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SOME ZERO ADJUSTED PROBABILITY MODEL FOR ADULT OUT MIGRATION

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ABSTRACT

The Present study is a development of some zero adjusted probability model for adult migrants from the households, with the help of poisson lindley distribution. The parameters have been estimated by moment estimation method. Its suitability is examined on the real survey data of migration.

Key words: Adult migration, probability model, Truncation, Size biasing.

Introduction

Migration is a term that encompasses a wide variety of movements and situations involving people of all dimensions of life and backgrounds. Migration is one of the major determinants affecting the distribution of population. In India most of the population lives in rural area where social amenities, job opportunities, education facilities are either absent or insufficient. For getting better above mentioned facilities people moved from one place to another, thus these are the possible reasons of migration and play a vital role in determination the flow of adult migration. Vice versa migration causes a certain effect on social and economic activities of the household as well as the entire community. Therefore to understand the dynamics of pattern of migration is important concern of the attraction of research.

A number of attempts have been made in the past for studying pattern in rural-urban migration through the use of probability models (Singh et al., 1980, 1985; Singh and Yadava, 1981; Sharma, 1985, 1987; Yadava & Yadava, 1988, Yadava et al. 1989 and 1994; Aryal, 2003; Singh & Singh, 2016; Singh et al. 2017). Inflated poisson lindley distributions has been studied (Borah et al. 2001) and further used by Singh et al. (2015) to explain the phenomena of the adult migration from the household. In this study an attempt has been made to use some simple mixture models for understanding the dynamics of adult out migration. Parameters of the models are estimated by method of moment and some real data sets are used to check the suitability of models.

The Model

During the literature review a paper entitled "An Inflated Probability Model for Adult Out Migration Pattern" by Pandey & Dubey (2016) is found. Perhaps Pandey & Dubey (2016) used the idea of model proposed by Singh et al. (2015) and a truncated probability distribution is used in zero adjusted model and named as inflated probability model (which is incorrect).

The probability mass function (pmf) of the model (Pandey & Dubey, 2016) is given as

$$\mathrm{P}\left(X=x\right)=\begin{cases}\left(1-\alpha\right) \\ \alpha.\dfrac{\theta^2\left(k+\theta+2\right)}{\left(\theta^2+3\theta+2\right)\left(\theta+1\right)} \quad , \quad k=1,2,....,\theta>0\end{cases}$$

Now

$$\sum \mathrm{P}\left(X=x\right)=\left(1-\alpha\right)+\sum_{k=1}^{\infty}\alpha\frac{\theta^2\left(k+\theta+2\right)}{\left(\theta^2+3\theta+2\right)\left(\theta+1\right)}$$

$$=\left(1-\alpha\right)+\alpha\frac{\theta^2}{\left(\theta^2+3\theta+2\right)\left(\theta+1\right)}\sum_{k=1}^{\infty}\left(k+\theta+2\right)$$

$$=\left(1-\alpha\right)+\alpha\frac{\theta^2}{\left(\theta^2+3\theta+2\right)\left(\theta+1\right)}\left[\left(\theta+3\right)+\left(\theta+4\right)+\left(\theta+5\right)............\right]$$

This is a divergent series hence the sum will not be equal to 1. Therefore the above probability function is not a probability mass function. It is worthwhile to mention here is that how they found the estimate and fittings.

Now we assume that at any point of time, α be the proportion of household in which adult migration occurs and the proportion of household having no adult migration is (1-α).

A number of models can be developing for the number of adult migrants from household in a society that households have varying number of adult migrants. If *X* be the number of adult migrants from a household, therefore we can consider different combination of poisson and lindley distribution.

1. Number of migrants follows truncated poisson lindley distribution with parameter theta.
2. Number of migrants follows size biased poisson lindley distribution with parameter theta.

The poisson lindley distribution with the pmf and $E(x)$ is as

$$p^*(x;\theta)=\frac{\theta^2\left(\theta+x+2\right)}{\left(\theta+1\right)^{x+3}} \qquad x=0,1,2,........,\ \theta>0 \tag{1}$$

$$E(x)=\frac{(\theta+2)}{\theta(\theta+1)}$$

Model-I

Now the truncated distribution at zero is defined as in case of poisson lindley distribution

$$p(x;\theta)=\frac{p^{*}(x;\theta)}{1-p^{*}(0;\theta)}=\frac{\theta^{2}\left(\theta+x+2\right)}{\left(\theta+1\right)^{x+3}}\cdot\frac{\left(\theta+1\right)^{3}}{\left(\theta^{2}+3\theta+1\right)}$$

Hence, the zero-truncated poisson lindley distribution is

$$p(x;\theta)=\frac{\theta^{2}\left(\theta+x+2\right)}{\left(\theta+1\right)^{x}\left(\theta^{2}+3\theta+1\right)}\quad x=1,2,.......\ \theta>0 \tag{2}$$

The model using truncated poisson lindley distribution is given as

$$P(X=x)=\begin{cases}(1-\alpha) & ;\ x=0\\ \alpha.\dfrac{\theta^{2}\left(\theta+x+2\right)}{\left(\theta+1\right)^{x}\left(\theta^{2}+3\theta+1\right)} & ;\ x=1,2,3,.........,\theta>0\end{cases} \tag{3}$$

This model contains two parameters α and θ.

Model-II

Now the size biased distribution is defined as in case of poisson lindley distribution

$$p(x;\theta)=\frac{xp^{*}(x;\theta)}{E(x)}=\frac{x.\theta^{2}\left(\theta+x+2\right)}{\left(\theta+1\right)^{x+3}}\cdot\frac{\theta\left(\theta+1\right)}{\left(\theta+2\right)}$$

Hence, the size biased poisson lindley distribution.

$$p(x;\theta)=\frac{x.\theta^{3}\left(\theta+x+2\right)}{\left(\theta+1\right)^{x+2}\left(\theta+2\right)}\quad x=1,2,.......\ \theta>0 \tag{4}$$

The model using size biased poisson lindley distribution is given as

$$P(X=x)=\begin{cases}(1-\alpha) & ;\ x=0\\ \alpha.\dfrac{x.\theta^{3}\left(\theta+x+2\right)}{\left(\theta+1\right)^{x+2}\left(\theta+2\right)} & ;\ x=1,2,3,.........,\theta>0\end{cases} \tag{5}$$

This model again contains two parameters α and θ.

Estimation

The method of zero-cell frequency has been considered to estimate the parameters involved in the models, because it is easy to use and has less computational complexity.

Parameter Estimation of Model-I

$$1-\alpha=\frac{f_{0}}{f}\quad\text{and}\quad E\left(x\right)=\alpha.\frac{\left(\theta+1\right)^{2}\left(\theta+2\right)}{\theta\left(\theta^{2}+3\theta+1\right)}$$

Where, f_0, *f,* and $E(x)$ denotes the zeroth cell frequency, total observation and mean respectively.

Parameter Estimation of Model-II

$$1-\alpha=\frac{f_0}{f} \quad \text{and} \quad E(x)=\alpha.\frac{\left(\theta^2+4\theta+6\right)}{\theta(\theta+2)}$$

Where, f_0, f, and $E(x)$ denotes the zero[th] cell frequency, total observation and mean respectively.

Table 1. Observed and expected frequency of the number of households according to the migrants in flooded area of Kosi River

Number of migrants	Observed number of households	Expected number of households	
		Model I	Model II
0	401	401.00	401.00
1	147	139.22	127.48
2	57	67.05	78.42
3	29	31.24	35.40
4	16	14.21	13.97
5	8	6.35	5.10
6	5	2.80	1.77
7	1	2.13	0.86
Total	664	664.00	664.00
Mean= 0.7365		χ^2 =2.98 (pooled)	χ^2 =15.38 (pooled)
		p-value=0.3942 (df=3)	p-value=0.0015 (df=3)
Estimated value of parameters		α =0.3961	α =0.3961
		θ =1.5343	θ =2.8110

Data source: Singh (2015)

Table 2. Observed and expected frequency of the number of households according to the migrants in Varanasi District

Number of migrants	Observed number of households	Expected number of households	
		Model I	Model II
0	1032	1032.00	1032.00
1	95	88.16	85.29
2	19	28.16	32.00
3	10	8.79	8.89
4	2	2.70	2.17
5	2	0.82	0.49
6	0	0.25	0.11
7	1	0.11	0.04
Total	1161	1161.00	1161.00
Mean= 0.1619		χ^2 =3.94 (pooled)	χ^2 =7.31 (pooled)
		p-value=0.0472 (df=1)	p-value=0.0069 (df=1)
Estimated value of parameters		α =0.1111	α =0.1111
		θ =2.6815	θ =4.997

Data source: Varanasi (1978)

Table 3. Observed and expected frequency of thenumber of households according to migrants in Nepal

Number of migrants	Observed number of households	Expected number of households	
		Model I	Model II
0	623	623.00	623.00
1	126	125.20	120.57
2	42	42.26	48.25
3	13	13.93	14.29
4	4	4.51	3.72
5	2	1.44	0.90
6	1	0.66	0.27
Total	811	811.00	811.00
Mean= 0.3465		χ^2 =0.09 (pooled)	χ^2 =3.38 (pooled)
		p-value=0.9552 (df=2)	p-value=0.1836 (df=2)
Estimated value of parameters		α =0.2318	α =0.2318
		θ =2.5011	θ =4.6507

Data source: Aryal (2002)

Table 4. Observed and expected frequency of thenumber of households according to migrants in Bangladesh

Number of migrants	Observed number of households	Expected number of households	
		Model I	Model II
0	1941	1941.00	1941.00
1	542	528.19	515.30
2	124	148.08	166.37
3	48	40.70	39.87
4	13	11.02	8.42
5	4	2.95	1.66
6	1	1.06	0.38
Total	2673	2673.00	2673.00
Mean= 0.3786		χ^2 =6.17 (pooled)	χ^2 =19.26 (pooled)
		p-value=0.0457 (df=2)	p-value=0.0000 (df=2)
Estimated value of parameters		α =0.2739	α =0.2738
		θ =3.1471	θ =3.1471

Data source: Hossain (2000)

Results and Discussion

In Table 1 the suitability of proposed model is examined by several sets of data collected under a survey entitled “Migration and related characteristics-a case study of North-Eastern Bihar” conducted during October 2009 to June 2010 used by Singh et al (2015). The same data is used by Pandey & Dubey (2016) without quoting the author. Again, the suitability of proposed model is examined by further more sets of data. Varanasi data was collected under a sample survey “Rural development and population growth (RDPG) survey” conducted in 1978 in Varanasi district and used by Sharma (1985) and Iwunor (1995). The Nepal data is taken from a sample survey of the Rupandhi and Palpa districts in Nepal and used by Aryal (2011). The Bangladesh data was collected under a sample survey "Impact of Migration on Fertility in Bangladesh: A study of Comilla district" conducted in 1997 and used by Hossain (2000). Here we found the observed and expected frequency and value of chi-square allow us to consider the truncated model provide the same fitting as given in Singh et al. (2015), however the size biased model provide different fitting poorer than truncated model. Thus the truncated model is somewhat better than size biased model. Although the fitting of the data by truncated and inflated model (Singh et al., 2015) is same but the value of mixing parameter is different, the interpretation of the mixing parameter in inflated model has a meaning i.e. this much amount is not governed by the plain distribution, actually this amount is extra and can be capture only considering any mixture distribution. Thus in such a situation the inflated distributions are more relevant that a model adjusted at a particular cell frequency.

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