Vol.9.Issue.3.2021 (July-Sept)
©KY PUBLICATIONS

http://www.bomsr.com
Email:editorbomsr@gmail.com

RESEARCH ARTICLE

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

ON THE HOMOGENEOUS CUBIC DIOPHANTINE EQUATION WITH FIVE UNKNOWNS $2(x^3+y^3)=31(z+w)P^2$

S.VIDHYALAKSHMI[1], **J.SHANTHI**[2], **M.A.GOPALAN**[3]

[1,2] Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.
email:vidhyasigc@gmail.com, shanthivishvaa@gmail.com

[3] Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.
email: mayilgopalan@gmail.com

DOI:10.33329/bomsr.9.3.50

ABSTRACT

The homogeneous cubic diophantine equation with five unknowns given by $2(x^3+y^3)=31(z+w)P^2$ is analyzed for its non-zero distinct integer solutions through applying the linear transformtions .

Keywords: Cubic equation with five unknowns, Homogeneous cubic, Integral solutions

Introduction:

The cubic diophantine equations are rich in variety and offer an unlimited field for research [1,2]. In particular, refer [3-24] for a few problems on cubic equation with 3 and 4 unknowns. This paper concerns with an interesting homogeneous cubic diophantine equation with five unknowns given by $2(x^3+y^3)=31(z+w)P^2$ for determining its infinitely many non-zero distinct integral solutions by employing linear transformations.

Method of Analysis:

The homogeneous cubic equation with five unknowns under consideration is

$$2(x^3+y^3)=31(z+w)P^2 \tag{1}$$

Employing the linear transformations

$$x = u + v, y = u - v, z = 2u + v, w = 2u - v, u \neq v \neq 0 \quad (2)$$

in (1), it reduces to the equation

$$u^2 + 3v^2 = 31P^2 \quad (3)$$

which is solved through different ways and thus,inview of (2),one obtains different sets of integer solutions to (1).

Way:1

Let

$$P = a^2 + 3b^2 \quad (4)$$

Write 31 as

$$31 = (2 + i3\sqrt{3})(2 - i3\sqrt{3}) \quad (5)$$

Substituting (4) and (5) in (3) and employing the method of factorization, define

$$u + i\sqrt{3}v = (2 + i3\sqrt{3})(a + i\sqrt{3}b)^2$$

On equating the real and imaginary parts, one obtains

$$u = 2(a^2 - 3b^2) - 18ab \ , \quad v = 3(a^2 - 3b^2) + 4ab$$

In view of (2), the values of x,y,z and w are given by

$$\left.\begin{aligned} x &= 5a^2 - 15b^2 - 14ab \\ y &= -a^2 + 3b^2 - 22ab \\ z &= 7(a^2 - 3b^2) - 32ab \\ w &= a^2 - 3b^2 - 40ab \end{aligned}\right\} \quad (6)$$

Thus (4) and (6) represent the integer solutions to (1).

Note :1

One may consider 31 on the R.H.S. of (3) as

$$31 = \frac{(7 + i5\sqrt{3})(7 - i5\sqrt{3})}{4}$$

The repetition of the above process leads to a different set of integer solutions to (1).

Way:2

(3) can be written as

$$3v^2 = 31P^2 - u^2 \quad (7)$$

Let

$$v = 31a^2 - b^2 \quad (8)$$

Write 3 as

$$3=(2\sqrt{31}+11)(2\sqrt{31}-11) \tag{9}$$

Substituting (8) and (9) in (7) and employing the method of factorization, define

$$\sqrt{31}P+u=(2\sqrt{31}+11)(\sqrt{31}a+b)^2$$

On equating the rational and irrational parts, one obtains

$$u=11(31a^2+b^2)+124ab$$

$$, \quad P=2(31a^2+b^2)+22ab \tag{10}$$

In view of (2), the values of x,y,z and w are given by

$$\left.\begin{aligned} x&=372a^2+10b^2+124ab\\ y&=310a^2+12b^2+124ab\\ z&=713a^2+21b^2+248ab\\ w&=651a^2+23b^2+248ab \end{aligned}\right\} , \quad , \tag{11}$$

Thus (10) and (11) represent the integer solutions to (1).

Note :2

One may consider 3 on the L.H.S. of (7) as

$$3=\frac{(\sqrt{31}+2)(\sqrt{31}-2)}{9}$$

$$3=\frac{(2\sqrt{31}+7)(2\sqrt{31}-7)}{25}$$

The repetition of the above process leads to two different sets of integer solutions to (1).

Way:3

(3) is written as

$$u^2+3v^2=31P^2*1 \tag{12}$$

Write 1 on the R.H.S. of (12) as

$$1=\frac{(1+i\sqrt{3})(1-i\sqrt{3})}{4} \tag{13}$$

Following the analysis presented in **Way:2,**the corresponding integer solutions to (1) are as below:

$$x=-4a^2+12b^2-88ab \, ,$$

$$y=-24a^2+72b^2-32ab \, ,$$

$$z=-18a^2+54b^2-148ab$$

$w = -38a^2 + 114b^2 - 92ab$,

$P = 4(a^2 + 3b^2)$

Note :3

1 on the R.H.S. Of (12) is also taken as

$$1 = \frac{(1 + i4\sqrt{3})(1 - i4\sqrt{3})}{49}$$

$$1 = \frac{(3r^2 - s^2 + i\sqrt{3}2rs)(3r^2 - s^2 - i\sqrt{3}2rs)}{(3r^2 + s^2)^2}$$

The repetition of the above process leads to two different sets of integer solutions to (1).

Way:4

(3) is written as

$$31P^2 - 3v^2 = u^2 * 1 \tag{14}$$

Let

$$u = 31a^2 - 3b^2$$

Write 1 on the R.H.S. of (14) as

$$1 = \frac{(\sqrt{31} + 3\sqrt{3})(\sqrt{31} - 3\sqrt{3})}{4}$$

Proceeding as above,one obtains a new set of integer solutions to (1).

Note :4

1 on the R.H.S. Of (14) is also taken as

$$1 = \frac{(2\sqrt{31} + 5\sqrt{3})(2\sqrt{31} - 5\sqrt{3})}{49}$$

The repetition of the above process leads to a different set of integer solutions to (1).

Conclusion

In this paper, an attempt has been made to obtain many non-zero distinct integral solutions to the homogeneous cubic equation with five unknowns represented by $2(x^3 + y^3) = 31(z + w)P^2$. As cubic equations are rich in variety, the readers may search for obtaining integer solutions to other choices of cubic equations.

REFERENCES

[1] L.E. Dickson, History of Theory of Numbers, Vol.2, Chelsea Publishing company, NewYork, 1952.

[2] L.J. Mordell, Diophantine equations, Academic press, New York, 1969.

[3] M.A. Gopalan, G. Sangeetha, "On the ternary cubic Diophantine equation $y^2 = Dx^2 + z^3$", Archimedes J.Math 1(1), 2011, 7-14.

[4] L.E. Dickson, History of Theory of Numbers, Vol.2, Chelsea Publishing company, NewYork, 1952

[5] M.A. Gopalan, B. Sivakami, "Integral solutions of the ternary cubic equation $4x^2 - 4xy + 6y^2 = ((k+1)^2 + 5)w^3$",Impact J.Sci.Tech, Vol.6, No.1, 2012, 15-22.

[6] M.A. Gopalan, B. Sivakami, "On the ternary cubic Diophantine equation $2xz = y^2(x+z)$", Bessel J.Math 2(3), 2012, 171-177.

[7] S. Vidyalakshmi, T.R. Usharani, M.A. Gopalan, "Integral solutions of non-homogeneous ternary cubic equation $ax^2 + by^2 = (a+b)z^3$", Diophantus J.Math 2(1), 2013, 31-38.

[8] M.A. Gopalan, K. Geetha, "On the ternary cubic Diophantine equation $x^2 + y^2 - xy = z^3$", Bessel J.Math., 3(2), 2013,119-123.

[9] M.A. Gopalan, S. Vidhyalakshmi, A.Kavitha "Observations on the ternary cubic equation $x^2 + y^2 + xy = 12z^3$", Antartica J.Math 10(5), 2013, 453-460.

[10] M.A. Gopalan, S. Vidhyalakshmi, K. Lakshmi, "Lattice points on the non-homogeneous cubic equation $x^3 + y^3 + z^3 + (x+y+z) = 0$", Impact J.Sci.Tech, Vol.7, No.1, 2013, 21-25.

[11] M.A. Gopalan, S. Vidhyalakshmi, K. Lakshmi "Lattice points on the non-homogeneous cubic equation $x^3 + y^3 + z^3 - (x+y+z) = 0$", Impact J.Sci.Tech, Vol.7, No1, 2013, 51-55,

[12] M.A. Gopalan, S. Vidhyalakshmi, S. Mallika, "On the ternary non-homogenous cubic equation $x^3 + y^3 - 3(x+y) = 2(3k^2 - 2)z^3$", Impact J.Sci.Tech, Vol.7, No.1, 2013, 41-45.

[13] S. Vidhyalakshmi, M.A. Gopalan, S. Aarthy Thangam, "On the ternary cubic Diophantine equation $4(x^2 + x) + 5(y^2 + 2y) = -6 + 14z^3$" International Journal of Innovative Research and Review (JIRR), Vol 2(3)., pp 34-39, July-Sep 2014

[14] M.A. Gopalan, N. Thiruniraiselvi and V. Kiruthika, "On the ternary cubic diophantine equation $7x^2 - 4y^2 = 3z^3$", IJRSR, Vol.6, Issue-9, Sep-2015, 6197-6199.

[15] M.A. Gopalan, S. Vidhyalakshmi, J. Shanthi, J. Maheswari, "On ternary cubic diophantine equation $3(x^2 + y^2) - 5xy + x + y + 1 = 12z^3$", International Journal of Applied Research, 1(8), 2015, 209-212.

[16] R. Anbuselvi, K. Kannaki, "On ternary cubic diophantine equation $3(x^2 + y^2) - 5xy + x + y + 1 = 15z^3$", IJSR, Vol.5, Issue-9, Sep 2016, 369-375.

[17] G. Janaki, C. Saranya, "Integral solutions of the ternary cubic equation $3(x^2 + y^2) - 4xy + 2(x + y + 1) = 972z^3$", IRJET, Vol.04, Issue 3, March 2017, 665-669.

[18] S. Vidhyalakshmi, T.R. Usha Rani, M.A. Gopalan, V. Kiruthika, "On the cubic equation with four unknowns $x^3 + y^3 = 14zw^2$", IJSRP, Volume 5, Issue 3, March 2015, 1-11.

[19] M.A. Gopalan, S. Vidhyalakshmi, G. Sumathi, "On the homogeneous cubic equation with four unknowns $X^3 + Y^3 = 14Z^3 - 3W^2(X+Y)$", Discovery, 2(4), 2012, 17-19.

[20] S. Vidhyalakshmi, M.A. Gopalan, A. Kavitha, "Observation on homogeneous cubic equation with four unknowns $X^3 + Y^3 = 7^{2n} ZW^2$", IJMER, Vol.3, Issue 3, May-June 2013, 1487-1492.

[21] M.A. Gopalan, S. Vidhyalakshmi, E. Premalatha, C. Nithya, "On the cubic equation with four unknowns $x^3 + y^3 = 31(k^2 + 3s^2)zw^2$", IJSIMR, Vol.2, Issue 11, Nov-2014, 923-926.

[22] M.A. Gopalan, S. Vidhyalakshmi, J. Shanthi, "On the cubic equation with four unknowns $x^3 + 4z^3 = y^3 + 4w^3 + 6(x-y)^3$", International Journal of Mathematics Trends and Technology, Vol 20, No.1, April 2015, 75-84.

[23] Dr. R. Anbuselvi, K.S. Araththi, "On the cubic equation with four unknowns $x^3 + y^3 = 24zw^2$", IJERA, Vol.7, Issue 11 (Part-I), Nov-2017, 01-06.

[24] E.Premalatha , M.A.Gopalan, "On Homogeneous Cubic Equation with Four Unknowns $x^3 + y^3 = 13zw^2$", International Journal of Advances in Engineering and Management (IJAEM), Volume 2, Issue 2, 2020, Pp: 31-41.