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ANALYSIS OF ABSOLUTELY CONVERGENCE PRESERVING TRANSFORMATION MATRICES

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ABSTRACT
We obtain the theorems of absolutely convergence preserving traneformation matrices .In this paper we prove the product GB of a β_A- matrix G and a δ_A- matrix B exists and is a β_A- matrix and every finite linear combination of δ_A- matrices is a δ_A- matrix. The product of two δ_A- matrix exists and is a δ_A- matrix.
Keywords: Matrix method for summability, Absolutely convergence etc.

1. INTRODUCTION

The subject of infinite matrices, being a recent one, is abounding in good research problems. A very important application of matrices, namely to the theory of summability of divergent sequence and the series was initiate by Toeplitz [1,12] in 1911. Since, then, it has attached almost all researchers in the fields of summability methods. Although, the concept of absolute summability was introduced as early as in 1911, by Fekete [14,2] in case of Cesaro [13]method, and the same for Reisz [5,6,8] and Abel [9,10,11] methods was defined by Obrechkoff [3,15] and Whittaker[4] in 1928 and 1932 respectively, for matrix transformation in general this was considered in 1937 by Mears [7]. Ordinary summability by the general matrix transformation has been investigated in considerable detail and thus has reached a stage where a sufficiently unified theory can be presented. But absolute summability on the other hand is still in its infancy.

1.1 We use following German abbreviation in this paper.

FF for sequence -to- sequence (1.1.1)

RF for series - to – sequence (1.1.2)

RR for series – to- series. (1.1.3)

Let A = (a_{nk}), (n, k = 1, 2,…) be a given matrix. We consider the transformation

$$t_n = \sum_{k=1}^{\infty} a_{nk} s_k \tag{1.1.4}$$

Then the matrix A provides an FF, RF or RR transformation according as it transform a sequence x = $\{S_k\}$ into the sequence y = $\{t_n\}$, the series $\sum s_n$ into the series $\sum t_n$, provided that each of the series (1.1.4) is convergent. The following definition in general use for the matrix A corresponding to FF – transformations each be made applicable with obvious changes o RF and RR transformation.

If a transformation (1.1.4) the sequences y = $\{t_n\}$ belongs to the space (c) of convergent sequences. We say that the sequences x = $\{S_k\}$ is summable by the matrix method A, or by the matrix method A, or by the matrix A or simply A- summable and we write either A- $lim\, x$ = $lim\, y$ or $lim\, s_n$ = $lim\, t_n$. The class $[A]$ of all A - summable sequences is called the convergence field of A. If for two methods A and B. We have relation [A] ⊂ $[B]$, we say that B is not weaker than A, A and B are said to be consistent if A- $lim\, x$ = B- $lim\, x$, whenever these limit exits. The matrix A is said to be convergence preserving if it transfers every convergent sequence y =$\{t_n\}$, with lim not necessary is same as that of$\{s_n\}$. The matrix A is said to be permanent if it is transform every convergent sequences x = $\{s_n\}$ into a convergent sequence y =$\{t_n\}$. And

$$\text{A-}lim\, x = lim\, y \tag{1.1.5}$$

Also, the matrix A is said to be absolutely convergence preserving if

$$\sum_{n=2}^{\infty} |s_n - s_{n-1}| < \infty \tag{1.1.6}$$

$$\Rightarrow \sum_{n=2}^{\infty} |t_n - t_{n-1}| < \infty$$

and if we have an addition to (1.1.6), (1.1.5) also, the matrix A is said to be absolutely permanent. The matrix A is *called* reversible if the equation A(x) = y, has exactly one solution x, convergent, or not for each value of y in (c).

If is too much to required from a matrix to give a permanent transformation. We demand little less and seek for conditions under which a matrix transformation is convergence preserving i.e. If the original sequence or series is convergent under what conditions that transformed sequence or series also converges, but necessarily to the same limit.

Kojima, in 1917 began the work in this direction. He proved the result for FF- transformation by lower semi- matrices. His result was generalized by Schur, who proved that an FF- transform matrix gives convergence preserving transformation iff it is a K- matrix, 1931, Basanquet [16,17,18 and 19] proved that a matrix of an RF- transcreation is convergence preserving iff it is a β- matrix. Vermes further studied the β- matrix and obtained the result that necessary and sufficient condition for a matrix to give a convergence preserving RR- transformation is that it is , a δ – matrix.

2. PRELIMINARIES

Theorem 2.1

$$\text{If} \quad g_{nk} = b_{1k} + b_{2k} + \cdots + b_{nk}\ , \ (n,k \geq 1) \tag{2.1.1}$$

then G = (g_{nk}) is a β_A – matrix iff B = (b_{nk}) is a δ_A- matrix.

Theorem 2.2

The product GB of β_A – matrix g and δ_A- matrix B exits and is a β_A – matrix.

Theorem 2.3

Every finite linear combination of δ_A – matrices (or β_A – matrices) is a δ_A – matrices (or β_A – matrix).

3. LEMMAS

For the proof of our theorem, the following lemmas are required.

Lemma 3.1

In order that RF- transformation given by the matrix G = (g_{nk}) be absolute convergence preserving, it is necessary and sufficient that the conditions

$$\sum_{n=2}^{\infty}\left|g_{nk}-g_{n-1,k}\right| < \text{M(G)} \tag{3.1.1}$$

$$|g_{nk}| < k_n(G) \tag{3.1.2}$$

be satisfied, where the absolute constants M(G) and $k_n(G)$ are independent of k.

Lemma 3.2

The RR transformation given by the matrix B = (b_{nk}) is absolute convergence preserving iff there exists constant M and k_n, both independent of k , such that the conditions

$$\sum_{n=1}^{\infty}|b_{nk}| < \text{M(B)} \tag{3.2.1}$$

$$|b_{nk}| < k_n(B) \tag{3.2.2}$$

are satisfied.

4. PROOF OF THE THEOREMS

4.1 Proof of the theorem 2.1

Suppose that the matrix B= (b_{nk}) in (2.1.1) is a δ_A- matrix. Then by lemma 3.2,

$$| g_{nk} | = | b_{1k}+b_{2k}+\cdots+b_{nk} |$$

$$\leq | b_{1k} |+| b_{2k} |+\ldots+| b_{nk} |$$

$$< nk_n(B) \leq k_n(G) \text{ say} \tag{4.1.1}$$

Where the constant k_n is independent of k. Also, by definition (2.1.1), we can write

$g_n - g_{n-1,k} = b_{nk}$, (n> $1, k \geq 1$)

$g_{1k} = b_{1k}$

Therefore

$$\sum_{n=2}^{\infty}\left|g_{nk}-g_{n-1,k}\right| = \sum_{n=2}^{\infty}|b_{nk}| < \text{M(B)} < \text{M(G)} \tag{4.1.2}$$

By condition (3.2.2). Thus, the inequalities (4.1.1) and (4.1.2) show that G = (g_{nk}) in (2.1.1) is a β_A- matrix. conversely, Let G = (g_{nk}) in (2.1.1) by a a β_A- matrix then it is easy to verify that B = (b_{nk}) so defined satisfies the condition of Lemma 3.2

This completes the proof.

4.2 Proof of the theorem 2.2

We write H = GB = (h_{nk}), so that

$$h_{nk} = \sum_{j=1}^{\infty} g_{nj} b_{jk} \tag{4.2.1}$$

Now,

$\left| \sum_{j=1}^{\infty} g_{nj} b_{jk} \right| \leq \sum_{j=1}^{\infty} |g_{nj}| \; |b_{jk}| < k_n(G).\sum_{j=1}^{\infty} |b_{jk}|$ from (3.1.2)

$$< k_n(G).M(B) < M(H) \tag{4.2.2}$$

Independent of k.

therefore, the matrix H = (h_{nk}) in (4.2.1) exists for all n and k. Also, we have from (4.2.2),

$$|h_{nk}| < k_n(H) \; \forall \; n, \tag{4.2.3}$$

Where k_n is independent of k.

Also, as in (4.2.2), we can prove that,

$$\sum_{n=2}^{\infty} |h_{nk} - h_{n-1,k}| < M(H), \tag{4.2.4}$$

Independent of k.

Thus, the result follows from (4.2.3) and (4.2.4) by virtual of lemma 3.1.

4.3 Proof of theorem 2.3

Let F and G be a β_A-matrices corresponding to δ_A – matrices A and B respectively i.e.

$g_{nk} = b_{1k} + b_{2k} + \cdots + b_{nk}$ (n, k ≥ 1),

$$f_{nk} = a_{1k} + a_{2k} + \cdots + a_{nk}, \text{ (n, k} \geq 1) \tag{4.3.1}$$

Also, let x and y be any two complex number writes

$$H = x^F + y^G \tag{4.3.2}$$

then from (3.1.2) we have

$$|h_{nk}| \leq |x||f_{nk}| + |y||g_{nk}| \leq |x||k_n(f)| + |y||k_n(G)| < k_n(H), \tag{4.3.3}$$

Where the constant k_n is independent of k.

By (4.3.1),

h_{1k}= x. f_{1k}+ y g_{1k} (k≥ 1)

$h_{nk} - h_{n-1,k}$= x$(f_{nk} - f_{n-1,k})$+ y$(g_{nk} - g_{n-1,k})$, (n> 1, $k \geq 1$)

Hence,

$\sum_{n=2}^{\infty} |h_{nk} - h_{n-1,k}| \leq |x||f_{1k}| + \sum_{n=2}^{\infty} |x|.|f_{nk} - f_{n-1,k}| + |y|.|g_{1k}|. + \sum_{n=2}^{\infty} |y|.|g_{nk} - g_{n-1,k}|$

$$< |x|.M(F) + |y|M(G) < M(H) \tag{4.3.4}$$

Independent of k.

The condition (4.3.3) and (4.3.4) are precisely, the condition of lemma 3.1, and hence, the matrix (H) = (h_{nk}) as defined in (4.3.2) is a β_A- matrix, this proved the result for β_A- matrices

Now,

$h_{1k} = xf_{1k} + yg_{1k},$ (k≥ 1)

$h_{nk} - h_{n-1,k} = xf_{nk} + yg_{nk},$ (n> 1 , $k \geq 1$)

Therefore, the matrix

C = (c_{nk}) defined as $c_{1k} = h_{1k}$ (k≥ 1)

$c_{nk} = h_{nk} - h_{n-1,k,}$ (n> 1, $k \geq 1$)

Which corresponds to the β_A- matrix it is a δ_A- matrix, and C = $x^A + y^B$

5. CONCLUSION

In this paper we found that product GB of a β_A- matrix G and a δ_A- matrix B exists and is a β_A-matrix and every finite linear combination of δ_A- matrices is a δ_A- matrix. The product of two δ_A-matrix exists and is a δ_A- matrix.

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