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AN INFLATED PROBABILITY MODEL FOR COMMUTATION PATTERN

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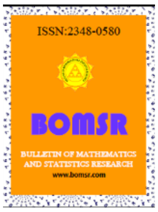

ABSTRACT

This present paper deals with an Inflated Probability Model for the commutation Pattern from the household under the certain assumptions. Parameters involved in the model have been estimated by MLE and MM. Suitability of this model has been checked by Demographic data.

Keyword: Inflated Model, Commutation Pattern, MLE and MM.

Introduction

The consideration of the Migration of population from one place to another is directly proportional to the population of these places and inversely proportional to the distance between two places. Migration is the seasonal movement of people from one region to another. It is a procedure of changing residence place once and permanently staying in a region or country. This is a movement involving two places, the origin to the destination. There is a proper influence of migration upon the society, economics, ethnicity and demographic characteristics of people at the place of origin to destination.

An important role plays of migration in distribution of population of the country and having made a firm decision of the labour force in the specific region.

There is a vast Concept for the reasons of migration, It has been assessed that the large scale movement of population from rural area to town area, from towns to other towns and from one country to another has been escorted the industrial and economic development.

India is the exemplar of the normal process of rural urban migration in developing countries. Simply put, migration is known as movement of population from one region to another with different intentions as settling temporary or permanently. According to this phenomena migration can be broadly classified into two points,

(i) Long term migration in which population move to a country other than that of its usual residence for a period of at least year so that the country of destination effectively becomes its new residence. (Pandey, H. and Dubey Kumar, A. (2016) Pandey, H. and Dubey Kumar, A. (2017)).

(ii) Short term migration in which population move to a country other than that of its usual residence for a short period at least three month but less than a year except in cases where the movement to that country is for purposes of recreations holidays visits to friends or relatives business or medical purpose this short term migration are known as commutation too, in which the individual migrate away from its residence in early morning to some distant place and return home late in evening the process involved in the type of migration is known as daily migrants or Commuters (Lee, S.H., Faucet, J.T. and R.G. Abad, (1985); Singh, R.B. (1986)). Commutation is a procedure of Daily migration between the place of residence and place of work. To developed inflated probability model for the pattern of commutation to test the acceptability of the model from household Data Collection as "rural development and population and growth: A Sample Survey 1978" that is organized by population Research Centre BHU Varanasi India.

There is a development of inflated probability model for the commutation pattern from the household, after this there is description of estimation procedure and applicability and suitability of the model has been tested through a demographic survey data.

Model:

Assumptions for the proposed model take the following forms:

1. Let μ be proportion of household from which at least one Commuter be exist.
2. Out of the μ- proportion of household, let ρ be the proportion of households from which only one Commuters be exist at the survey point.
3. Out of (1-ρ) μ- proportion of household Commuters exist which follows the displaced Possion distribution.

Under these above assumption and with the help of Johnson and Kotz (1969) our model takes following forms:

$$\left.\begin{aligned} P[x=0] &= 1-\mu, & k=0 \\ P[x=1] &= \mu\rho, & k=1 \\ P[x=k] &= \frac{(1-\rho)\mu[\eta^{k-1}(e^{\eta}-1)^{-1}]}{\lfloor k-1}, & k=2,3\ldots. \end{aligned}\right\} \quad (1)$$

The above model involves three parameter μ ρ and η to be estimated from observed distribution of Commuters.

Estimation:

Method of Moments: The present model consist three parameters μ ρ and η. The parameters μand ρ are estimated by equating$Zero^{th}$, First cell theoretical frequencies to observed frequencies, and theoretical mean to the observed mean.

$$\frac{f_0}{f} = 1 - \mu \tag{2}$$

$$\frac{f_1}{f} = \mu\rho \tag{3}$$

$$E(X) = \bar{X} = \mu\rho + (1-\rho)\mu\,\{\eta e^{\eta}(e^{\eta}-1)^{-1} + 1\,\} \tag{4}$$

Where,

f_0 =Number of observations in Zeroth cell.

f_1 =Number of observations in first cell.

f =Total number of observations.

$\bar{X}$ =Observed mean.

Maximum Likelihood Method: Consider a sample consisting of N observations of random variable X with probability function in which $f_0$designates the number of zero observation ; $f_1$the number of one observation and $f_2$the number of second observation and f the total number of observations. The values chosen as estimates and those which maximize the expression is as follows,

$$P(x_{1,}x_{2\,\ldots\ldots\ldots,}x_{f,}\mu,\rho,\eta) = (1-\mu)^{f_0}(\mu\rho)^{f_1}[(1-\rho)\mu\eta(e^{\eta}-1)^{-1}]^{f_2}[\mu\{1-\rho - (1-\rho)\eta(e^{\eta}-1)^{-1}\}]^{f-f_0-f_1-f_2}$$

Now, taking logarithmic of above equation and partially differentiating w.r.to μ, ρ, and ηin turn, and equating to zero yields the estimating equation:

$$\frac{\partial logL}{\partial\mu} = -\frac{f_0}{(1-\mu)} + \frac{f_1}{\mu} + \frac{f_2}{\mu} + \frac{f-f_0-f_1-f_2}{\mu} = 0$$

$$\frac{\partial logL}{\partial\rho} = \frac{f_1}{\rho} + \frac{f_2}{1-\rho} + \frac{f-f_0-f_1-f_2}{1-\rho} = 0$$

$$\frac{\partial logL}{\partial\eta} = f_2[-(e^{\eta}-1)^{-1}e^{\eta}+1] - \frac{(f-f_0-f_1-f_2)(e^{\eta}-1)^{-1}[-(e^{\eta}-1)^{-1}e^{\eta}\eta+1]}{[1-\eta(e^{\eta}-1)^{-1}]} = 0$$

A solution of equation provides the estimator of μ as:

$$\mu = \frac{f-f_0}{f}$$

Now, solving the equation provides the estimator of ρ and η as:

$$\mu = \frac{f_1}{f-f_0}$$

$$\eta(e^{\eta}-1)^{-1} = \frac{f_2}{f-f_0-f_1}$$

The second partial derivatives of Log L can be obtained as:

$$\frac{\partial^2 logL}{\partial\mu^2} = -\frac{f_0}{(1-\mu)^2} - \frac{f_1}{\mu^2} - \frac{f_2}{\mu^2} - \frac{f-f_0-f_1-f_2}{\mu^2}$$

$$\frac{\partial^2 logL}{\partial \rho^2} = -\frac{f_1}{\rho^2} - \frac{f_2}{(1-\rho)^2} - \frac{f - f_0 - f_1 - f_2}{(1-\rho)^2}$$

In the case of η we have taken approximation of e^{η} at one place upto three terms and then partially differentiating of log L we obtained as follows:

$$\frac{\partial^2 logL}{\partial \eta^2} = -f_2(e^{\eta}-1)^{-1}\{1 - e^{\eta}(e^{\eta}-1)^{-1}\} - \frac{f_2}{\eta^2} - \frac{(f - f_0 - f_1 - f_2)\left[\frac{3\eta^2}{4} + \eta\right]}{\left[\frac{\eta^3}{4} + \frac{\eta^2}{2}\right]^2}$$

Now ,

$$\frac{\partial^2 logL}{\partial \mu \partial \eta} = \frac{\partial^2 logL}{\partial \eta \partial \mu} = 0$$

$$\frac{\partial^2 logL}{\partial \rho \partial \eta} = \frac{\partial^2 logL}{\partial \eta \partial \rho} = 0$$

$$\frac{\partial^2 logL}{\partial \mu \partial \rho} = \frac{\partial^2 logL}{\partial \rho \partial \mu} = 0$$

Using the fact we get,

$$E(f_0) = f(1-\mu)$$

$$E(f_1) = f\mu\rho$$

$$E(f_2) = f(1-\rho)\mu(e^{\eta}-1)^{-1}\eta$$

$$E(f - f_0 - f_1 - f_2) = f\mu\{1 - \rho - (1-\rho)\eta(e^{\eta}-1)^{-1}\}$$

Where, E is denoted for the expectation.

The expected value of second partial derivatives of Log l can be obtained by three different cases as.

Case 1: When $\rho$is taking know from the method of moment then

$$\phi_{11} = E\left[\frac{\left(\frac{-\partial^2 logL}{\partial \mu^2}\right)}{f}\right] = \left[\frac{1}{1-\mu} + \frac{1}{\mu}\right]$$

$$\phi_{22} = E\left[\frac{\left(-\frac{\partial^2 logl}{\partial \eta^2}\right)}{f}\right]$$

$$= (1-\rho)\mu\eta e^{\eta}(e^{\eta}-1)^{-2}\{1 - e^{\eta}(e^{\eta}-1)^{-1}\} + \frac{(1-\rho)\mu(e^{\eta}-1)^{-1}}{\eta}$$

$$+ \frac{\mu\{1 - \rho - (1-\rho)\eta(e^{\eta}-1)^{-1}\}\left[\frac{3\eta^2}{4} + \eta\right]}{\left[\frac{\eta^3}{4} + \frac{\eta^2}{2}\right]^2}$$

$$\phi_{12} = \phi_{21} = \frac{E\left[\frac{-\partial^2 logL}{\partial \mu \partial \eta}\right]}{f} = 0$$

$$V(\hat{\mu}) = \frac{1}{f}\left[\frac{\phi_{22}}{\phi_{11}\phi_{22} - {\phi_{21}}^2}\right]$$

$$V(\hat{\eta}) = \frac{1}{f}\left[\frac{\phi_{11}}{\phi_{11}\phi_{22} - {\phi_{21}}^2}\right]$$

Case 2: When $\eta$is taking know from the method of moment then

$$\phi_{11} = E\left[\frac{\left(\frac{-\partial^2 logL}{\partial\mu^2}\right)}{f}\right] = \left[\frac{1}{1-\mu} + \frac{1}{\mu}\right]$$

$$\phi_{22} = E\left[\frac{\left(\frac{-\partial^2 logL}{\partial\rho^2}\right)}{f}\right] = \left[\frac{1}{\rho} + \frac{1}{1-\rho}\right]$$

And

$$\phi_{12} = \phi_{21} = \frac{E\left[\frac{-\partial^2 logL}{\partial\mu\partial\rho}\right]}{f} = 0$$

$$V(\hat{\mu}) = \frac{1}{f}\left[\frac{\phi_{22}}{\phi_{11}\phi_{22} - {\phi_{21}}^2}\right]$$

$$V(\hat{\rho}) = \frac{1}{f}\left[\frac{\phi_{11}}{\phi_{11}\phi_{22} - {\phi_{21}}^2}\right]$$

Case 3: Whenμ is taking know from the method of moment then:

$$\phi_{11} = E\left[\frac{\left(\frac{-\partial^2 logL}{\partial\rho^2}\right)}{f}\right] = \mu\left[\frac{1}{\rho} + \frac{1}{1-\rho}\right]$$

$$\phi_{22} = E\left[\frac{\left(-\frac{\partial^2 logl}{\partial\eta^2}\right)}{f}\right]$$

$$= (1-\rho)\mu\eta e^{\eta}(e^{\eta}-1)^{-2}\{1 - e^{\eta}(e^{\eta}-1)^{-1}\} + \frac{(1-\rho)\mu(e^{\eta}-1)^{-1}}{\eta}$$

$$+ \frac{\mu\{1-\rho-(1-\rho)\eta(e^{\eta}-1)^{-1}\}\left[\frac{3\eta^2}{4} + \eta\right]}{\left[\frac{\eta^3}{4} + \frac{\eta^2}{2}\right]^2}$$

And

$$\phi_{12} = \phi_{21} = \frac{E\left[\frac{-\partial^2 logL}{\partial\rho\partial\eta}\right]}{f} = 0$$

$$V(\hat{\rho}) = \frac{1}{f}\left[\frac{\phi_{22}}{\phi_{11}\phi_{22} - {\phi_{12}}^2}\right]$$

$$V(\hat{\eta}) = \frac{1}{f}\left[\frac{\phi_{11}}{\phi_{11}\phi_{22} - {\phi_{21}}^2}\right]$$

Application

The proposed probability model in a survey entitled "Rural Development and population Growth – A Sample Survey-1978". Which was conducted by the population research center (PRC) Varanasi (India) has been applied to commutation data collection in survey. The used data in this survey collected from rural areas of Varanasi District and some from its just near district Azamgarh of eastern Uttar Pradesh, India under the research project conduct by center of population studies, Banaras Hindu University Varanasi, India entitled "Evaluation of impact of Development Activities and fertility regulation programs on population growth rates in rural areas." All the household numbering 3514 from the 19 villages covering the population of 23321 are included in the survey personal interview method were used in data collection from the selected household. The information were collected the three type of villages repressing deferent level of soil and economic development semi urban, remote and growth rate ware termed of these three type of villages. Singh 1986 gave the details about the survey. The distribution of the observed and expected frequency of three type of villages have shown in the table 1, 2, and 3 respectively in significant at 5% level of significant with appropriate degree of fredom in MLE and method of moment , therefore commutation pattern are provided by the proposed probability model. Hence the According to the total number of commuter the observed and expected frequency of household are given in table 1, 2 and 3. The estimated value of the parameter is given in table 4 for the three deferent types of villages.

The table 4 indicated that risk of commutation in high in semi urban then the other region the estimated value in also higher in semi urban then the other region.

TABLE 1: Observed and Expected Distribution of household according to total number of commuters from the Semi-Urban type of villages.

Total number of Commuters	Semi-Urban type of villages.		
	Observed	Expected	
		Method of likelihood Estimation	Method Of Moment
0	507	507	507
1	398	397.89	397.96
2	150	149.93	148.48
3	65	66.79	67.43
4	20	19.83	20.08
5^+	07	5.56	6.05
Total	1147	1147	1147
μ		0.5579	0.5579
ρ		0.6218	0.6219
η		0.8909	0.9083
χ^2		0.4226	0.4221

TABLE 2: Observed and Expected Distribution of Household according to total number of Commuters from the Remote type of Villages.

Total number of Commuters	Remote type of villages.		
	Observed	Expected	
		Method of likelihood Estimation	Method Of Moment
0	806	806	806
1	230	229.97	229.37
2	48	44.98	48.68
3	17	18.46	16.76
4	05 }	3.59	5.25
5^+	00 }		
Total	1106		
μ		0.2712	0.2713
ρ		0.7667	0.7665
η		0.7125	0.68884
χ^2		0.6885	1.8

TABLE: 3: Observed and Expected Distribution of household according to total number of commuters from the Growth Centre type of villages.

Total number of Commuters	Growth Centre type of villages.		
	Observed	Expected	
		Method of likelihood Estimation	Method Of Moment
0	947	947	947
1	190	189.98	189.98
2	35	35.0002	33.41
3	06		9.48
4	04 }	10.02	
5^+	00 }		1.796
Total	1182		
μ		0.1988	0.1988
ρ		0.8085	0.8085
η		0.483145	0.567990
χ^2		0.004011	3.22

TABLE: 4: Estimated value of the parameters from the three different types of the villages.

Parameters	Types of the villages.		
	Semi-Urban	Remote	Growth Center
μ	0.5579	0.2712	0.1988
ρ	0.6218	0.7667	0.8085
η	0.89099	0.7125	0.48315

Conclusion

Above proposed probability model may be kept as a useful tool in calculating various probability of commuter connected by the process of commutation from the household and as well as for the predication in a specific population that leads to fresh environment and quality of the social life.

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