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STUDY OF A NEW POWER DISTRIBUTION AS A FAILURE MODEL : ITS STATISTICAL AND MATHEMATICAL PROPERTIES

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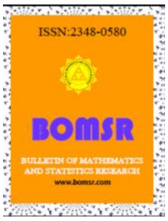


ABSTRACT

A finite range probability distribution has been developed and some of its statistical and structural properties have been discussed.Goodness of fit has been successfully tested on the Transmitter tube failure data of Davis(1952).
Keywords: Probability distribution, statistical properties, structoral properties, Goodness of fit.

Introduction

Introducing new probability distribu have been an intresting field of researchers for last many decades. Weibull (1951), Folks and Chikara (1978), Mukherji and islam (1983), Siddiqui et al (1992, 1994, 1995, 2016), Cha (2007), Madal and Yakov (2010) and Shradha et al (2017) and Syed A.A. et.al (2016).

The proposed probability distribution is also useful in many real life data so it can be added in family of probability distributions.

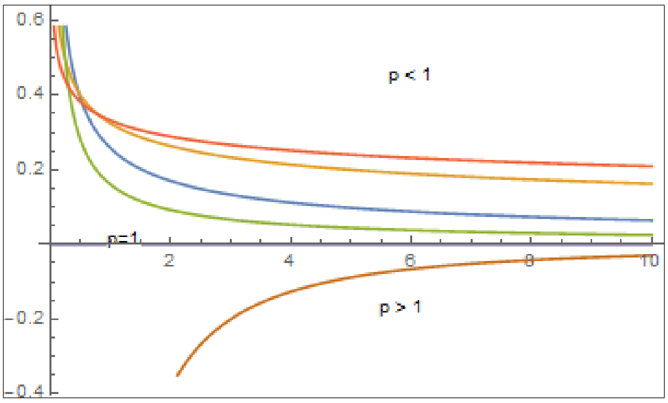
1 Proposed distribution

The probability density function of the proposed distribution is:

$$f\,(x) = (1 - p)x^{-p}\ \theta^{p-1} \tag{1}$$

here p is the shape parameter and θ is the scale parameter.

$0 < x < \theta;\ \theta > 0;\ 0 < p < 1$



Graph(1): pdf

Conditions: $p < 1 \rightarrow$ Shape is reverse J shape

$p = 1 \rightarrow$ Shape is straight line

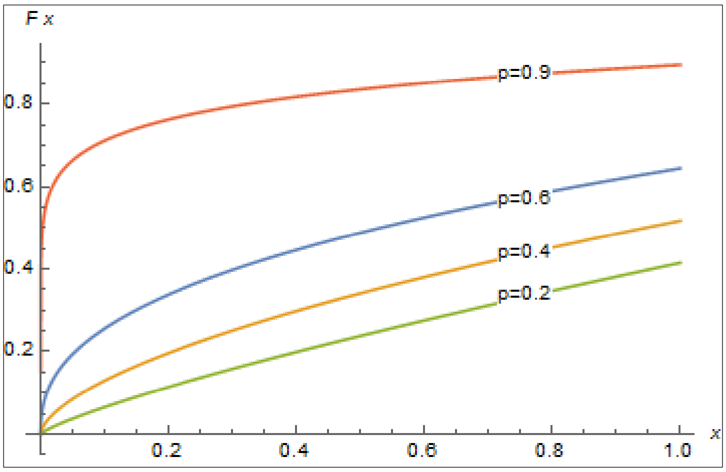
$p > 1 \rightarrow$ shape is inverse bath tub

For lesser value of parameter p proposed distribution is a growth function and for higher value of p it is treated as a decay function. Therefore it can be used for profit and loss both in financial data.

The cumulative distribution is defined as:

$$F(x) = \theta^{p-1} \quad x^{1-p} \tag{2}$$

where,
$0 < x < \theta;\ \theta > 0;\ 0 < p < 1$



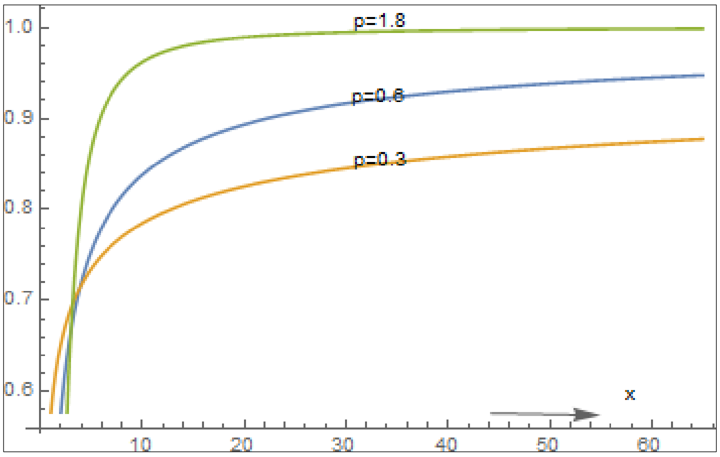
Graph(2) :CDF

1.1 Reliability Function

Reliability function of proposed model is

$R(x) = 1 - F(x) = 1 - \theta^{p-1}\ x^{1-p}$ (3)

where θ and p are scale and shape parameters respectively. at $t = 0$; eqn 3 gives $R(0) = 1$ and as time t increases reliability decreases.



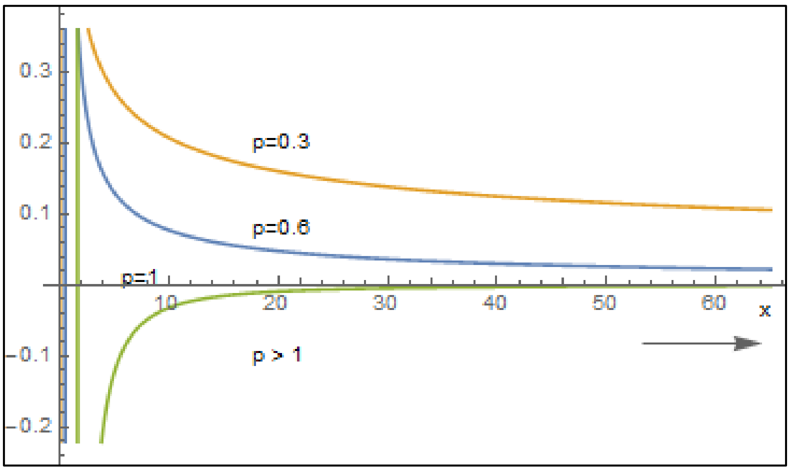
Graph(3) : *Reliability function*

1.2 Hazzard Rate Function

Hazard rate or Failure rate shows the failures of engineered system per unit of time.For the new distribution it is shown by

$$\frac{f(x)}{1-F(x)} = \frac{(1-p)x^{-p}\theta^{p-1}}{1-\theta^{p-1}x^{1-p}}$$

$$= \frac{(1-p)x^{-p}}{\theta^{1-p}x^{1-p}} \ldots\ldots\ldots\ldots\ldots\ldots..(4)$$



Graph(4) : *Hazard Rate Function*

Graph shows the given distribution has only reverse J-shaped [DFR (Burn in) < CFR (Useful life) Hazard rate it means hazard rate is monoton- ically decreasing.The reverse-J hazard rate is a better tool over hazard rate for the characterization of any failure mode.D De- sai,V.Maariapan et.al [2010].

1.3 Moment Generating function (mgf)

Moment generating function is an important statistical property of a probability distribution function as it provides the basis of an alternative route to analytical results compared to directly with pdf or cdf .

If any random variable x has new proposed distribution,then the moment generating (*mgf*) will be:

$$M_x(t) = E(e^{tx})$$

$$=\int_o^\theta (1-p)\,\theta^{p-1}x^{-p}\,dx$$

$$= (1-p)\sum_{r=0}^{\infty}\frac{t^r\theta^r}{r!\,[(r+1)-p]}$$

$$M_x(t) = (1-p)\sum_{r=0}^{\infty}\frac{t^r\theta^r}{r!\,[(r+1)-p]} \qquad (5)$$

which is the desired moment generating function.

1.4 Mean

The *Mean* μ_r a continuous random variablex of proposed proba- bility density function pdf is

$$\mu'_r = E(X) = \int_o^\theta xf(x)\,dx$$

$$\mu'_r = E(X) = \int_o^\theta (1-p)\theta^{p-1}x^{-p}\,dx$$

where, $0 < x < \theta;\ \theta > 0;\ 0 < p < 1$

$$\mu'_1 = \frac{1-p}{(r+1)-p}\theta^r \qquad (6)$$

Using above relation(6),we get first and second moment as:

$$\mu'_1 = \mathrm{E(X)} = \frac{1-p}{2-p}\theta \quad (7)$$

$$\mu'_1 = \mathrm{E(X^2)} = \frac{1-p}{3-p}\theta^2 \quad (8)$$

μ'_1 refers to the mean of the proposed distribution.

1.5 Variance

Variance the measure of spread can be find by below:

$$V_{(x)} = \mu'_2 - (\mu'_1)^2$$

$$= \mathrm{E(X^2)} - [\mathrm{E}(X)]^2$$

$$= \left[\frac{1-p}{3-p} - \left(\frac{1-p}{2-p}\right)^2\right]\theta^2$$

$$V_{(x)} = \frac{(1-p)\theta^2}{(3-p)(2-p)^2} \quad (9)$$

above equation (9) shows the variance of the proposed distribution.

1.6 Median

$$\int_o^{Me} (1-p)\,x^{-p}\,\theta^{p-1}dx = \frac{1}{2}$$

$$(x^{1-p}\ \theta^{p-1})_0^{Me} = \frac{1}{2}$$

$$(Me)^{1-p}\ \theta^{p-1} = \frac{1}{2}$$

$$(Me)^{1-p} = \frac{\frac{1}{2}}{\theta^{p-1}}$$

$$\ln(Me)^{1-p} = \ln\frac{1}{2} - ln\,\theta^{p-1}$$

$$Me = \frac{\ln\frac{1}{2} - ln\theta^{p-1}}{1-p} \qquad (10)$$

1.7 Mode

For Mode on differentiating $f\ (x)$ with respect to x ,we get

$$f'(x) = (1-p)(-p)x^{-p-1}\theta^{p-1}$$

$$f''(x) = (1-p)(-p-1)x^{-p-2}\theta^{p-1}$$

$$f''(x) = -(p+2)(p+1)p(1-p)x^{-p-3}\theta^{p-1} > 0$$

it indicates that mode for the new distribution does not exixts.

1.8 Forgetfulness Property

The distribution is forgetful or has no memory.Means if a unit sur- vived for the t hours,than the probability of its surviving an addi- tional h hours is exactly same

$$P(X \geq t+h|X \geq t) = \frac{\int_t^{t+h}(1-p)\,x^{-p}\ \theta^{p-1}dt}{\int_o^t(1-p)\,x^{-p}\ \theta^{p-1}dt}$$

$$= \left(1+\frac{h}{t}\right)^{1-p} - 1$$

which is free from x,which shows the forgetfulness property of the proposed distribution.

2 Estimation of parameters

2.1 Maximum Likelihood estimation(mle)

Let $x_1, x_2 \ldots\ldots x_n$ be a random sample of size n, Likelihood estimation of the parameters can be wrriten as

$$L(p) = \prod_{t=1}^{n}(1-p)x_i^{-p}\ \theta^{p-1}$$

$$= (1-p)^n\theta^{n(p-1)}\prod_{t=1}^{n}x^{-p}$$

$$\frac{\partial \ln(p)}{\partial p} = 0 \Leftrightarrow \frac{-n}{1-p} + nln\theta - \sum_{t=1}^{n}\ln(x)$$

$$\hat{p} = \frac{n}{nln\theta - \Sigma lnx} \quad (11)$$

above equation(11) shows estimation for the shape parameter p.

$$\hat{\theta} = t_n = Max\ (t_1\,, t_2\,, \ldots\ldots t_n\,)(12)$$

above equation(12) shows estimation for the shape parameter θ.

2.2 Moments method

$$\boldsymbol{Mean} = \frac{1-p}{2-p}\theta = GM$$

$$(1-p)\theta = (2-p)GM$$

$$p(GM-\theta) = 2GM-1$$

$$\hat{p} = \frac{2GM-1}{GM-\theta} \qquad (13)$$

3 Order statistics

Here the density fi:n of the ith order statistics,for $i = 1, 2, \ldots n$ from independent identically distributed random variable $X1, X2, \ldots\ldots Xn$ is given by

$$f_{i:n}(x) = \frac{f(x)}{B(i, n-i+1)} = F(x)^{i-1}[1-F(x)]^{n-1}\ (14)$$

$$f_{i:n}(x) = \frac{(-p)x^{-p}\theta^{p-1}}{B(i, n-i+1)}\sum_{k=0}^{n-1} \quad (-1)^k \begin{bmatrix} n-1 \\ k \end{bmatrix} [x^{1-p}\ \theta^{p-1}]^{(1+k)-1}$$

where $0\ <\ F\ (x)\ <\ 1\, for\, x\ >\ 0$

by above equation we can express K_{th} ordinary moment of the ith order statistics $X_{i:n}$ as a linear combination of the K_{th} moment of the new distribution with different shape parameter. Therefore, the measure of skewness and kurtosis of the distribution of $X_{i:n}$ can be calculated.

The L-moments are analogous to the ordinary moments but can be estimated by linear combinations of order statistics.

$$\lambda_1 = E[X_{1:1}] \qquad (15)$$

$$\lambda_2 = \frac{1}{2}E[X_{2:2} - X_{1:2}] \qquad (16)$$

$$\lambda_3 = \frac{1}{3}E[X_{3:3} - 2X_{2:3} + X_{1:3}] \qquad (17)$$

$$\lambda_4 = \frac{1}{4}E[X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}] \qquad (18)$$

4 Fisher information Matrix of new distribution

The Fisher information is that a random variable 'X' contains about the parameter θ is given by

$$I(\theta) = E[\frac{\partial}{\partial x log f(x;\theta)}]^2 \quad (19)$$

Now if $logf$ (x; θ) is twice diffrentiable w.r.t. θ under certain regu- larity conditions, Fisher's information is given by;

$$\boldsymbol{I}(\theta) = \boldsymbol{E}_{\theta}[\partial^2/\partial\theta^2 log(\boldsymbol{f}(x;\theta,p)] \quad (20)$$

Where

$$f(x;\theta,\ p = (1-p)x^{-p}\theta^{p-1}$$

$$log(f(x;\theta,p) = log(1-p) - plogx + (p-1)log\theta \qquad (21)$$

differentiating w.r.t. p , θ and taking Expectations on both sides of the those equations we get following equations:

$$-E\left[\frac{\partial logf(x;\theta,p)}{\partial p^2}\right] = \frac{1}{(1-p)^2} \quad (22)$$

$$-E\left[\frac{\partial^2 logf(x;\theta,p)}{\partial p\partial\theta}\right] = \frac{-1}{\theta} \quad (23)$$

$$-E\left[\frac{\partial^2 logf(x;\theta,p)}{\partial\theta\partial p}\right] = \frac{-1}{\theta} \quad (24)$$

$$-E\left[\frac{\partial^2 logf(x;\theta,p)}{\partial\theta^2}\right] = \frac{(1-p)}{\theta^2} \quad (25)$$

Now we have

$$I_{(1,1)} = \frac{1}{(1-p)^2}$$

$$I_{(1,1)} = \frac{-1}{\theta}$$

$$I_{(1,2)} = \frac{-1}{\theta}$$

$$I_{(1,1)} = \frac{p-1}{\theta^2}$$

Above equation is the Fisher's information Matrix of the new distribution.

5 Entropies

5.1 R'enyi and q-entropy

The entropy of a random variable X is a measure of the uncertain variations in the system.The R'enyi entropy is defined by

$$I_R(\delta) = \frac{1}{(1-\delta)}\log[I(\delta)]$$

Where

$$I(\delta) = \int_R f^{\delta}(x)dx\ f\ or\ \delta > 0\ and\ \delta \neq 1$$

$$f^{\delta} = (1-p)^{\delta}\,(x^{-p})^{\delta}(\theta^{p-1})^{\delta}$$

$$I(\delta) = \int_R f^{\delta} = (\theta^{p-1})^{\delta}\,(x^{1-p})^{\,\delta}$$

Hence R'enyi entropy reduces to

$$I_R(\delta) = \frac{1}{(1-\delta)} \log[(\theta^{p-1})^{\delta}\,(x^{1-p})^{\,\delta}]$$

5.2 q-entropy

q $-$ *entropy say* $Hq\ (f\ )$ *is defined by*

$$H_q(f) = \frac{1}{(q-1)} \log[1 - I_q(f)]$$

Where

$$I_q(f) = \int_R f^{q}\,(x)dx\ f\ or\ q > 0\ and\ q \neq 1$$

$$H_q(f) = \frac{1}{(q-1)} \log[1 - (\theta^{p-1})^{q}\,(x^{1-p})^{\,q}]$$

5.3 Shannon's entropy

$$f(x) = (1-p)x^{-p}\theta^{p-1}$$

$$\log L*(x,\theta,p) = n\log(1-p) + n(p-1)log\theta - p\sum_{i=1}^{n}\log x_i$$

$$l(x,\theta,p) = n\,[\log(1-p) + (p-1)log\theta] - p\sum_{i=1}^{n}\log x_i$$

$$\frac{l(x,\theta,p)}{n} = \log(1-p) + (p-1)log\theta - \overline{p\log x_\iota}$$

The shannon's entropy of new distribution will be

$$\widehat{H} = -[\log(1-p) + (p-1)log\theta - \overline{p\log x_\iota}$$

$$\widehat{H} = \frac{-l(x;\theta,p)}{n} \quad (26)$$

above equation shows the shannos' entropy of new distribution.

6 Goodness of Fit

To check the goodness of fit of the proposed distribution a data set of Radar set component failure is taken for a Transmitter Tube failure (D.J.Davis-1952).

1-Transmitter Tube

$t(hr.)$	Observed freq.	Expected freq.	$(o-e)^2/e$
0-25	109	105	0.1524
25-50	42	30	4.88
50-75	17	21	0.7619
75-100	7	17	5.8823
100-up	13	15	0.2666
	188	188.0	11.9432

$$d.f. = 4; \chi^2 = 11.9432 ; p = 0.6$$

χ^2_{Cal} = = 11.9432 and χ^2_{tab}= 13.28 at level of significance. it shows that the proposed data fits well to this data of failure.

7 References

[1]. Abdelkarim Ra'ed, Abuiyada Reem and Siddiui S.A. ."Undergraduate Students' Attitude Towards Mathematics After Peer Teaching Experience ",Global Journal of Pure and Applied Mathematics. , Volume 12 , No.2 , pp.1501-1517 (2016)

[2]. Cha,J.H, and Mi,J.: Study of stochastic failure model in a random environment, Journal of Applied probabil- ity,Vol.44,No. 1,pp.151-163,(2007).

[3]. D Desai,V. Mariapaan,M.Sahardande:Nature of Reversed Hazzard Rate:An Investigation,Intenational Journal of Performability,Vol 7.No. 2,pp.165-171.(2011).

[4]. Davis,D.J.:The analysis of some failure data, J.Am.Statistics association,vol 47,113-160,(1952).

[5]. Dwivedi Shradha ,Sabir Ali Siddiqui:Bayes Estimator Of Scale Parameter Of Size-Biased Mukherjii- Islam (Sbmid) By Lindleys Approach ,Int. J. Agricult. Stat. Sci. ,vol 13, No. 2, ISSN : 0973-1903,(2017).

[6]. Folks,J.F. and Chikara,R.s.:The inverse Gaussian distribution and its statistical application-A Review,Journal of Royal Stat. Soc.,Vol B40,pp.263-289,(1978).

[7]. Mandel,M.and Ritov Yaakov:The accerelated failure time model under bised sampling,Biometrics,Vol 66,No.4. pp.1306- 1308,(2010).

[8]. Mukerjee,S.P.and Islam,A"A finite range distribution of faliure times",Naval research logistics Quaterly,Vol.30,pp.487- 491,(1983).

[9]. Siddiqui,S.A, Kumar,S. Bayesian Estimation of reliabil- ity and hazard rate functions,Micro electron Reliab., Vol. 13. No. 1, 53-57, (1991).

[10]. Siddiqui,S.A, Jain,S and Chauhan,R.K. Bayesian Analysis of reliability and hazard rate functions of a Mixture model, Micro electron Reliab., Vol. 37. No. 6, 935-941, (1997).

[11]. Sidiqui S.A.,Dwivedi Shradha,Dwivedi Peeyush Dwivedi ,San- jay Jain and Siddiqui Irfan."Development of Size-Biased MUKHERJEE-ISLAM Distribution",International Jour- nal of Pure and Applied Mathematics, Volume 107 No. 2 , 505-515 (2016).

[12]. Sidiqui S.A.,Sanjay Jain,Irfaan sidiqui,Khaliquzamman khan and Masood Aalam."Characterization and De- Velopment Of A New Failure Model ",Jour- nal of Theoretical and Applied Information Technology ,Vol- ume 86,No.1, (2016)

[13]. Sidiqui S.A.,Syed NasserAndrabi and sanjay Jain, Masood Aalam and RaedAbdelkarim."Concomitants of Order Statis- tics of a New Bivariate Finite Range

Distribution (NBFRD) ”,Global Journal of Pure andApplied Mathemat- ics. ,Volume 12,No.2, pp. 1691-1697 (2016)

[14]. Sidiqui .”CHARACTERIZATION OF A BASIC FI- NITE RANGE PROBABILITY DISTRIBUTION ”,Int. J. Agricult. Stat. Sci. ,Volume 12,No.1, pp. 153-156, (2016)

[15]. Sidiqui Sabir Ali, Dwivedi Shradha, Dwivedi Peeyush, Siddiqui Irfan.”Development Of Size-Biased Mukherjee-Islam Distribution ”,International Journal of Pure and Applied Mathematics , Volume 107, No. 2, pp. 505-515, (2016)

[16]. Syed Aqeel Ashraf, Brian G. Stewart,KhaliquzzamanKhan, D. M. Hepburn,Sabir Ali Siddiqui.” Stochastic Behaviour of Propagation of Partial Discharge Acoustic Signals in Transformer Oil”, International Journal of Applied En- gineering Research , Volume 11 , No.12 , pp.7702-7707 (2016)

[17]. Weibull,W:A Statistical distribution of wide applicabil- ity,Journal of applied Mechanics,Vol.18,pp.293-297,(1951).