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RESEARCH ARTICLE

ESTIMATING AGRICULTURAL YIELDS MORE ACCURATELY WHEN GROWN UNDERGROUND

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ABSTRACT

It is relatively more difficult to estimate agricultural produces accurately when grown underground. This problem leads to exploitation of farmers by many agencies which, in turn, cause severe hardships to them. Under these scenarios, we have made an attempt to estimate the agricultural produces more accurately as a solution to the national problem. Ranked Set Sampling (RSS) is one such cost-effective method that could be employed for the estimation of agricultural yields grown underground more accurately. According to McIntyre's (1952) RSS method, m^2 units are randomly selected from an infinite population and the selected units are arranged in m sets each consisting of m units. After ranking the units of each set separately with respect to the variable of interest, the unit with the lowest rank is quantified from the first set, the unit with the second lowest rank is measured from the second set, and the process is continued until the unit possessing rank m is quantified from the m^{th} *set*. This process results in m quantifications out of the m^2 units selected randomly. In order to get a large sample size n the whole process is repeated r times. This provides a ranked set sample of size $n = mr$. We call m as the set size and r as the number of cycles. This method could be used with equal as well as unequal allocations. It performs better under the latter than the former. For skewed distributions, the optimum gain in precision is obtained through an unequal allocation based on Neyman's approach. These procedures are illustrated using a real agricultural data set related to the yields of potato. We have obtained the relative precision (RP) under equal and unequal allocations as 1.70 and 2.21 respectively in perfect ranking scenario and as 1.52 and 1.72 respectively in concomitant ranking scenario. These suggest that the relative savings (RS) under equal and unequal allocations are 41% and 55% respectively in perfect ranking scenario and 35% and 42% respectively in concomitant ranking scenario. This is based on the set size *m* = 4, the number of cycles *r* = 3 and the sample size *n* = 12. The technique appears to be quite useful to those who look for cost-effective methods for estimating the farm yields grown under the land surface such as potato, ginger, turmeric, garlic, onion, beet, beetroot, carrot, peanut, etc.

Keywords: Cost-effectiveness, ranked set sampling, relative precision and savings, simple random sampling, skewed distribution

1. Introduction

Ranked set sampling (RSS) was developed by McIntyre (1952) for estimating pasture yields. It is a cost effective procedure when compared to the commonly used simple random sampling (SRS) in the situations where the actual measurement of the characteristics of interest is difficult and expensive but the ordering of the sampling units can be done easily without using their actual measurements. He proposed the sample mean based on RSS as an estimator of the population mean and found that estimator based on RSS is more efficient than SRS. Many modifications on RSS have been done since McIntyre (1952). It has also been used to estimate the forage yield by Halls and Dell (1966), the mass herbage in a paddock by Cobby *et al*. (1985), the tree volume in a forest by Stokes and Sager (1988) and the bone mineral density in a human population by Nahhas *et al*. (2002). Some other conditions where RSS procedures are used have been enumerated by Kaur *et al.* (1997) and Chen *et al.* (2004). For an overview of RSS, see Patil, Sinha and Taillie (1994).

In the present paper we describe the problem of both equal as well as an unequal allocation in RSS for skewed distributions. In the case of equal allocation, it has been shown by Takahasi and Wakimoto (1968) that the relative precision (RP) of RSS in comparison to simple random sampling (SRS) lies between 1 and $(m+1)/2$, where m is the set size whereas in case of an unequal allocation, Takahasi and Wakimoto (1968) also showed that the RP of RSS relative to SRS lies between 0 and *m*. Here it has been shown that after the use of an unequal allocation the RP of RSS in comparison to SRS increased over the use of equal allocation. In the situation of RSS with an unequal allocation, McIntyre (1952) suggested the use of optimum allocation based on Neyman's approach where the sampling units are allocated to various ranks order in proportional to the standard deviation of that rank order. For skewed distributions, Takahasi (1970) proposed the use of random allocations in RSS and this idea was further developed and discussed by Yanagawa and Shirahata (1976) and Yanagawa and Chen (1980). In case of positively skewed or right-tailed distributions on (0, ∞), the variances of order statistics typically increase with the rank orders. Thus, more quantifications at the right tail confirm with Neyman's optimum allocation approach. Kaur *et al.* (1997) suggested two models of unequal allocation for positively skewed distributions: the '*t*-model' and '(*s*, *t*)-model'. They found that the '*t*-model' performs better than the equal allocation model for an appropriate choice of t $(t \geq 1)$, '(*s*, *t*)-model' performs better than '*t*-model'. The performance of Neyman's allocation model is better than '(*s*, *t*)-model', the gain remains moderate to marginal. They also propose rules-of-thumb to determine appropriate allocation factor(s) *t* for the '*t*-model' and (*s*, *t*) for '(*s*, *t*)-model' based on the knowledge of skewness, kurtosis or coefficient of variation (CV). Tiwari and Chandra (2011) proposed a simple and systematic approach for unequal allocation for RSS with positively skewed distributions. The proposed approach performs better than SRS and RSS with equal allocation for positively skewed distributions. The proposed approach also performs better than the RSS with unequal allocation using '*t*-model' and quite close to the '(*s*, *t*)-model'.

Section 2 describes the RSS with equal allocation with the RP as compared with SRS. Further Section 3 discusses the RSS with an unequal allocation for asymmetrical population along with the RP as compared with SRS. In section 4, we illustrate the use of RSS with equal and unequal allocation for estimating the tuber weight based on real data set taken from The Central Potato Research Station (Indian Council of Agricultural Research), Sahay Nagar, Patna – 801506 (Bihar), India. In section 5, the results of the present work are included.

2. RSS with Equal Allocation

In RSS procedure with equal allocation, m^2 units are randomly selected from an infinite population and then the selected units are arranged in *m* sets each consisting of *m* units. After ranking the units of each set separately with respect to the characteristic of interest, the unit with the lowest rank is quantified from the first set, the unit with the second lowest rank is quantified from the second set and the process is continued until the unit possessing rank *m* is quantified from the m^{th} set. Thus, in this way *m* units are quantified out of the initially m^2 units selected from an infinite population. The whole process is repeated *r* times in order to get a large sample size *n*. This provides a ranked set sample of size *n* = *mr*. We call *m* as the set size and *r* as the number of cycles. As one gets the same number of quantifications *r* for each rank order, this procedure is called RSS with the equal allocation (RSSWEA). For the above results, see Norris, Patil and Sinha (1995).

In general, suppose that $X_{(i:m)j}$ denotes the i^{th} order statistic based on perfect ranking in the j^{th} cycle, for $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,r$. It is to be noted that these are not *iid* in general, but for a given value of *i* these are so with $E(X_{(i:m)j}) = \mu_{(i:m)}$ and $Var(X_{(i:m)j}) = \sigma^2_{(i:m)}$. The McIntyre's estimator $\hat{\mu}_{MRSS}$ of the population mean μ is defined as:

$$\hat{\mu}_{MRSS} = \frac{1}{m}\sum_{i=1}^{m}\hat{\mu}_{(i:m)}$$

where, $$\hat{\mu}_{(i:m)} = \frac{1}{r}\sum_{j=1}^{r} X_{(i:m)j}$$

Here, $E(\hat{\mu}_{(i:m)}) = \mu_{(i:m)}$; $E(\hat{\mu}_{MRSS}) = \mu$ and $Var(\hat{\mu}_{(i:m)}) = \dfrac{\sigma^2_{(i:m)}}{r}$.

Thus we get, $$Var(\hat{\mu}_{MRSS}) = Var\left[\frac{1}{m}\sum_{i=1}^{m}\hat{\mu}_{(i:m)}\right]$$

$$= \frac{1}{m^2}\sum_{i=1}^{m} Var(\hat{\mu}_{(i:m)}) = \frac{1}{m^2}\sum_{i=1}^{m}\frac{\sigma^2_{(i:m)}}{r}$$

Therefore, $Var(\hat{\mu}_{MRSS}) = \frac{1}{m^2 r}\sum_{i=1}^{m}\sigma^2_{(i:m)}$.

The estimate of σ^2 for simple random sampling is given by,

$$\hat{\sigma}^2_{MRSS} = \left[\frac{mr-m+1}{m^2 r(r-1)}\right]\sum_{i=1}^{m}\sum_{j=1}^{r}(X_{(i:m)j} - \hat{\mu}_{(i:m)})^2 + \left(\frac{1}{m}\right)\sum_{i=1}^{m}(\hat{\mu}_{(i:m)} - \mu_{MRSS})^2$$

Relative Precision (RP) for an Equal Allocation

The relative precision (RP) of the MRSS estimator $\hat{\mu}_{MRSS}$ as compared with SRS estimator $\hat{\mu}_{SRS}$ with the same sample size *n* is obtained as:

$$RP = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{MRSS})}$$

As $\mathrm{var}(\hat{\mu}_{SRS}) = \frac{\sigma^2}{mr}$, this leads to

$$RP = \frac{\sigma^2}{\overline{\sigma^2}} \qquad \text{where } \overline{\sigma^2} = \frac{\sum_{i=1}^{m}\sigma^2_{(i:m)}}{m}$$

An equivalent and useful measure could be the relative cost (RC) and the relative savings (RS). These are defined as

$RC = \frac{1}{RP}$ and *RS = 1- RC*

The range of *RP* is $1 \le RP \le \frac{m+1}{2}$ and that of *RS* is $0 \le RS \le \frac{m-1}{m+1}$.

3. RSS with Unequal Allocation

For unequal allocation McIntyre (1952) suggested that to take sample size of each rank order proportional to its standard deviation while sampling with asymmetrical populations. It implies that if r_i denotes the number of the sets having quantified units with rank *i*, then $r_i \alpha \sigma_{(i:m)}$ for $i = 1,2,\ldots,m$. Thus the expression for r_i is given as

$$r_i = \frac{n\sigma_{(i:m)}}{\sum_{i=1}^{m}\sigma_{(i:m)}}; \quad i = 1,2,\ldots,m.$$

The RSS estimator $\hat{\mu}_{MRSSUA}$ of the population mean μ based on an unequal allocation of samples is given by

$$\hat{\mu}_{MRSSUA} = \frac{1}{m}\sum_{i=1}^{m}\frac{T_i}{r_i} \quad \text{and} \quad Var(\hat{\mu}_{MRSSUA}) = \frac{1}{m^2}\sum_{i=1}^{m}\frac{\sigma^2_{(i:m)}}{r_i}$$

where T_i represents the sum of the quantifications of the r_i units having i^{th} rank order. On putting the value of r_i into the expression for $Var(\hat{\mu}_{MRSSUA})$ we have

$$Var(\hat{\mu}_{MRSSUA}) = \frac{(\bar{\sigma})^2}{n}; \qquad \text{where, } \bar{\sigma} = \frac{\sum_{i=1}^{m}\sigma_{(i:m)}}{m}$$

The estimate of σ^2 for simple random sampling in case of unequal allocation is given by,

$$\hat{\sigma}^2{}_{MRSSUA} = \sum_{i=1}^{m}\left[\frac{m(r_i-1)+1}{m^2 r_i(r_i-1)}\right]\sum_{j=1}^{r_i}(X_{(i:m)j} - \bar{X}_{(i:m)})^2 + \left(\frac{1}{m}\right)\sum_{i=1}^{m}(\bar{X}_{(i:m)} - \bar{X}_{(m)r})^2$$

Relative precision (RP) for unequal allocation

The relative precision (RP_{ua}) of $\hat{\mu}_{MRSSUA}$ relative to $\hat{\mu}_{SRS}$ with the same number of quantification is given by

$$RP_{ua} = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{MRSSUA})}$$

This yields that $$RP_{ua} = \frac{\sigma^2/n}{\sum_{i=1}^{m}\frac{\sigma^2_{(i:m)}}{r_i}/m}$$

This could also be written as $$RP_{ua} = \left(\frac{\sigma}{\bar{\sigma}}\right)^2.$$

This proves that $RP_{ua} \geq RP$. Takahasi and Wakimoto (1968) show that $0 \leq RP_{ua} \leq m$. For the above results see Sinha (2005).

4. Illustration

For an illustration, we have taken a real data set of four characteristics fresh haulm weight (in kg), fresh root weight (in kg), tuber (potato) weight (in kg) and chlorophyll measurements of potato plant each having 96 observations, see Kumar and Sinha (2015). For estimating tuber weight, we have used perfect (retrospective study) and concomitant ranking scenario both. For this, the plots have been ranked using the exact weight of potato which is supposed to be known. We have randomly selected sixteen plots from the given data set and weights of the tuber are presented in four rows and four columns. This process is carried out for all the three sets. This number is considered only for our convenience. One could take any number of sets with the set size four. For using RSS with unequal allocation, we compute r_i using the formula $r_i = \frac{n\sigma_{(i:m)}}{\sum_{i=1}^{m}\sigma_{(i:m)}}$ where $i = 1,2,\ldots,m$. The results obtained are given below. Table 1 represents the weights of tuber in four plots of equal size in each row for three sets.

Table 1: Weights of tuber in four plots of equal size in each row for three sets

Set	Observed weights of tuber in four plots of equal size in each row			
Set 1	26.316	25.570	25.272	33.253
	20.714	28.248	16.206	16.782
	21.918	34.928	41.226	28.566
	23.648	26.010	17.142	34.970
Set 2	17.852	23.616	12.950	13.646
	35.628	27.000	25.862	23.122
	17.124	32.382	17.170	11.840
	33.477	24.006	16.694	19.670
Set 3	19.200	29.241	23.598	38.379
	25.760	15.640	9.914	19.394
	18.986	25.912	15.628	24.604
	30.414	32.940	29.894	19.370

Equal Allocation:

Table 2 shows the weights of tuber for four ranks in three sets. Again, we consider set size, $m = 4$ and number of cycles, $r = 3$.

Table 2: Weights of tuber for four ranks in three sets

Set	Rank of tuber weights for four plots in each set			
	Rank 1	Rank 2	Rank 3	Rank 4
Set 1	25.272	16.782	34.928	34.970
Set 2	12.950	25.862	17.170	33.477
Set 3	19.200	15.640	24.604	32.940
Mean	19.140	19.430	25.570	33.796
S. D.	6.160	5.600	8.920	1.052
Variance	37.960	31.370	79.530	1.106

Now, we have,

$\hat{\mu}_{(1:4)}$ = 19.140 *kg*, $\hat{\mu}_{(2:4)}$ = 19.430 *kg*, $\hat{\mu}_{(3:4)}$ = 25.570 *kg*, $\hat{\mu}_{(4:4)}$ = 33.796 *kg*,

$\sum_i \hat{\mu}_{(i:4)}$ = 97.936 *kg*

$$\hat{\mu}_{RSS} = \frac{\sum_i \hat{\mu}_{(i:4)}}{m} = \frac{97.936}{4} = 24.484kg$$

$\hat{\sigma}^2_{(1:4)}$ = 37.96, $\hat{\sigma}^2_{(2:4)}$ = 31.37, $\hat{\sigma}^2_{(3:4)}$ = 79.53, $\hat{\sigma}^2_{(4:4)}$ = 1.106, $\sum_i \hat{\sigma}^2_{(i:4)}$ = 149.966

$$Var(\hat{\mu}_{RSS}) = \frac{\sum_i \hat{\sigma}^2_{(i:4)}}{m^2 r} = 3.12$$

The estimate of σ^2 for simple random sampling is given by,

$$\hat{\sigma}^2_{MRSS} = \left[\frac{mr - m + 1}{m^2 r(r-1)}\right] \sum_{i=1}^{m} \sum_{j=1}^{r} (X_{(i:m)j} - \hat{\mu}_{(i:m)})^2 + \left(\frac{1}{m}\right) \sum_{i=1}^{m} (\hat{\mu}_{(i:m)} - \mu_{MRSS})^2$$

$$= 63.6184$$

Therefore, $Var(\hat{\mu}_{SRS}) = \frac{\hat{\sigma}^2}{n} = \frac{\hat{\sigma}^2}{mr} = \frac{63.6184}{12} = 5.30$

$$R.P. = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{RSS})} = 1.70 > 1$$

Thus, $Var(\hat{\mu}_{RSS}) < Var(\hat{\mu}_{SRS})$

The Relative Cost (RC) and the Relative Savings (RS) are given as:

$RC = \frac{1}{RP} = 0.59$, $RS = 1 - RC = 0.41$ or, *RS* = 41 %

Unequal allocation:

For an unequal allocation we first calculate the standard deviation (S.D.) for each rank order as given in Table 2.

$\sigma_{(1:4)}$ = 6.16, $\sigma_{(2:4)}$ = 5.60, $\sigma_{(3:4)}$ = 8.92, $\sigma_{(4:4)}$ = 1.052, $\sum_{i=1}^{4} \sigma_{(i:4)}$ = 21.732

Here we compute r_i using the formula $r_i = \frac{n\sigma_{(i:m)}}{\sum_{i=1}^{m} \sigma_{(i:m)}}$ where $i = 1,2,\ldots,m$.

$r_1 = 3,\ r_2 = 3,\ r_3 = 4,\ r_4 = 2,\ \sum_{i=1}^{4} r_i = n = 12$

With these values of r_i we have the following values of mean and variance which are shown in Table 3.

Table 3: Tuber weights at 4 ranks under unequal allocation

Rank	Rank 1	Rank 2	Rank 3	Rank 4
Values	25.272	16.782	34.928	34.970
	12.950	25.862	17.170	33.477
	19.200	15.640	24.604	
			30.414	
Mean	19.140	19.430	26.780	34.224
Variance	37.960	31.370	58.890	1.115

$\hat{\mu}_{(1:4)}$ = 19.14 *Kg*, $\hat{\mu}_{(2:4)}$ =19.43 *Kg*, $\hat{\mu}_{(3:4)}$ = 26.78 *Kg*, $\hat{\mu}_{(4:4)}$ = 34.224 *Kg*,

$\sum_i \hat{\mu}_{(i:4)}$ = 99.574 *Kg*

$$\hat{\mu}_{MRSSUA} = \frac{1}{m}\sum_{i=1}^{4}\frac{T_i}{r_i} = \frac{99.574}{4} = 24.8935\,Kg$$

Thus, estimates of the population mean, $\hat{\mu}_{MRSSUA} = 24.8935$

$\hat{\sigma}^2_{(1:4)}$ = 37.96, $\hat{\sigma}^2_{(2:4)}$ = 31.37, $\hat{\sigma}^2_{(3:4)}$ = 58.89, $\hat{\sigma}^2_{(4:4)}$ = 1.115

$$Var(\hat{\mu}_{MRSSUA}) = \frac{1}{m^2}\sum_{i=1}^{4}\frac{\sigma^2_{(i:4)}}{r_i} = \frac{1}{16}\left(\frac{37.96}{3} + \frac{31.37}{3} + \frac{58.89}{4} + \frac{1.115}{2}\right) = 2.39937$$

The estimate of σ^2 for simple random sampling in case of unequal allocation is given by,

$$\hat{\sigma}^2_{MRSSUA} = \sum_{i=1}^{m}\left[\frac{m(r_i-1)+1}{m^2 r_i(r_i-1)}\right]\sum_{j=1}^{r_i}(X_{(i:m)j} - \overline{X}_{(i:m)})^2 + \left(\frac{1}{m}\right)\sum_{i=1}^{m}(\overline{X}_{(i:m)} - \overline{X}_{(m)r})^2$$

= 63.5296

Therefore, $Var(\hat{\mu}_{SRS}) = \frac{\hat{\sigma}^2}{n} = \frac{\hat{\sigma}^2}{mr} = \frac{63.5296}{12} = 5.29413$

$$R.P. = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{MRSSUA})} = 2.21 > 1$$

Thus, $Var(\hat{\mu}_{MRSSUA}) < Var(\hat{\mu}_{SRS})$

The Relative Cost (RC) and Relative Savings (RS) are given as:

$$RC = \frac{1}{RP} = 0.45, \qquad RS = 1 - RC = 0.55 \quad \text{or, } RS = 55\ \%$$

Concomitant Ranking Scenario of tuber weight Based on Chlorophyll Measure

The concomitant ranking is performed using the ranks on the basis of chlorophyll level, fresh haulm weight and on the basis of two characteristics chlorophyll level and fresh haulm weight jointly. The main characteristic of interest is tuber weight while other characteristics are used as concomitant variables for ranking the plots with respect to the main characteristic of interest. But for selecting concomitant variable for ranking we need to choose those variables which are highly correlated with the tuber weight. For this we performed the correlation analysis using Statistical Software, Minitab 16. The output of Correlation Analysis is given below.

Correlation Matrix:

Correlations: Fresh Haulm Weight (Kg), Fresh Root Weight (gm), Tubers Weight (Kg), Chlorophyll Measure

	Fresh Haulm Weight(kg)	**Fresh Root Weight(gm)**	**Tubers Weight(kg)**
Fresh Root Weight(gm)	0.085		
	0.410		
Tubers Weight(kg)	0.219	0.016	
	0.032	0.875	
Chlorophyll Measure	0.049	0.007	0.349
	0.637	0.945	0.000

Cell Contents: Pearson correlation
P-Value

Table 4: Data in Cell: (Tuber weight, Chlorophyll measure) (Rank)

Set	Observed weights of tuber and chlorophyll measure in four plots of equal size in each row			
Set 1	(17.868, 40.3) (1)	(27.7984, 48.5)	(21.642, 49.3)	(25.760, 50.8)
	(15.640, 52.9)	(8.960, 46.8) (2)	(19.394, 42.1)	(29.787, 53)
	(25.838, 39.7)	(27.908, 49.4)	(16.206, 43.8)	(19.200, 44.5) (3)
	(17.124, 50.5)	(27.408, 50.2)	(20.714, 45.7)	(19.768, 58.8) (4)
Set 2	(29.082, 47.7) (1)	(35.628, 58.3)	(17.170, 54.9)	(13.646, 49.9)
	(26.010, 54.5)	(28.566, 53.4)	(17.200, 53.2) (2)	(38.379, 52.4)
	(16.782, 49.1) (3)	(15.032, 40.2)	(27.000, 44.4)	(23.232, 49.8)
	(41.226, 54.4)	(33.477, 60.8) (4)	(32.382, 52.8)	(16.800, 43.8)
Set 3	(12.048, 41.7) (1)	(12.950, 44.5)	(10.322, 47)	(19.670, 44.4)
	(22.130, 57.7)	(23.979, 48.2)	(26.262, 51.4) (2)	(31.861, 52.2)
	(15.302, 56)	(19.370, 36.3)	(18.986, 42.1)	(25.912, 47.6) (3)
	(28.166, 52.5)	(16.652, 43.4)	(18.542, 56) (4)	(19.440, 55.8)

Table 5: Rank of tuber weights based on chlorophyll measure for four plots in each set

Set	Rank of tuber weights based on chlorophyll measure for four plots in each set			
	Rank 1	Rank 2	Rank 3	Rank 4
Set 1	17.868	8.960	19.200	19.768
Set 2	29.082	17.200	16.782	33.477
Set 3	12.048	26.262	25.912	18.542
Mean	19.670	17.470	20.630	23.930
S. D.	8.660	8.650	4.730	8.290
Variance	74.960	74.900	22.380	68.750

For Equal Allocation,

$$Var(\hat{\mu}_{RSS}) = 1.73 \qquad Var(\hat{\mu}_{SRS}) = 2.63$$

$$R.P. = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{RSS})} = 1.52 > 1$$

Thus, $Var(\hat{\mu}_{RSS}) < Var(\hat{\mu}_{SRS})$

The Relative Cost (RC) and the Relative Savings (RS) are given as:

$$RC = \frac{1}{RP} = 0.65, \qquad RS = 1 - RC = 0.35 \quad \text{or, } RS = 35\ \%$$

For Unequal Allocation,

$r_1 = 4$, $r_2 = 3$, $r_3 = 2$, $r_4 = 3$, $\sum_{i=1}^{4} r_i = n = 12$

With these values of r_i we have the following values of mean and variance which are shown in Table 6.

Table 6: Tuber weights at 4 ranks under unequal allocation

<table>
<tr><th rowspan="2">Set</th><th colspan="4">Rank of tuber weights based on chlorophyll measure for four plots in each set</th></tr>
<tr><th>Rank 1</th><th>Rank 2</th><th>Rank 3</th><th>Rank 4</th></tr>
<tr><td rowspan="4">Values</td><td>17.868</td><td>8.960</td><td>19.200</td><td>19.768</td></tr>
<tr><td>29.082</td><td>17.200</td><td>16.782</td><td>33.477</td></tr>
<tr><td>12.048</td><td>26.262</td><td rowspan="2"></td><td>15.302</td></tr>
<tr><td>16.652</td><td></td><td></td></tr>
<tr><td>Mean</td><td>18.91</td><td>17.470</td><td>17.99</td><td>22.85</td></tr>
<tr><td>Variance</td><td>52.25</td><td>74.900</td><td>2.92</td><td>89.70</td></tr>
</table>

$$Var(\hat{\mu}_{MRSSUA}) = 1.49, \qquad Var(\hat{\mu}_{SRS}) = 2.56$$

$$R.P. = \frac{Var(\hat{\mu}_{SRS})}{Var(\hat{\mu}_{MRSSUA})} = 1.72 > 1$$

Thus, $Var(\hat{\mu}_{MRSSUA}) < Var(\hat{\mu}_{SRS})$

The Relative Cost (RC) and the Relative Savings (RS) are given as:

$$RC = \frac{1}{RP} = 0.58, \qquad RS = 1 - RC = 0.42 \quad \text{or, } RS = 42\ \%$$

5. Conclusion

The results obtained for RP, RC and RS with equal and unequal allocation under perfect and concomitant ranking scenarios for the same set size, say, m = 4 and the same sample size, say, n = 12 are enumerated in Table 7 given below.

Table 7: RP, RC and RS under equal and unequal allocation under Perfect and Concomitant Ranking for the same set size, *m = 4* and the same sample size, *n = 12*

Allocation and Method	RP	RC	RS
Perfect Ranking (Equal Allocation) (m = 4, r = 3, n = 12)	1.70	0.59	0.41 (41%)
Perfect Ranking (Unequal Allocation), (m = 4, n = 12)	2.21	0.45	0.55 (55%)
Concomitant Ranking (Equal Allocation) (m = 4, r = 3, n = 12)	1.52	0.65	0.35 (35%)
Concomitant Ranking (Unequal Allocation) (m = 4, n = 12)	1.72	0.55	0.42 (42%)

From Table 7, we observe that the RP is obtained higher in case of unequal allocation than that of equal allocation under both the perfect and concomitant ranking scenario for the same set size m = 4 and the same sample size n = 12. It suggests that RSS with unequal allocation performs better than SRS and RSS with equal allocation. In this way we also see that, RC is less in case of unequal allocation than that of equal allocation under both the perfect and concomitant ranking scenario and consequently we observed that RS is higher in case of unequal allocation than that of equal allocation under both the perfect and concomitant ranking scenario for the same set size m = 4 and the same sample size n = 12. Thus, the procedure appears to be more useful to those who look for estimating farm produces that are grown under the land surface like potato, ginger, turmeric, garlic, onion, beetroot, peanut, etc.

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References

Chen, Z., Bai, Z. D. and Sinha, B. K. (2004). Ranked Set Sampling: Theory and Application. *Springer-Verlag*, New York.

Cobby, J. M., Ridout, M. S., Bassett, P. J. and Large, R. V. (1985). An investigation into the use of ranked set sampling on grass and grass-clover swards. *Grass and Forage Science*, **40**, 257-263.

Halls, L. K. and Dell, T. R. (1966). Trials of ranked set sampling for forage yields. *Forest Science*, **12**, 22-26.

Kaur, A., Patil, G. P. and Taillie, C. (1997). Unequal allocation models for ranked set sampling with skew distributions. *Biometrics*, **53 (1)**, 123-130.

Kumar, V. and Sinha, A. K. (2015). Estimating Underground Farm Produce by Ranked Set Sampling with Unequal Allocations, *International Journal of Agricultural and Statistical Sciences*, Vol. 11(1), June 2015, ISSN: 0973-1903, e-ISSN: 0976-3392, 47-52.

McIntyre, G. A. (1952). A method for unbiased selective sampling, using ranked sets. *Australian Journal of Agricultural Research*, **3**, 385-390.

Nahhas, R. W., Wolfe, D. A. and Chen, H. (2002). Ranked set sampling: Cost and optimal set size. *Biometrics*, **58**, 964-971.

Norris, R. C., Patil, G. P. and Sinha, A. K. (1995). Estimation of multiple characteristics by ranked set sampling methods. *Coenoses* (The journal of the International Center for Theoretical and Applied Ecology, Italy), **10** (2-3), 95-111.

Patil, G. P., Sinha, A. K. and Taillie, C. (1994b). Ranked set sampling: In *Handbook of Statistics, Environmental Statistics*, G. P. Patil, and C. R. Rao. (eds), Elsevier Science B. V., Amsterdam: North-Holland, **12**, 167-198.

Sinha, A. K. (2005). On some recent developments in ranked set sampling. *Bulletin of Informatics and Cybernetics, Research Association of Statistical Sciences*, **37**, 135-160.

Stokes, S. L. and Sager, T. W. (1988). Characterization of a ranked set sample with application to estimating distribution functions. *Journal of the American Statistical Association*, **83**, 374-381.

Takahasi, K. (1970). Practical note on estimation of population means based on samples stratified by means of ordering. *Annals of the Institute of Statistical Mathematics*, **22**, 421-428.

Takahasi, K. and Wakimoto, K. (1968). On unbiased estimates of the population means based on the sample stratified by means of ordering. *Annals of the Institute of Statistical Mathematics*, **20**, 1-31.

Tiwari, N. and Chandra, G. (2011). A systematic approach for unequal allocations under ranked set sampling with skew distributions. *Journal of the Indian Society of Agricultural statistics*, **65(3)**, 331-338.

Yanagawa, T. and Chen, S. (1980). The MG-procedure in ranked set sampling. *Journal of Statistical Planning and Inference*, **4**, 33-44.

Yanagawa, T. and Shirihata, S. (1976). Ranked set sampling theory with selective probability matrix. *Australian Journal of Agricultural Research*, **18**, 45-52.