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DEVOLPMENT OF A NEW TRANSFORMATION TO SOLVE A NEW TYPE OF ORDINARY LINEAR DIFDERENTIAL EQUATION

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ABSTRACT

The purpose of this paper is to present a definition of a novel integral transform, which we refer to as the AMK-Transform. Also, properties, transformations of logarithmic, trigonometric, constants and other functions are also introduced. In addition, the use of transform and inverse of transform to solve ordinary linear differential equations is discussed.

Keywords: AMK-Transformation, Sine coefficient, Inverse of AMK-Transformation, AZ-Equation, L.O.D.E with sine coefficient.

Introduction:

Laplace transformation [1] is one of the known techniques to solve the ordinary linear differential equation with constant coefficient with initial conditions and which is presented as

$$\delta_0 \frac{d^n y(\varsigma)}{d\varsigma^n} + \delta_1 \frac{d^{n-1} y(\varsigma)}{d\varsigma^{n-1}} + \delta_2 \frac{d^{n-2} y(\varsigma)}{d\varsigma^{n-2}} + \ldots + \delta_{n-1} \frac{dy(\varsigma)}{d\varsigma} + \delta_n y(\varsigma) = f(\varsigma) \quad (1)$$

Where $\delta_0, \delta_1, \delta_2, \ldots, \delta_n$ are constant. In this paper we are going to apply a new transformation to solve the linear differential equation (L.O.D.E) with sine coefficient, which have general form

$$\delta_0 \left(sin\varsigma\right)^n \frac{d^n y\left(sin\varsigma\right)}{d\varsigma^n} + \delta_1 \left(sin\varsigma\right)^{n-1} \frac{d^{n-1} y(sin\varsigma)}{d\varsigma^{n-1}} + \delta_2 \left(sin\varsigma\right)^{n-2} \frac{d^{n-2} y(sin\varsigma)}{d\varsigma^{n-2}} + \ldots + \delta_n y\left(sin\varsigma\right)$$
$$= f\left(sin\varsigma\right) \quad (2)$$

The above equation is known as **AZ (Ali and Zafar) equation**. The transformation is introduced for some functions.

Definition (1): [2]

Let f is a function described on (a,b), then the integral transform for function f that is $F(\varpi)$ described as

$$F(\varpi)=\int_a^b k(\varpi,\varsigma)\, f(\varsigma)\, d\varsigma\,, \tag{3}$$

Where k is a fix kernel of the transformation with two variable, $a,b\in R$, the above integral is convergent.

Definition (2):

Ali Moazzam and Kashif-Transformation (AMK-transformation)

The AMK-transform for the $f(sin\varsigma)$ where $\varsigma\in\left[0,\frac{\pi}{2}\right]$ is described as

$$AMK\{f(sin\varsigma)\}=\int_0^{\frac{\pi}{2}}(sin\varsigma)^{\varpi}\, cos\varsigma\, f(sin\varsigma)\, d\varsigma=F(\varpi) \tag{4}$$

Where ϖ is constant and the integral is convergent in interval $\left[0,\frac{\pi}{2}\right]$.

Property (1): (Linear Property)

$AMK\{\alpha f(sin\varsigma)\pm\beta g(sin\varsigma)\}=\alpha AMK\{f(sin\varsigma)\}\pm\beta AMK\{g(sin\varsigma)\}$, where α,β are constants.

Theorem (1):

If $F(\varpi)$ is defined as $AMK\{f(sin\varsigma)\}=F(\varpi)$ and a is a constant then

$$AMK\{(sin\varsigma)^a\, f(sin\varsigma)\}=F(\varpi+a). \tag{5}$$

Definition (2):

Let $f(sin\varsigma)$ be a function and $AMK\{f(sin\varsigma)\}=F(\varpi)$ then $f(sin\varsigma)$ is said to be the inverse of $F(\varpi)$ and defined as $AMK^{-1}\{F(\varpi)\}=f(sin\varsigma)$. (6)

Theorem (2):

Let $F(\varpi)=AMK\{f(sin\varsigma)\}$ and a be a constant then

$$AMK^{-1}\{F(\varpi+a)\}=(sin\varsigma)^a\, AMK^{-1}\{F(\varpi)\} \tag{7}$$

Transformation for some functions:

We are going to find the AMK-transformation for some functions, like logarithmic, fix function, trigonometric function and other functions.

Function on $f(sin\varsigma)$	$AMK\{f(sin\varsigma)\}=\int_0^{\frac{\pi}{2}}(sin\varsigma)^{\omega}\cos\varsigma\, f(sin\varsigma)d\varsigma=F(\omega)$	Region of convergence
$f(sin\varsigma)=1$	$\frac{1}{\varpi+1}$	$\varpi>-1$
$f(sin\varsigma)=k$	$\frac{k}{\varpi+1}$	$\varpi>-1$
$f(sin\varsigma)=sin^n\varsigma$	$\frac{1}{\varpi+(n+1)}$	$\varpi>-(n+1)$
$f(sin\varsigma)=cos(alnsin\varsigma)$	$\frac{\varpi+1}{(\varpi+1)^2+(a)^2}$	$\varpi>-1$
$f(sin\varsigma)=sin(alnsin\varsigma)$	$\frac{a}{(\varpi+1)^2+(a)^2}$	$\varpi>-1$
$f(sin\varsigma)=cosh(aln(sin\varsigma))$	$\frac{\varpi+1}{(\varpi+1)^2-(a)^2}$	$\varpi>-1$
$f(sin\varsigma)=sinh(a(sin\varsigma))$	$\frac{-a}{(\varpi+1)^2-(a)^2}$	$\varpi>-1$
$f(sin\varsigma)=[ln(sin\varsigma)]^n$	$\frac{n!(-1)^n}{(1+\varpi)^{1+n}}$	$\varpi>-1$

Theorem (3):

If the $f(sin\varsigma)$ is described for $\xi>0$ and its derivatives $\frac{d(sin\varsigma)}{d\varsigma},\frac{d^2(sin\varsigma)}{d\varsigma^2},\frac{d^3(sin\varsigma)}{d\varsigma^3},\ldots,\frac{d^n(sin\varsigma)}{d\varsigma^n}$ are exit then $AMK\left[(sin\varsigma)^n y^{(n)}(sin\varsigma)\right]$

$$=y^{(n-1)}(1)+(-1)^n(n+\varpi)y^{(n-2)}(1)+(-1)^{n-1}(n+\varpi)((n-1)+\varpi)y^{(n-3)}(1)+\ldots+(\varpi+n)((n-1)+\varpi)+\ldots+(\varpi+2)y(1)-(\varpi+n)!.F(\varpi) \quad (8)$$

Example (1): Solution of linear ODE of first order

$$(sin\varsigma)y'(sin\varsigma)-4y(sin\varsigma)=sinh(2ln(sin\varsigma)), \qquad y(1)=-3$$

By using AMK-transformation

$$AMK\{(sin\varsigma)y'(sin\varsigma)-4y(sin\varsigma)\}=AMK\{sinh(2ln(sin\varsigma))\}$$

$$y(1)-(\varpi+1)F(\varpi)-4F(\varpi)=\frac{-2}{(\varpi+1)^2-4}$$

$$-3-(\varpi+5)F(\varpi)=\frac{-1}{(\varpi+1)^2-4}$$

$$F(\varpi)=\frac{-3\varpi^2-6\varpi+11}{\left((\varpi+1)^2-4\right)(\varpi+5)}$$

By using partial fraction

$$AMK^{-1}\{F(\varpi)\}=AMK^{-1}\left[\frac{a_1}{(\varpi+5)}+\frac{a_2\varpi+a_3}{\left((\varpi+1)^2-4\right)}\right]$$

$$a_1+a_2=-3$$

$$2a_1+5a_2+a_3=-6$$

$$-3a_1+5a_3=11$$

From above equation,

$$a_1=\frac{-17}{6}, \qquad a_2=\frac{-1}{6}, \qquad a_3=\frac{1}{2}$$

$$y(sin\varsigma)=\frac{-17}{6}(sin\varsigma)^4-\frac{1}{6}cosh(2lnsin\varsigma)-\frac{1}{3}sinh(2ln(sin\varsigma))$$

Example (2): Solution of second order ordinary differential equation

$$-(sin\varsigma)y'(sin\varsigma)+(sin\varsigma)^2\,y''(sin\varsigma)+y(sin\varsigma)=ln(sin\varsigma)$$

Where $y(1)=-1$ and $y'(1)=2$

Sol:

By using AMK-transformation

$$AMK\left\{(sin\varsigma)^2\,y''(sin\varsigma)-(sin\varsigma)y'(sin\varsigma)+y(sin\varsigma)\right\}=AMK\{ln(sin\varsigma)\}$$

$$y'(1)-(\varpi+2)y(1)+(\varpi+2)(\varpi+1)F(\varpi)-y(1)+(\varpi+1)F(\omega)+F(\varpi)=\frac{-1}{(\varpi+1)^2}$$

$$F(\varpi)=\frac{-\varpi^3-7\varpi^2-11\varpi-10}{(\varpi+1)^2(\varpi+2)^2}$$

By taking inverse of transformation

$$AMK^{-1}\left\{F\left(\varpi\right)\right\} = AMK^{-1}\left[\frac{a_1}{(\varpi+1)} + \frac{a_2}{(\varpi+1)^2} + \frac{a_3}{(\varpi+2)} + \frac{a_4}{(\varpi+2)^2}\right]$$

$a_1 + a_3 = -1$

$5a_1 + a_2 + 4a_3 + a_4 = -7$

$8a_1 + 4a_2 + 5a_3 + 2a_4 = -11$

$4a_1 + 4a_2 + 2a_3 + a_4 = -6$

By solving we get

$a_1 = 2, \quad a_2 = -1, \quad a_3 = -3, \quad a_4 = -4$

$$y\left(sin\varsigma\right) = 2 + ln\left(sin\varsigma\right) - 3sin\varsigma + 4sin\varsigma ln\left(sin\varsigma\right)$$

CONCLUSION

In this article, we have introduced a new transformation which is able to solve the ordinary linear differential equation with given condition.

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