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RESEARCH ARTICLE

ON FUZZY SEMI PRIME QUASI IDEALS IN TOPOLOGY

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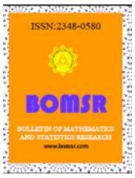

ABSTRACT

We give some properties of fuzzy quasi Idalas in Semi Topolgical spaces and characterized a completely regular topological spaces of semigroup in terms of fuzzy semiprime quasi ideals in topology.

Introduction

A non empty sub set Q of a semigroup S is called quasi ideal of S if $QS \subset SQ \subset Q$. This concept was introduced by Steinfield [8] and systematically discuused in [7].It was extended to fuzzy quazi-ideal and discuused by the author[5]. A nonempty subset A of the semigroup S is called semiprime . This concept was also extended by [3].,that is, a fuzzy subset "f" of a semiprime of S is called fuzzy semiprime if $f(a) > f(a^2)$ for all $a \in S$. A fuzzy subsemigroup f of S is called a fuzzy biideal of S if $f \circ S \circ f \subseteq f$. which is dicussed in [4] . Any fuzzy quasi-ideal of S is a fuzzy bi-ideal of S. But the converse does not hold in genral. Iseki[2] gave the following definition of semigroup S is regular iff $RL = R \cap L$,for every right ideal R and every idel L and S. This was considered in fuzzy ideals,that is, a semigroup S is regular iff $f \circ g = f \cap g$ for every fuzzy right ideal f and every fuzzy left ideal g of S.From this ,in a regular semiprime S,any fuzzy bi-ideal of S is a fuzzy quasi-ideal of S. In this note, we characterized a completely regular semigroup for the subgroup which is disjoint union groups and a semigroup that a semilattice group in terms of its fuzzy semiprime quasiideals.

Definitions and Preliminary

Let S be semiprime , A function "f" from S to the unit interval [0,1] is a fuzzy subset of S. Let "f" and "g" be two fuzzy subsets of S.Then the inclusion $f \subseteq g$ is defined by $f(x) \le g(x)$ for all $x \in S$.and interaction of $f \cap g$ is defined by $(f \cap g)(x) = \min\{f(x), g(x)\}$ for all $x \in S$.The

Product of $(f \circ g)(x) = \{ \begin{array}{ll} \sup_{x \to yz} & [\min\{f(y), g(x)] \\ 0 & if\ x\, is\ not\ expressible\ as\, at\ x = yz. \end{array}$

A fuzzy subset f of S is called a fuzzy sub semigroup of S if $f * f \subseteq f$, and a fuzzy left (right) ideal of S if $S \circ f \subseteq f (f \circ S \subseteq f)$. A fuzzy subsets emigroup f of S is called a fuzzy biideal of S if $f \circ S \circ f \subseteq f$.

A non empty subset Q of S is called a quasi –ideal of S and any fuzzy quasi –ideal of S is a fuzzy biideal of S. We denote by f_A the characteristic function of the subset A of S.

Lemma 2.1 : Let Q be a nonempty subset of a semi group S, Then Q is a quasi –ideal of S iff f_Q is a fuzzy quasi –ideal of S.

Proof : Let us consider that Q is a quasi –ideal of S. Let a be any element of S . If $a \in Q$,then

$$\{(f_Q \circ S) \cap (S \circ f_Q)\}(a) \leq 1 = f_Q(a).$$

If $a \notin Q$, then $f_Q(a) = 0$. On the other hand ,assume that

$$\{(f_Q \circ S) \cap (S \circ f_Q)\}(a) = 1.$$

Then,

$$Sup_{a \to pq}\{\min\{f_Q(p).S(q)\} = (f_Q \circ S)(a) = 1.$$

And

$$Sup_{a \to pq}\{\min\{S(p), f_Q(q)\} = (S \circ f_Q)(a) = 1.$$

This implies that there exist an element b,c,d and e of S with a=bc=de such that $f_Q(b) = 1$. Then a=bc=de$\in QS \cap SQ \subseteq Q$.which contradicts that $a \in Q$. Thus we have

$$\{(f_Q \circ S) \cap (S \circ f_Q)\}(a) = 0 = f_Q(a).$$

In either case , we have

$$(f_Q \circ S) \cap (S \circ f) \subseteq f_{Q.}.$$

And so f_Q is a fuzzy quasi-ideal of S ,conversely ,assume that f_Q is a fuzzy quasi ideal of S. Let 'a' be any element of $QS \cap SQ$. Then there exists an element s and t of S and elements of b and c of Q Such that a=bs=tc.

Then we have

$(f_Q \circ S)(a) = 1$. similarly ,we have $(S \circ f_Q)(a) = 1$. Then

$$f_Q \geq \{(f_Q \circ S) \cap (S \circ f_Q)\}(a) = \min\{(f_Q \circ S)(a).(S \circ f_Q)(a)\} = 1.$$

Thus ,we have $a \in Q$ and so that $QS \cap SQ \subseteq Q$. Therefore , Q is a quasi-ideal of S .This completes the lemma .

Lemma 2.2 : Let f and g be two fuzzy quasi-ideals of a semigroup S. Then $f \cap g$ is a fuzzy quasi-ideal of a semigroup S.

Proof : $[(f \cap g) \circ S] \cap (S \circ (f \cap g)] \subseteq (f \circ S) \cap (S \circ g) \subseteq f \cap g.$

Lemma 2.3 : Let f and g be two fuzzy quasi-ideals of a semi-group S. Then the Product of $f \circ g$ is a fuzzy bi-ideal of S .

Proof : Because f is a fuzzy quasi –ideal of S.

$f \circ g \circ f \subseteq S \circ f$ and $f \circ S \circ f \subseteq f \circ S \circ S \subseteq f \circ S.$

And also $f \circ S \subseteq (f \circ S) \cap (S \circ f) \subseteq f.$

There for we have

$$(f \circ g) \circ S \circ (f \circ g) = (f \circ (g \circ S) \circ f) \circ g \subseteq (f \circ (S \circ S) \circ f) \circ g \subseteq (f \circ S \circ f) \circ g \subseteq f \circ g.$$

This implies that $f \circ g$ is a fuzzy bi –ideal of S.

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