occurs. In other words recruitment becomes inevitable.
- The process, which generates the exits, the sequence of depletions and the threshold are mutually independent.

NOTATIONS

X_i : A discrete random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i[th] occasion of policy announcement, $i = 1,2,3, \dots k$and $X_i^{'}s$are i.i.d and $X_i = X$for all i.

Y : A discrete random variable denoting the threshold level having the Exponentiated Exponential Binomial Distribution.

$g(.)$: The probability mass functions (p.m.f) of X_i

$g_k(.)$: The k- fold convolution of $g(.)$i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

$g*(.)$: Laplace transform of$g(.)$;$g_k^*(.)$: Laplace transform of $\mathrm{g_k}(.)$

$h($. : The probability mass functions of random threshold level which has the Exponentiated Exponential Binomial Distribution and$H(.)$is the corresponding probability generating functions.

U : A continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: P.m.f. of random variable U with corresponding Probability generating function.

$V_k(t)$: $F_k(t) - F_{k+1}(t)$

$F_k(t)$: Probability that there are exactly *'k'* policies decisions in (0, t]

$S(.)$: The survivor function i.e. $P[T > t]$;$1 - S(t) = L(t)$

MODEL DESCRIPTION AND SOLUTION

The Cumulative Distribution Function (CDF) of the Distribution is

$$F(x) = \frac{1 - \left(1 - \theta\left(1 - e^{-\lambda x}\right)^{\alpha}\right)^{n}}{1 - \theta^{-n}}$$

$$= \frac{\theta\left(1-e^{-\lambda x}\right)^{\alpha}}{1-\theta^{-1}}$$

$$= \frac{\theta}{1-\theta^{-1}} \sum_{r=0}^{\alpha} (-1)^r \binom{\alpha}{r} e^{-\lambda r x}$$

The corresponding Survival Function is (SF)

$$\bar{H}(x) = 1 - F(x)$$

$$= 1 - \frac{\theta}{1-\theta^{-1}} \sum_{r=0}^{n} (-1)^r \binom{\alpha}{r} e^{-\lambda r x}$$

In general that the threshold Y follows Exponentiated Exponential Binomial Distribution with parameterθ. It can be shown that,

$$P(x<y) = \int_0^{\infty} g_k(x)\bar{H}(x)$$

$$= \int_0^{\infty} g_k(x)\left(1 - \frac{\theta}{1-\theta^{-1}} \sum_{r=0}^{n} (-1)^r \binom{\alpha}{r} e^{-\lambda r x}\right) dx$$

$$= \int_0^{\infty} g_k(x)dx - \frac{\theta}{1-\theta^{-1}} \int_0^{\infty} g_k(x) \sum_{r=0}^{n} (-1)^r \binom{\alpha}{r} e^{-\lambda r x}\, dx$$

$$= g_k^*(1) - \frac{\theta}{1-\theta^{-1}} \sum_{r=1}^{n} (-1)^r \binom{\alpha}{r} g_k^*(\lambda r)$$

$$= [g^*(1)]^k - \frac{\theta}{1-\theta^{-1}} \sum_{r=1}^{\alpha} (-1)^r \binom{\alpha}{r} [g^*(\lambda r)]^k$$

The survival function which gives the probability that the cumulative threshold will fail only after time t.

$\mathrm{S(t)} = \mathrm{P(T} > t) =$ Probability that the total damage survives beyond t

$$= \sum_{\mathrm{k}=0}^{\infty} \mathrm{P}\,\{\text{there are exactly k decisions in } (0,\mathrm{t}] * \mathrm{P}\ (\text{the total cumulative threshold } (0,\mathrm{t}]\}$$

The survival function $S(t)$ which is the probability that an individual survives for a time t

It is also known from renewal process that

$$P(T>t) = \sum_{k=0}^{\infty} F_k(t)P(X_i<y)$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\left[(g^*(1))^k - \frac{\theta}{1-\theta^{-1}} \sum_{r=1}^{n} (-1)^r \binom{\alpha}{r} [g^*(\lambda r)]^k\right]$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)](g^*(1))^k$$

$$- \left[\frac{\theta}{1-\theta^{-1}} \sum_{k=0}^{\infty} \sum_{r=1}^{n} (-1)^r \binom{\alpha}{r} [g^*(\lambda r)]^k\, [F_k(t) - F_{k+1}(t)]\right]$$

$$= 1 - \left(1 - g^*(1)\right) \sum_{k=1}^{\infty} F_k(t)\,(g^*(1))^{k-1}$$

$$- \sum_{r=1}^{n} (-1)^r \binom{\alpha}{r}\left[1 - \left(1 - g^*(\lambda r)\right) \sum_{k=1}^{\infty} F_k(t)\,(g^*(\lambda r))^{k-1}\right]$$

$$= 1 - \left(1 - g^*(1)\right) \sum_{k=1}^{\infty} F_k(t)\,(g^*(1))^{k-1}$$

$$- \sum_{r=0}^{n} (-1)^r \binom{\alpha}{r} \frac{\theta}{1-\theta^{-1}} + \left[\sum_{k=1}^{\infty} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{\theta}{1-\theta^{-1}} (g^*(\lambda r))^{k-1}\right]$$

$$= 1 - \left(1 - g^*(1)\right) \sum_{k=1}^{\infty} F_k(t)\,(g^*(1))^{k-1} - \frac{\theta}{1-\theta}$$

$$+ \sum_{k=1}^{\infty} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{\theta}{1-\theta^{-1}} (1 - g^*(\lambda r)) F_k(t) (g^*(\lambda r))^{k-1}$$

Now, the life time is given by

$P(T < t) = L(t) =$ the distribution function of life time (t)

Using convolution theorem for Laplace transforms, $F_0(t) = 1$ and on simplification, it can shown that,

$$= \left(1 - g^*(1)\right) \sum_{k=1}^{\infty} F_k(t)\,(g^*(1))^{k-1} + \frac{\theta}{1-\theta^{-1}}$$

$$+ \sum_{k=1}^{\infty} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{\theta}{1-\theta^{-1}} (1 - g^*(\lambda r)) F_k(t) (g^*(\lambda r))^{k-1}$$

$$= \left(1 - g^*(1)\right) \sum_{k=1}^{\infty} F_k(t)\,(g^*(1))^{k-1} + \frac{\theta}{1-\theta^{-1}}$$

$$+ \frac{\theta}{1-\theta^{-1}} \sum_{k=1}^{\infty} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} (1 - g^*(\lambda r))^{k-1} F_k(t) (g^*(\lambda r))^{k-1}$$

By taking Laplace-Stieltjes transform, it can be shown that

$$l^*(s) = \frac{\left[\left(1 - g^*(1)\right) f^*(s)\right]}{\left[1 - g^*(1) f^*(s)\right]} + \frac{\theta}{1-\theta^{-1}} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{\left[\left(1 - g^*(\lambda r)\right) f^*(s)\right]}{\left[1 - g^*(\lambda r) f^*(s)\right]}$$

Let the random variable Udenoting inter arrival time which follows exponential with parameter. Now$f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation we get,

$$= \frac{\left[\left(1 - g^*(1)\right) \frac{c}{c+s}\right]}{\left[1 - g^*(1) \frac{c}{c+s}\right]} + \frac{\theta}{1-\theta^{-1}} \sum_{r=1}^{\alpha} (-1)^{r+1} \binom{\alpha}{r} \frac{\left[\left(1 - g^*(\lambda r)\right) \frac{c}{c+s}\right]}{\left[1 - g^*(\lambda r) \frac{c}{c+s}\right]}$$

$$= \frac{\left[\left(1 - g^*(1)\right) c\right]}{\left[c + s - g^*(1) c\right]} + \frac{\theta}{1-\theta^{-1}} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{\left[\left(1 - g^*(\lambda r)\right) c\right]}{\left[c + s - g^*(\lambda r) c\right]}$$

$$E(T) = \frac{-d}{ds} l^*(s) \Big/ s = 0$$

$$= \left[\frac{1}{c[1 - g^*(1)]} + \frac{\theta}{1-\theta^{-1}} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{1}{c[1 - g^*(\lambda r)}\right]$$

$$E(T^2) = \frac{d^2}{ds^2} l^*(s)$$

$$= \frac{1}{[1 - g^*(1)]^2 c^2} + \frac{\theta}{1-\theta^{-1}} \sum_{r=1}^{n} (-1)^{r+1} \binom{\alpha}{r} \frac{1}{c^2 [1 - g^*(\lambda r)]^2}$$

$$g^*(1) \sim \frac{1}{\mu} g^*(\lambda r) \sim \frac{\mu}{\mu + 2r}$$

$$E(T)=\frac{1}{c\left[1-\frac{1}{\mu}\right]}+\frac{\theta}{1-\theta^{-1}}\sum_{r=1}^{n}(-1)^{r+1}\binom{\alpha}{r}\frac{1}{c\left[1-\frac{\mu}{\mu+\lambda r}\right]}$$

$$E(T)=\frac{\mu}{c[\mu-1]}+\frac{\theta}{1-\theta^{-1}}\sum_{r=1}^{n}(-1)^{r+1}\binom{\alpha}{r}\frac{\mu+\lambda r}{c[\lambda r]}$$

$$E(T^2)=\frac{\mu^2}{c^2[\mu-1]^2}+\frac{\theta}{1-\theta^{-1}}\sum_{r=1}^{n}(-1)^{r+1}\binom{\alpha}{r}\frac{[\mu+\lambda r]^2}{c^2[\lambda r]^2}$$

From which $V(T)$ can be obtained

$$V(T)=E(T^2)-[E(T)]^2$$

$$V(T)=\frac{\mu^2}{c^2[\mu-1]^2}+\frac{\theta}{1-\theta^{-1}}\sum_{r=1}^{n}(-1)^{r+1}\binom{\alpha}{r}\frac{[\mu+\lambda r]^2}{c^2[\lambda r]^2}-\left(\frac{\mu}{c[\mu-1]}+\frac{\theta}{1-\theta^{-1}}\sum_{r=1}^{n}(-1)^{r+1}\binom{\alpha}{r}\frac{\mu+\lambda r}{c[\lambda r]}\right)^2$$

NUMERICAL ILLUSTRATION

The theory developed was tested using stimulated data in MathCAD software. The total number of leavers in the organization was decisively rejected that the numbers come from exponentiated exponential binomial distribution.

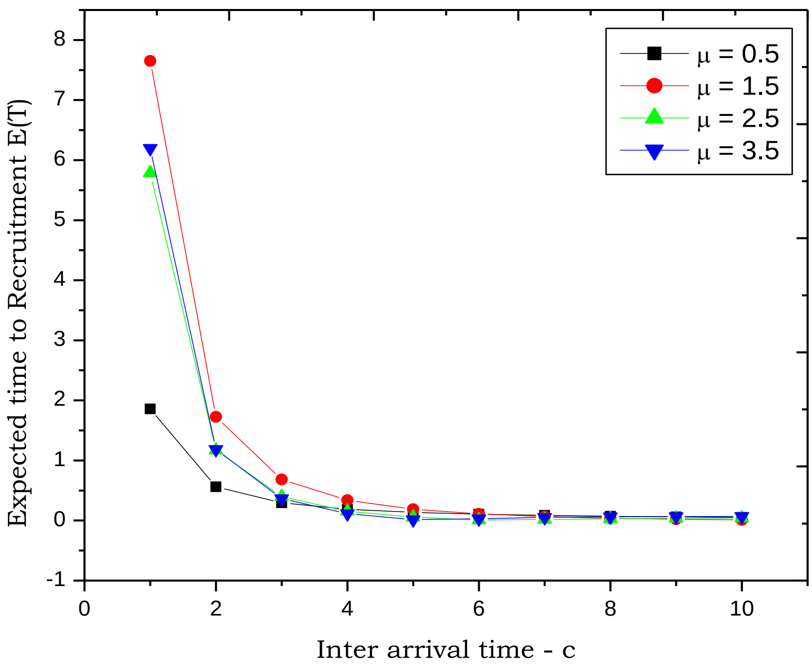

Figure - 1a

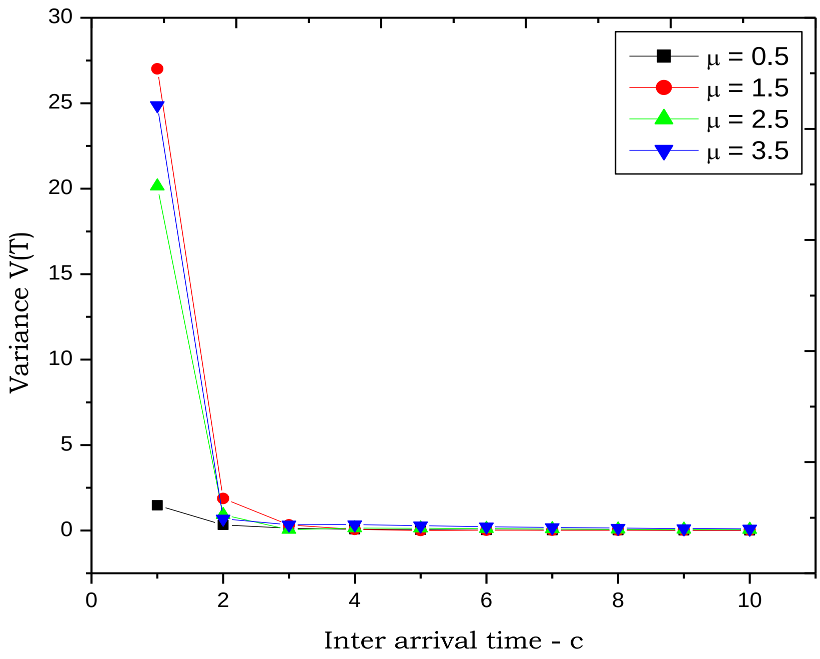

Figure –1b

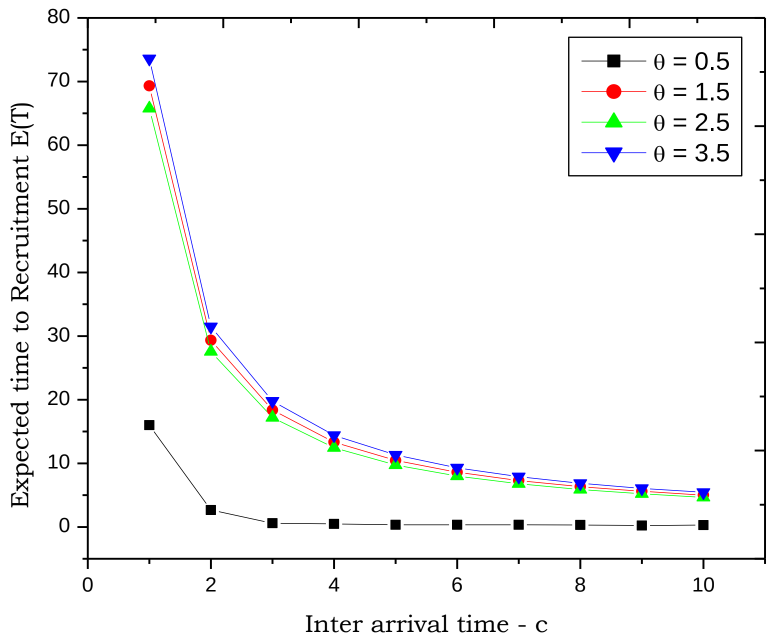

Figure - 2a

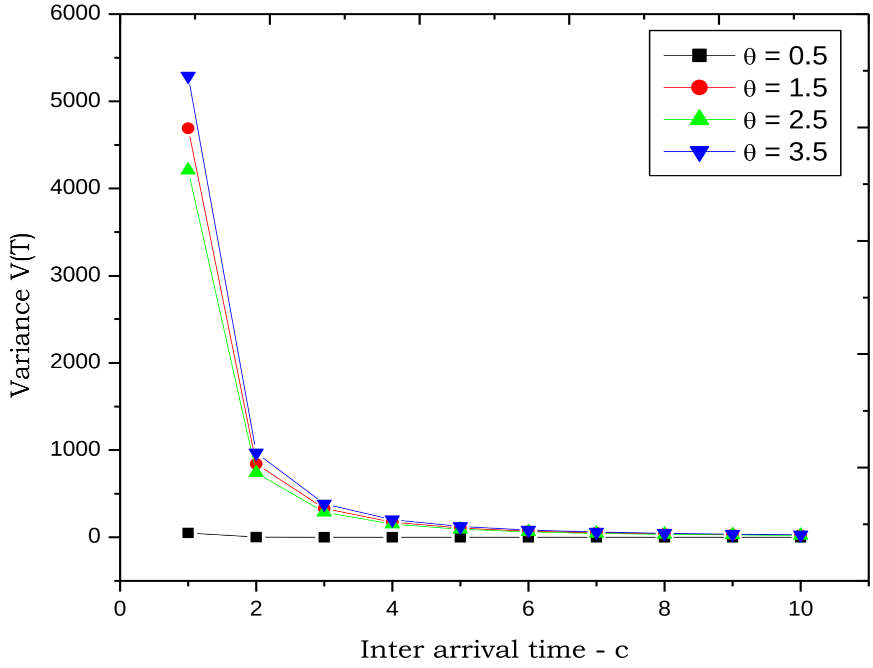

Figure –2b

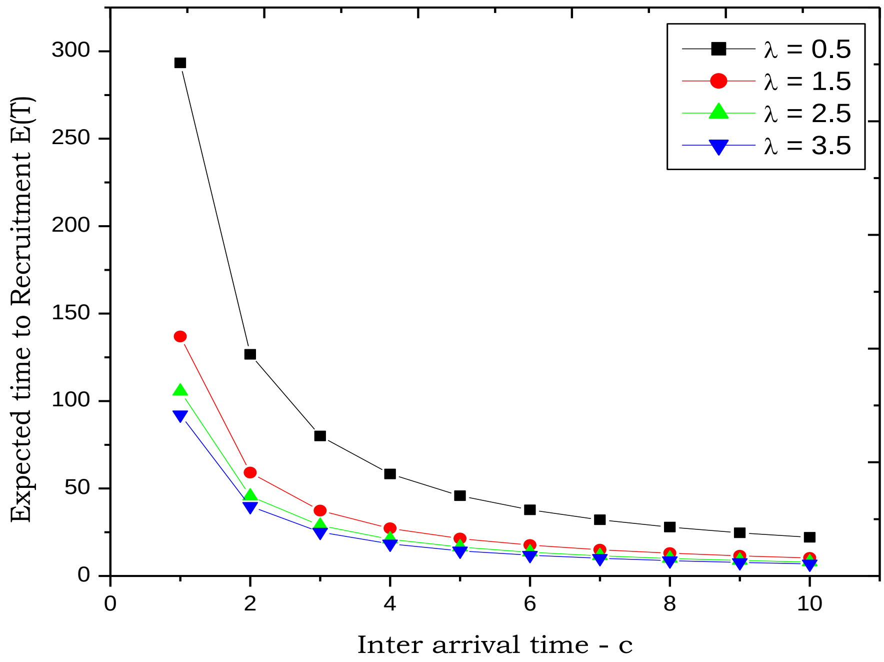

Figure –3a

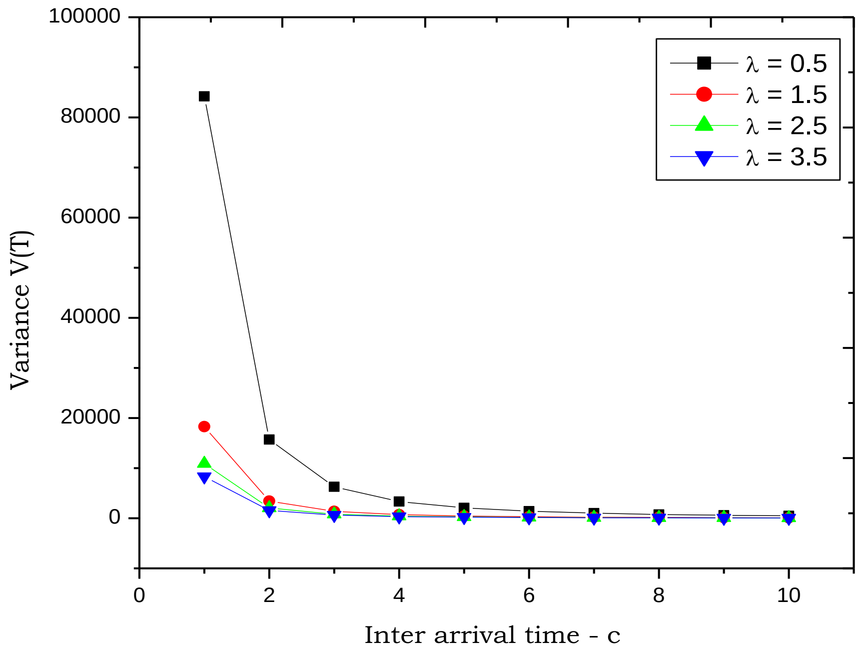

Figure – 3b

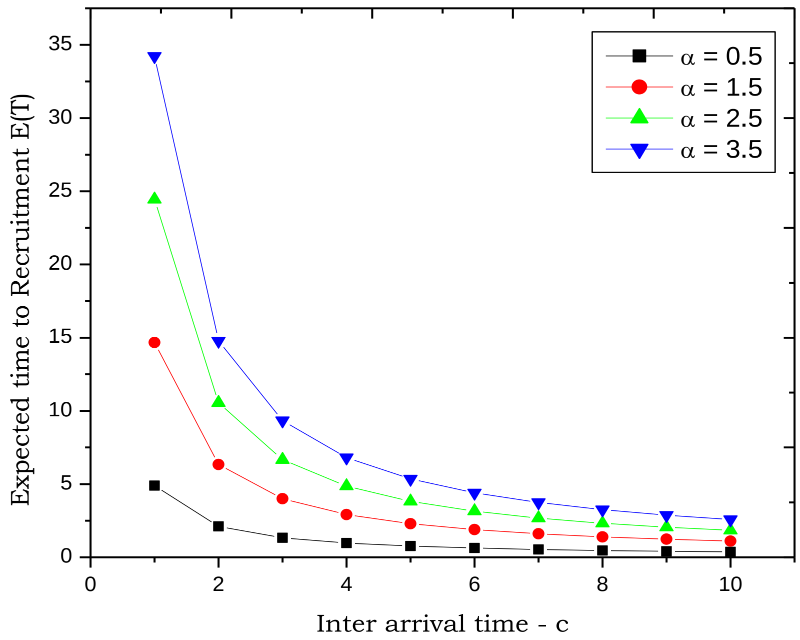

Figure –4a

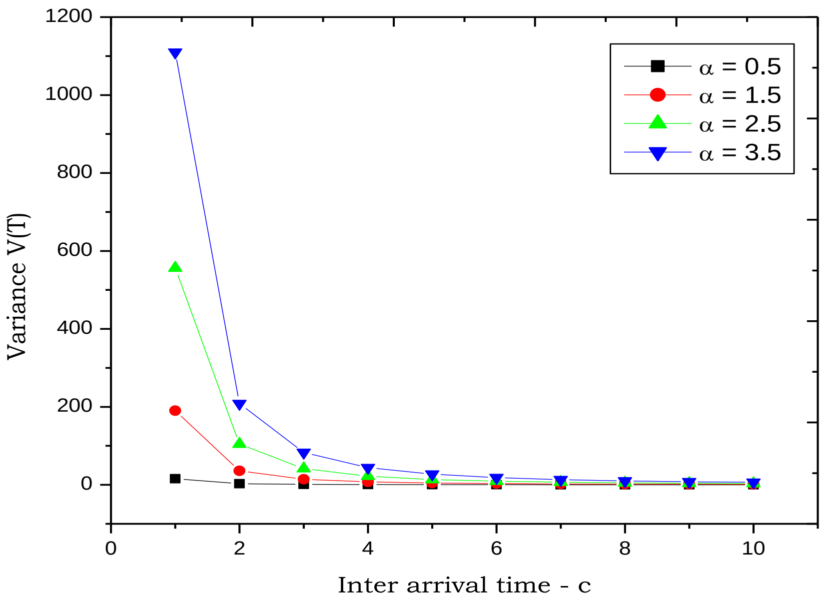

Figure – 4b

CONCLUSION

We have introduced a new generalization of the Exponentiated Exponential Binomial distribution. The four parameter Exponentiated Exponential Binomial distribution is embedded in the proposed distribution. Some mathematical properties along with estimate the expected time and variance is discussed. We believe that the subject distribution can be used in several different areas. We expect that this study will serve as a reference and help to advance future research in the subject area.

When μ is kept fixed the inter-arrival time $'c'$ which follows exponential distribution, is an increasing case by the process of time to recruitment. Therefore, the value of the expected time $E(T)$ to cross the time to recruitment is found to be decreasing, in all the cases of the parameter value $\mu = 0.5, 1.5, 2.5, 3.5$ when the value of the parameter μ increases, the expected time is also found decreasing, this is observed in Figure 1a. The same case is found in Variance $V(T)$ which is observed in Figure 1b.

When θ is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the time to recruitment is found to be decreasing, in all the cases of the parameter value$\theta = 0.5, 1.5, 2.5, 3.5$ When the value of the parameter θ increases, the expected time is found increasing, this is indicated in Figure 2a. The same case is observed in the variance $V(T)$ which is observed in Figure 2b.

When λ is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the time to recruitment is found to be decreasing, in all the cases of the parameter value$\lambda = 0.5, 1.5, 2.5, 3.5$ When the value of the parameter λ increases, the expected time is found increasing, this is indicated in Figure 3a. The same case is observed in the variance $V(T)$ which is observed in Figure 3b.

Whenα is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the time to recruitment is found to be decreasing, in all the cases of the parameter value$\alpha = 0.5, 1.5, 2.5, 3.5$ When the value of the parameter α increases, the expected time is found increasing, this is indicated in Figure 4a. The same case is observed in the variance $V(T)$ which is observed in Figure 4b.

REFERENCE

[1]. Esary, J.D., Marshall, A.W and Prochan, F. (1973). “Shock Models and Wear Process”, *Annals of probability*, Vol.1, No.4, pp.627 - 650.

[2]. Gupta, R., Kundu, D., (1999). “Generalized exponential distributions”. *Aust. N. Z. J. Stat.* Vol.41 (2), pp.173–188.

[3]. K.Kannadasan, and R.Vinoth (2015), “Calculating the Expected time to control the Recruitment When system fails”, International *Journal of Multidisciplinary Research and Modern Education*, Vol.1, No.1, pp.288-292.

[4]. Karlis, D., (2009). “A note on the exponential Poisson distribution: a nested EM algorithm”. *Comput. Statist. Data Anal*. Vol.53 (4), pp.894–899.

[5]. Kus, C., (2007). “A new lifetime distribution”. *Comput. Statist. Data Anal*. Vol.51 (9), pp.4497–4509.

[6]. P.Pandiyan, K.Kannadasan, R.Vinoth and R.Saminathan (2010), “Stochastic Model for Mean and Variance to Recruit Manpower in Organization”, *Journal of Advanced in Applied Mathematical Analysis*.Vol.5, pp.77-82.

[7]. Sathiyamoorthi, R. (1980). “Cumulative Damage model with Correlated Inter arrival Time of Shocks”, IEEE Transactions on Reliability, R-29, No.3.