Vol.8.Issue.4.2020 (Oct-Dec)
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RESEARCH ARTICLE

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

ON A SIZE BIASED TWO-PARAMETER QUASI-LINDLEY MIXTURE OF SIZE BIASED POISSON DISTRIBUTION

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DOI: 10.33329/bomsr.8.4.59

ABSTRACT
A size biased two-parameter quasi Lindley mixture of size biased Poisson distribution has been obtained by size biasing the two-parameter quasi-Lindley mixture of Poisson distribution of Sah (2015). This distribution has also been obtained by mixing size biased Poisson distribution with the size biased two-parameter quasi-Lindley distribution of Sah (2015). A general expression of the r^{th} factorial moment of this distribution has been derived and hence its first four moments about origin have been obtained. The method of moments of estimation of its parameter has been discussed. The distribution has been fitted to some discrete data sets to test its goodness of fit. It is expected that this distribution gives better fit to similar data-sets than the size-biased Poisson-Lindley distribution of Ghitany and Al Mutairi (2008).

Keywords: Two-parameter quasi-Lindley distribution, Size-biased distribution, Poisson-Lindley distribution, Compounding, Estimation of parameters, Goodness of fit.

1. INTRODUCTION

Let x be a non negative variable having probability function $f(x,\phi)$, with unknown parameter ϕ, where ϕ belongs to parameter space and E(x) is the expected value of x. Then $f^*(x,\phi)$ is known as size-biased distribution with probability function as

$$f^{*}(x,\ \phi)=\frac{x\,f(x,\ \phi)}{E(x)} \tag{1.1}$$

Such that $E(x)>0$ and E(x) must be exist.

Size-biased distribution arises in practice when observations from a sample are recorded with unequal probabilities, having probability proportional to some measure of unit size. The idea of weighted distribution was first given by Fisher (1934). He used these distributions to model ascertainment bias which were later formalized by Rao in a unifying theory. Van Deusen (1986) discussed size-biased distribution theory and applied it to fitting distributions of diameter at breast height (DBH) data arising from horizontal point sampling (HBS).

A size-biased Poisson-Lindley distribution (SBPLD) was obtained by Ghitany and Al Mutairi (2008) and its probability mass function (pmf) has been given by

$$P_1(x,\ \phi)=\frac{\phi^3}{\phi+2}.\frac{x(x+\phi+2)}{(\phi+1)^{x+2}};\ \phi>0\ ;\ x=1,2,3,\ldots \tag{1.2}$$

The expression (1. 2) has been obtained by size biasing the Poisson-Lindley distribution (PLD) of Sankaran (1970) having pmf

$$P_2(x,\ \phi)=\frac{\phi^2(x+\phi+2)}{(\phi+1)^{x+3}};\ \ \phi>0\ ;\ x=0,1,2,\ldots \tag{1.3}$$

It is to be noted that Sankaran obtained the distribution (1.3) by mixing the Poisson distribution with the Lindley (1958) distribution having probability density function (pdf)

$$f_1(x,\ \phi)=\frac{\phi^2}{\phi+1}(1+x)e^{-\phi x};x>0,\phi>0 \tag{1.4}$$

The first four moments about origin of the SBPLD (1.2) have been obtained as

$$\mu_2'=1+\frac{2(\phi+3)}{\phi(\phi+2)} \tag{1.5}$$

$$\mu_2'=1+\frac{6(\phi+3)}{\phi(\phi+2)}+\frac{6(\phi+4)}{\phi^2(\phi+2)} \tag{1.6}$$

$$\mu_3'=1+\frac{14(\phi+3)}{\phi(\phi+2)}+\frac{36(\phi+4)}{\phi^2(\phi+2)}+\frac{24(\phi+5)}{\phi^3(\phi+2)} \tag{1.7}$$

$$\mu_4'=1+\frac{30(\phi+3)}{\phi(\phi+2)}+\frac{126(\phi+4)}{\phi^2(\phi+2)}+\frac{240(\phi+5)}{\phi^3(\phi+2)}+\frac{120(\phi+6)}{\phi^4(\phi+2)} \tag{1.8}$$

and its variance has been obtained as

$$\mu_2=\frac{2\left(\phi^3+6\phi^2+12\phi+6\right)}{\phi^2(\phi+2)^2} \tag{1.9}$$

Sah (2015) obtained a two-parameter quasi Lindley mixture of Poisson distribution given by its pmf

$$P_3(x;\phi,\alpha)=\frac{\phi^3}{\left(\phi^2+\phi+2\alpha\right)}.\left[\frac{(1+\phi)^2+(x+1)(1+\phi)+\alpha(x+1)(x+2)}{(1+\phi)^{x+3}}\right]$$

$$x=0,1,2,\ldots..;\phi>0,\ \alpha>-\frac{\phi(1+\phi)}{2} \qquad \ldots\ (1.10)$$

It can be observed that the PLD (1.3) is a particular case of (1.10) at $\alpha = 0$. Sah (2015) has shown that (1.10) is a better model than the PLD (1.3) for analyzing different type of count data. The distribution (1.10) arises from the Poisson distribution when its parameter λ follows two-parameter quasi Lindley distribution of Sah (2015) having its probability density function (pdf)

$$f_2(x;\phi,\alpha) = \frac{\phi^3}{(\phi^2+\phi+2\alpha)}(1+x+\alpha x^2)e^{-\phi x} \quad ; x>0;\phi>0;\alpha>-\left[\frac{\phi(1+\phi)}{2}\right] \tag{1.11}$$

In this paper, a size-biased two-parameter quasi Lindley mixture of size biased Poisson distribution (SBTPQLMSBPD), of which the SBPLD (1.2) is a particular case, has been obtained. A general expression for the r^{th} factorial moment of this distribution has been obtained and hence its first four moments about origin of this distribution have been obtained. The method of maximum likelihood and the method of moments of estimation of its parameters have been discussed. The distribution has been fitted to some discrete data-sets having variance greater than the mean and it has been found that to almost all these data-sets, it provides closer fits than the SBPLD (1.2). Hence, it is expected that the SBTPQLMSBPD (2.2) is more flexible than SBPLD (1.2) of the Ghitany et al (2008) for analysing different types of count data.

2 Size-Biased Two-Parameter Quasi Lindley Mixture of Size Biased Poisson Distribution (SBTPQLMSBPD)

The probability mass function (pmf) of size-biased two-parameter quasi Lindley mixture of size biased Poisson distribution with parameters ϕ and α is obtained as

$$P_4(x;\phi,\alpha) = \frac{x P_3(x;\phi,\alpha)}{\mu_1^{/}} \tag{2.1}$$

Taking the pmf $P_3(x;\phi,\alpha)$ from (1.10) and its mean $\mu_1^{/} = \frac{(\phi^2+2\phi+6\alpha)}{\phi(\phi^2+\phi+2\alpha)}$ of Sah (2015), we get

$$P_4(x;\phi,\alpha) = \frac{\phi^4}{(\phi^2+2\phi+6\alpha)} \frac{\left[x(1+\phi)^2 + x(x+1)(1+\phi) + \alpha x(x+1)(x+2)\right]}{(1+\phi)^{x+3}}$$

$$x = 1,2,3,.....;\phi>0;\alpha>-\left[\frac{\phi(2+\phi)}{6}\right] \tag{2.2}$$

The expression (2.2) is reduced to (1.2) at $\alpha = 0$. The SBTPQLMSBPD can also be obtained by mixing size biased Poisson distribution with size biased TPQLD having pdf

$$f_3(x;\phi,\alpha) = \frac{\phi^4}{(\phi^2+2\phi+6\alpha)}(x+x^2+\alpha x^3)e^{-\phi x} \quad ; x>0,\phi>0,\alpha>-\left[\frac{\phi(\phi+2)}{6}\right] \tag{2.3}$$

The pmf of size-biased Poisson distribution with parameter λ is given by

$$P_5(x;\lambda) = \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!} \quad ; x>1,\ \lambda>0 \tag{2.4}$$

Where λ follows SBTPQLD (2.3).

SBTPQLMSBPD can also be obtained as follows

$$P(X=x) = \int_0^\infty \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}\cdot\frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\cdot(\lambda+\lambda^2+\alpha\lambda^3)e^{-\phi\lambda}\,d\lambda \tag{2.5}$$

$$=\frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\cdot\frac{1}{(x-1)!}\int_0^\infty e^{-(\phi+1)\lambda}\left(\lambda^x+\lambda^{x+1}+\alpha\lambda^{x+2}\right)d\lambda$$

$$=\frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\cdot\frac{1}{(x-1)!}\left[\frac{\Gamma(x+1)}{(\phi+1)^{x+1}}+\frac{\Gamma(x+2)}{(\phi+1)^{x+2}}+\frac{\alpha\Gamma(x+3)}{(\phi+1)^{x+3}}\right]$$

$$=\frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\cdot\frac{1}{(x-1)!}\left[\frac{x!}{(\phi+1)^{x+1}}+\frac{(x+1)!}{(\phi+1)^{x+2}}+\frac{\alpha(x+2)!}{(\phi+1)^{x+3}}\right]$$

$$=\frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\cdot\frac{1}{(1+\phi)^{x+1}}\left[\frac{x(1+\phi)^2+x(x+1)(1+\phi)+\alpha x(x+1)(x+2)}{(1+\phi)^2}\right] \quad (2.6)$$

Which is the pmf of SBTPQLMSBPD is as obtained in the expression (2.2).

Where $x=1,2,3,.....,\phi>0,\alpha>-\left[\frac{\phi(\phi+2)}{6}\right]$.

At $\alpha=0$, the expression (2.6) is reduced to SBPLD (1.2). Hence, the pmf of SBTPQLMSBPD is obtained as

$$P(X=x)=\frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\cdot\frac{1}{(1+\phi)^{x+3}}\cdot\left[x(1+\phi)^2+x(x+1)(1+\phi)+\alpha x(x+1)(x+2)\right]$$

Where, $x=1,2,3,...;\phi>0,\alpha>-\left[\frac{\phi(\phi+2)}{6}\right]$ (2.7)

3. MOMENTS OF SBTPQLMSBPD

The r^{th} factorial moments about origin is defined as

$$\mu'_{(r)}=\frac{\sum_{i=1}^{n} f_i x_i^{(r)}}{N} \quad (3.1)$$

Where, $x^{(r)}=x(x-1)(x-2)......(x-r+1)$

The r^{th} factorial moment of the SBTPQLMSBPD can be obtained as

$$\mu'_{(r)}=E\left[E\left(x^{(r)}/\lambda\right)\right] \quad (3.2)$$

Where, $x^{(r)}=x(x-1)(x-2)......(x-r+1)$

From (2.5), we get

$$\mu'_{(r)}=\int_0^\infty\left[\sum_{x=0}^{\infty}\frac{x^{(r)}e^{-\lambda}\lambda^{x-1}}{(x-1)!}\right]\frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\left(\lambda+\lambda^2+\alpha\lambda^3\right)e^{-\phi\lambda}d\lambda \quad (3.3)$$

$$=\int_0^\infty\left[\sum_{x=0}^{\infty}\frac{x(x-1)...(x-r+1)}{(x-1)(x-2)...(x-r+1)(x-r)!}.e^{-\lambda}\lambda^{x-r}.\lambda^{r-1}\right]\cdot\frac{\phi^4}{(\phi^2+2\phi+6\alpha)}(\lambda+\lambda^2+\alpha\lambda^3)e^{-\phi\lambda}d\lambda$$

$$=\frac{\phi^4}{\phi^2+2\phi+6\alpha}\int_0^\infty\left[\lambda^{r-1}\sum_{x=r}^{\infty}\frac{x\,e^{-\lambda}\lambda^{x-r}}{(x-r)!}\right]\left(\lambda+\lambda^2+\alpha\lambda^3\right)e^{-\phi\lambda}d\lambda$$

Taking (x+r) in place of x, we get

$$\mu'_{(r)} = \frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\int_0^\infty \lambda^{r-1}\left[\sum_{x=0}^{\infty}\frac{(x+r)e^{-\lambda}\lambda^x}{x!}\right](\lambda+\lambda^2+\alpha\lambda^3)e^{-\phi x}\,d\lambda$$

The expression under bracket is clearly (λ +r) and hence we have

$$= \frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\int_0^\infty (\lambda+r)\lambda^r(1+\lambda+\alpha\lambda^2)d\lambda$$

$$= \frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\int_0^\infty (\lambda^{x+r}+r\lambda^r)(1+\lambda+\alpha\lambda^2)d\lambda$$

$$= \frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\int_0^\infty \left[r\lambda^r+(1+r)\lambda^{r+1}+(1+r\alpha)\lambda^{r+2}+\alpha\lambda^{r+3}\right]e^{-\phi\lambda}\,d\lambda$$

$$\therefore \mu'_{(r)} = \frac{\phi^4}{(\phi^2+2\phi+6\alpha)}\left[r.\frac{\Gamma(r+1)}{\phi^{r+1}}+\frac{(1+r)\Gamma(r+2)}{\phi^{r+2}}+\frac{(1+r\alpha)\Gamma(r+3)}{\phi^{r+3}}+\alpha\frac{\Gamma(r+4)}{\phi^{r+4}}\right]$$

$$= \frac{\phi^4\Gamma(r+1)}{(\phi^2+2\phi+6\alpha)}\left[\frac{r}{\phi^{r+1}}+\frac{(1+r)(1+r)}{\phi^{r+2}}+\frac{(1+r\alpha)(r+2)(r+1)}{\phi^{r+3}}+\alpha\frac{(r+3)(r+2)(r+1)}{\phi^{r+4}}\right] \quad (3.4)$$

The expression (3.4) is a general expression for the r^{th} factorial moment about origin of the SBTPQLMSBPD (2.7). After obtaining the first four factorial moments, by substituting r=1, 2, 3 and 4 in (3.4) and then using the relationship between factorial moments and moments about origin, the first four moments about origin of the SBTPQLMSBPD are obtained as

$$\mu'_1 = 1+\frac{(2\phi^2+6\phi+24\alpha)}{\phi(\phi^2+2\phi+6\alpha)} \quad (3.5)$$

$$\mu'_2 = 1+\frac{6(\phi^2+3\phi+12\alpha)}{\phi(\phi^2+2\phi+6\alpha)}+\frac{6(\phi^2+4\phi+20\alpha)}{\phi^2(\phi^2+2\phi+6\alpha)} \quad (3.6)$$

$$\mu'_3 = 1+\frac{14(\phi^2+3\phi+12\alpha)}{\phi(\phi^2+2\phi+6\alpha)}+\frac{36(\phi^2+4\phi+20\alpha)}{\phi^2(\phi^2+2\phi+6\alpha)}+\frac{24(\phi^2+5\phi+30\alpha)}{\phi^3(\phi^2+2\phi+6\alpha)} \quad (3.7)$$

$$\mu'_4 = 1+\frac{30(\phi^2+3\phi+12\alpha)}{\phi(\phi^2+2\phi+6\alpha)}+\frac{150(\phi^2+4\phi+20\alpha)}{\phi^2(\phi^2+2\phi+6\alpha)}+\frac{240(\phi^2+5\phi+30\alpha)}{\phi^3(\phi^2+2\phi+6\alpha)}+\frac{120(\phi^2+6\phi+42\alpha)}{\phi^4(\phi^2+2\phi+6\alpha)} \quad (3.8)$$

Variance of the SBTPQLMSBPD is obtained as

$$\mu_2 = \mu'_2 - {\mu'_1}^2$$

$$= \frac{2(\phi^2+3\phi+12\alpha)}{\phi(\phi^2+2\phi+6\alpha)}\left[1-\frac{2(\phi^2+3\phi+12\alpha)}{\phi(\phi^2+2\phi+6\alpha)}\right]+\frac{6(\phi^2+4\phi+20\alpha)}{\phi^2(\phi^2+2\phi+6\alpha)} \quad (3.9)$$

It can be easily verified that at $\alpha = 0$, these moments are reduced to the respective moments, (1.5) to (1.9), of the SBPLD.

4. ESTIMATION OF PARAMETERS

4.1 Maximum Likelihood Estimates

Let (x_1, x_2, …, x_n) be a random sample of size n from the SBTPQLMSBPD (2.2) and let f_x be the observed frequency in the sample corresponding to X=x such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency. The likelihood function L of the SBTPQLMSBPD is given by

$$L=\left(\frac{\phi^4}{\phi^2+2\phi+6\alpha}\right)^n . \frac{1}{(1+\phi)^{\sum_{x=1}^{k}(x+3)f_x}} \prod_{x=1}^{k}\left[x(1+\phi)^2+x(x+1)(1+\phi)+\alpha x(x+1)(x+2)\right]^{f_x} \qquad (4.1)$$

and hence the log likelihood function is obtained as

$$\log L = n\log\left(\frac{\phi^4}{\phi^2+2\phi+6\alpha}\right)-\left[\sum_{x=1}^{k} f_x(x+3)\right].\log(1+\phi)+\sum_{x=1}^{k} f_x \log\left[x(1+\phi)^2+x(x+1)(1+\phi)+\alpha\, x(x+1)(x+2)\right] \qquad (4.2)$$

The two log likelihood equations are thus obtained as

$$\frac{\partial \log L}{\partial \phi}=\frac{4n}{\phi}-\frac{2n(\phi+1)}{\left(\phi^2+2\phi+6\alpha\right)}-\sum_{x=1}^{k} f_x \frac{(x+3)}{(1+\phi)}+\sum_{x=1}^{k} f_x \frac{[2x(1+\phi)+x(x+1)]}{\left[x(1+\phi)^2+x(x+1)(1+\phi)+\alpha x(x+1)(x+2)\right]} \qquad (4.3)$$

$$\frac{\partial \log L}{\partial \alpha}=\frac{-6n}{\left(\phi^2+2\phi+6\alpha\right)}+\sum_{x=1}^{k} f_x \frac{[x(x+1)(x+2)]}{\left[x(1+\phi)^2+x(x+1)(1+\phi)+\alpha x(x+1)(x+2)\right]} \qquad (4.4)$$

The two equations (4.3) and (4.4) are difficult to solve directly. However, the Fisher's scoring method can be applied to solve these equations. We have,

$$\frac{\partial^2 \log L}{\partial \phi^2}=-\frac{4n}{\phi^2}-\frac{2n}{\left(\phi^2+2\phi+6\alpha\right)}+\frac{4n(1+\phi)^2}{\left(\phi^2+2\phi+6\alpha\right)^2}+\sum_{x=1}^{k} f_x \frac{(x+3)}{(1+\phi)^2}$$
$$+\sum_{x=1}^{k}\frac{2x f_x}{\left[x(1+\phi)^2+x(x+1)(1+\phi)+\alpha x(x+1)(x+2)\right]}-\sum_{x=1}^{k}\frac{[2x(1+\phi)+x(x+1)]^2}{\left[x(1+\phi)^2+x(x+1)(1+\phi)+\alpha x(x+1)(x+2)\right]^2} \qquad (4.5)$$

$$\frac{\partial^2 \log L}{\partial \phi \partial \alpha}=\frac{12n(1+\phi)}{\left(\phi^2+2\phi+6\alpha\right)^2}-\sum_{x=1}^{k}\frac{[x(x+1)(x+2)][2x+(1+\phi)+x(x+1)]f_x}{\left[x(1+\phi)^2+x(x+1)(1+\phi)+\alpha x(x+1)(x+2)\right]^2} \qquad (4.6)$$

$$\frac{\partial^2 \log L}{\partial \alpha^2}=\frac{36n}{\left(\phi^2+2\phi+6\alpha\right)^2}-\sum_{x=1}^{k}\frac{[x(x+1)(x+2)]^2 f_x}{\left[x(1+\phi)^2+x(x+1)(1+\phi)+\alpha x(x+1)(x+2)\right]^2} \qquad (4.7)$$

Using which the equations can be solved for the estimates $\hat{\phi}$ and $\hat{\alpha}$, where ϕ_0 and α_0 are the initial values of ϕ and α respectively.

$$\begin{bmatrix} \frac{\partial^2 \log L}{\partial \phi^2} & \frac{\partial^2 \log L}{\partial \phi\, \partial \alpha} \\ \frac{\partial^2 \log L}{\partial \phi\, \partial \alpha} & \frac{\partial^2 \log L}{\partial \alpha^2} \end{bmatrix}_{\substack{\hat{\phi}=\phi_0 \\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \hat{\phi}-\phi_0 \\ \hat{\alpha}-\alpha_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial \log L}{\partial \phi} \\ \frac{\partial \log L}{\partial \alpha} \end{bmatrix}_{\substack{\hat{\phi}=\phi_0 \\ \hat{\alpha}=\alpha_0}} \qquad (4.8)$$

These equations are solved numerically and iteratively till sufficiently close estimates $\hat{\phi}$ and $\hat{\alpha}$ are obtained.

4.2 Method of Moments

The SBTPQLMSBPD has two parameters to be estimated and so its first two moments are required to get the estimates of its parameters by the method of moments.

From (3.5) and (3.6), we have

$$\frac{\left(\mu_2^{/}-1\right)-3\left(\mu_1^{/}-1\right)}{\left(\mu_1^{/}-1\right)}=\frac{3\left(\phi^2+4\phi+20\alpha\right)}{\phi\left(\phi^2+3\phi+12\alpha\right)} \tag{4.9}$$

Which gives a polynomial equation in ϕ as

$$g(\phi)=K_1K\ \phi^3-(7-K)K_1\phi^2-8(K_1-1)\phi+12=0 \tag{4.10}$$

Where $K=\frac{\left(\mu_2^{/}-1\right)-3\left(\mu_1^{/}-1\right)}{\left(\mu_1^{/}-1\right)}$ & $K_1-\mu_1^{/}-1$

Replacing the first two population moments by respective sample moments value of K and K_1 can be estimated. Substituting the value of K & K_1 in (4.10) and solving it by Regula-Falsi method or Newton-Rapsion method, an estimate $\hat{\phi}$ of ϕ is obtained.

Again, an estimate $\hat{\alpha}$ of α can be obtained by using

$$\hat{\alpha}=\frac{K\phi^3+3\phi^2(K-1)-12\phi}{12(5-K\phi)} \tag{4.11}$$

5. GOODNESS OF FIT

The SBTPQLMSBPD has been fitted to a number of data-sets related to a number of observations of the size distribution of 'freely-forming' small group in various public situations reported by James(1953), Coleman and James(1961), and Simonoff(2003) and it was found that to all these data-sets the SBTPQLMSBPD provides closer fits than the SBPLD. The fittings of the SBTPQLMSBPD to only three data-sets have been presented in the following tables.

The expected frequencies according to the SBPLD have also been given in these tables for ready comparison with those obtained by the SBTPQLMSBPD. The estimates of the parameters have been obtained by the method of moments. It can be seen that the SBTPQLMSBPD gives much closer fits than the SBPLD and thus provides a better alternative to the SBPLD for the similar types of data-sets.

Table 1: Counts of groups of people in public places on a spring afternoon in Portland

Size of Groups	Observed Frequency	Expected Frequency	
		SBPLD	**SBTPQLMSBPD**
1	1486	1532.5	1500.8
2	694	630.6	664.4
3	195	191.9	201.1
4	37	51.3	48.0
5	10	12.8	8.2
6	1	3.9	0.5
Total	**2423.0**	**2423.0**	**2423.0**
$\mu_1^{/} = 1.511762$			
$\mu_2^{/} = 2.845645$			
$\mu_2 = \mu_2^{/} - \mu_1^{/2} = 0.560220655$			
$\hat{\phi}$:		4.5224	3.354374
$\hat{\alpha}$:			-0.861161
χ^2 :		13.766	4.778
d.f. :		3	2
P(χ^2):		< 0.01	0.094

Table 2: Counts of Shopping Groups- Eugene, spring, Department Store and Public Market.

Size of Groups	**Observed Frequency**	**Expected Frequency**	
		SBPLD	**SBTPQLMSBPD**
1	316	313.0	317.8
2	141	132.5	138.5
3	44	40.2	41.8
4	5	10.7	10.1
5	4	3.6	2.0
Total	**510.0**	**510.0**	**510.0**
$\mu_1^{/} = 1.509803922$			
$\mu_2^{/} = 2.854901961$			
$\hat{\phi}$:		4.5224	3.493027
$\hat{\alpha}$:		-	-0.840579
d.f. :		2	1
χ^2 :		3.021	0.973
P(χ^2):		0.40	0.615

Table 3: Counts of Play Groups-Eugene, spring, Public Playground 'D'

Size of Groups	Observed Frequency	Expected Frequency	
		SBPLD	**SBTPQLMSBPD**
1	305	314.4	307.5
2	144	134.4	141.4
3	50	42.5	44.7
4	5	11.8	11.2
5	2	3.1	2.1
6	1	0.8	0.1
Total	**507**	**507.0**	**507.0**
$\mu_1' = 1.536489152$			
$\mu_2' = 2.952662722$			
$\hat{\phi}$:		4.3179	3.222461
$\hat{\alpha}$:		-	0.807855
χ^2 :		6.4115	2.872
d.f. :		2	1
P(χ^2):		0.043	0.093

6. CONCLUSION

In this paper, a size-biased two-parameter quasi Lindley mixture of size biased Poisson distribution (SBTPQLMSBPD) of which the size-biased Poisson-Lindley distribution (SBPLD) is a particular case has been introduced to model count data. A general expression for the r^{th} factorial moment of this distribution has been obtained and hence its first four moments about origin of this distribution have been obtained. The method of maximum likelihood and the method of moments of estimation of its parameters have been discussed and the obtained distribution is fitted to some data-sets which were used by Shanker and Mishra (2013). It has been found that to all these data-sets it provides closer fits to observed data-sets than those provided by the SBPLD and hence it should be preferred to the SBPLD while modeling count data-sets.

ACKNOWLEDGEMENTS

The authors express their thankfulness to the learned referee for his valuable comments and suggestions which improved the quality of the paper.

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