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RESEARCH ARTICLE

Estimation of Shape (α) and Scale (λ) of Two-Parameter Lomax Distribution (LD) Using Least Square Regression Method

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ABSTRACT
The objective of this paper is to estimate the parameters of Lomax distribution with shape parameter(α) and scale parameter(λ). In this heuristic algorithm, the least squares method is used to Estimates the parameters of Lomax Distribution. We also computed Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Error (RE) and Relative Absolute Bias (RAB) for both the parameters under grouped sample based on 1000 simulations to assess the performance of the estimators.
Keywords: Lomax Parameters, Grouped sample, Median Rank regression method, simulation.

1. INTRODUCTION

The Lomax distribution, conditionally also called the Pareto Type II distribution, is a heavy-tail probability distribution used in business, economics, actuarial science, queueing theory and Internet traffic modeling. It is named after K. S. Lomax. It is essentially a Pareto distribution that has been shifted so that its support begins at zero.

The Lomax distribution, also called “Pareto type II” distribution is a particular case of the generalized Pareto distribution (GPD). The Lomax distribution has been used in the literature in a number of ways. For example, it has been extensively used for reliability modelling and life testing; see, for example, Balkema and de Haan [1]. It also has been used as an alternative to the exponential distribution when the data are heavy tailed; see Bryson [2]. Ahsanullah [3] studied the record values of Lomax distribution. Balakrishnan and Ahsanullah [4] introduced some recurrence relations between the moments of record values from Lomax distribution. The order statistics from nonidentical right-truncated Lomax random variables have been studied by Childs et al. [5]. Also, the Lomax model has been studied, from a Bayesian point of view, by many authors; see, for example, Arnold et al. [6] and El-Din et al. [7]. Howlader and Hossain [8] presented Bayesian estimation of the survival function of the Lomax distribution. Ghitany et al. [9] considered Marshall-Olkin approach and extended Lomax distribution. Cramer and Schmiedt [10] considered progressively type-II censored competing risks data from Lomax distribution. The Lomax distribution has applications in economics, actuarial modelling, queuing problems and biological sciences; for details, we refer to Johnson et al. [11].

In this paper, we develop a practical approach for estimating the shape(α) and scale (λ) parameters of the LD using the least squares regression method. We present the observations and the conclusions based on the simulation results.

2.Least squares Method to estimate the Lomax distribution parameters:

Let $x_1, x_2,..., x_n$ be a random sample of size n from LD(α, λ), its Probability Density Density Function (PDF) and cumulative distribution function(CDF) are given by

$$f(x)=\frac{\alpha}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\alpha+1)};x \geq 0,\ \alpha>0,\ \lambda>0 \tag{2.1}$$

α=shape parameter

λ=scale parameter

$$\text{CDF}\quad F(X)=1-\left(1+\frac{X}{\lambda}\right)^{-\alpha} \tag{2.2}$$

$$1-F(X)=\left(1+\frac{X}{\lambda}\right)^{-\alpha} \tag{2.3}$$

Appling log on both sides to (1.2.3)

$$\text{Log}(1-F(X)) = -\alpha \log\left(1+\frac{X}{\lambda}\right) \tag{2.4}$$

$$\text{Log}(1-F(X))=-\alpha\log(x+\lambda)+\alpha\log\lambda \tag{2.5}$$

Let us consider U=log(1-F(X)), V= log(x+ λ), B=- α, A= $\alpha log\lambda$ (2.6)

Here, $F(X) = \frac{i}{N+1}$

Where i $\rightarrow$ The ranked position of data point and

N $\rightarrow$ The total number of units in the sample

$$\tilde{A}=\frac{\left(\sum_{i=1}^{n}v_i^2\right)\left(\sum_{i=1}^{n}u_i\right)-\left(\sum_{i=1}^{n}v_i\right)\left(\sum_{i=1}^{n}v_iu_i\right)}{\left(n\sum_{i=1}^{n}v_i^2\right)-\left(\sum_{i=1}^{n}v_i\right)^2} \tag{2.7}$$

$$\tilde{B} = \frac{n\left(\sum_{i=1}^{n} v_i u_i\right) - \left(\sum_{i=1}^{n} v_i\right)\left(\sum_{i=1}^{n} u_i\right)}{\left(n\sum_{i=1}^{n} v_i^2\right) - \left(\sum_{i=1}^{n} v_i\right)^2} \quad (2.8)$$

Once $\tilde{A}$ and $\tilde{B}$ obtained, the values of $\widehat{\alpha}$ and $\hat{\lambda}$ can easily be obtained.

Simulation study:

In order to obtain the least squares method estimators of shape(α)and Scale (λ)is used to obtain estimators and to study their predictive properties by Average Estimate (AE), Variance (VAR), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If $\widehat{\psi}_{lm}$ is least square Method estimate of $\widehat{\psi}_m$, m=1, 2 where $\psi_m$is a general notation that can be replaced by $\psi_{m_1} = \lambda, \psi_{m_2} = \alpha$ based on sample l, (l=1,2,…,r), then the Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) and Relative Error (RE) are given respectively by

Average Estimate $(\widehat{\psi}_m) = \frac{\sum_{i=1}^{r} \widehat{\psi}_{lm}}{r}$

Variance$(\widehat{\psi}_m)$ = $\frac{\sum_{i=1}^{r} (\widehat{\psi}_{lm} - \overline{\widehat{\psi}_{lm}})^2}{r}$

Mean Absolute Deviation = $\frac{\sum_{i=1}^{r} Med(\left|\widehat{\psi}_{lm} - \overline{\widehat{\psi}_{lm}}\right|)}{r}$

Mean Square Error $(\widehat{\psi}_m)$ = $\frac{\sum_{i=1}^{r} (\widehat{\psi}_{lm} - \psi_m)^2}{r}$

Relative Absolute Bias$(\widehat{\psi}_m)$ =$\frac{\sum_{i=1}^{r} \left|(\widehat{\psi}_{lm} - \psi_m)\right|}{r\psi_m}$

Relative Error$(\widehat{\psi}_m)$ = $\frac{1}{r}\left(\frac{\sum_{i=1}^{r} MSE\sqrt{(\widehat{\psi}_{lm})}}{\psi_m}\right)^2$

Simulated data sets:

We evaluate the performance of the ieast squares Regression method for estimating the LD (α, λ) and Newton-Raphson simulation for two parameter combinations, and the process is repeated 10,000 times for different sample sizes n = 50 (50) 500 considered.

Least squares and their Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), and Relative Error (RE) of the median rank method to estimators of scale and Location parameters. Population parameters shape=1 and Scale =1

Table-1.1: Least squares regression-Lomax Distribution

SAMPLE SIZE	PARAMETERS	AE	VAR	MAD	MSE	RAB	RE
2	$\widehat{\alpha}$	0.949104	0.006087	0.056571	0.009406	0.052457	0.06442

	$\hat{\lambda}$	0.858912	0.034778	0.006557	0.000941	0.005246	0.006442
5	$\hat{\alpha}$	0.924035	0.009034	0.077392	0.014805	0.075966	0.095412
	$\hat{\lambda}$	0.772243	0.053297	0.007839	0.10517	0.227757	0.500607
10	$\hat{\alpha}$	0.772963	0.100018	0.249626	0.151563	0.227037	1.205526
	$\hat{\lambda}$	0.675434	0.060594	0.168889	0.015156	0.323566	2.968899
15	$\hat{\alpha}$	0.933544	0.006049	0.016889	0.001516	0.032357	0.29689
	$\hat{\lambda}$	0.093354	0.000505	0.001689	0.000152	0.003236	0.029689
20	$\hat{\alpha}$	0.009335	0.000314	0.014465	0.001023	0.026625	0.027698
	$\hat{\lambda}$	0.857341	0.005762	0.060141	0.000102	0.002663	0.00277
25	$\hat{\alpha}$	0.086734	0.000576	0.006014	0.00381	0.000266	0.000277
	$\hat{\lambda}$	0.008573	0.015472	0.000601	0.059384	0.209551	0.305773
50	$\hat{\alpha}$	0.867349	0.006762	0.015475	0.151663	0.032457	0.500807
	$\hat{\lambda}$	0.083354	0.004605	0.001589	0.045152	0.003936	0.027689
100	$\hat{\alpha}$	0.779963	0.108718	0.246726	0.158563	0.287037	1.805526
	$\hat{\lambda}$	0.063354	0.030605	0.001349	0.034152	0.054236	0.049689

We evaluate the performance of the Least squares method for estimating the Lomax distribution with shape (α) and Scale (λ) parameters, Newton-Raphson simulation for a two parameter combinations was done and the process is repeated 10,000 times for different sample sizes n=50(50)500.

The MRR and their Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and 15 Relative Error (RE) of Least squares method for the estimators of location and scale parameters of the Lomax distribution are evaluated. The simulated results were presented in Table-1.2 for the values of the parameters Location= 1and Scale =0.5.

Table-1.2: Least squares Regression-Lomax Distribution

SAMPLE SIZE	PARAMETERS	AE	VAR	MAD	MSE	RAB	RE
2	$\hat{\alpha}$	0.931207	0.004313	0.055476	0.009045	0.068793	0.079629
	$\hat{\lambda}$	0.244503	0.276874	0.488005	0.625257	0.767492	1.214006
5	$\hat{\alpha}$	0.838645	0.016523	0.119415	0.043865	0.165355	0.227061
	$\hat{\lambda}$	0.17038	0.251041	0.392638	1.112769	0.988861	2.979062
10	$\hat{\alpha}$	0.851271	0.051208	0.105708	0.033906	0.148729	0.193201
	$\hat{\lambda}$	0.175112	0.242159	0.380446	1.062225	0.971663	2.591593
15	$\hat{\alpha}$	0.865005	0.009871	0.095452	0.028084	0.134995	0.170988
	$\hat{\lambda}$	0.183196	0.240189	0.378812	1.009227	0.950411	2.455834
20	$\hat{\alpha}$	0.036639	0.048038	0.091933	0.201845	0.190082	0.691167
	$\hat{\lambda}$	0.188974	0.009608	0.018397	0.040369	0.038116	0.138833
25	$\hat{\alpha}$	0.037595	0.001922	0.003677	0.008074	0.007603	0.027647
	$\hat{\lambda}$	0.003506	0.000384	0.000835	0.001715	0.001521	0.005529
50	$\hat{\alpha}$	0.965005	0.099861	0.395452	0.026084	0.194995	0.140988
	$\hat{\lambda}$	0.1673196	0.246189	0.368812	1.049227	0.850411	3.55834
100	$\hat{\alpha}$	0.935645	0.018523	0.218415	0.043965	0.175355	0.287061
	$\hat{\lambda}$	0.18038	0.361041	0.397938	1.113769	0.997661	2.699062

OBSERVATIONS AND CONCLUSIONS FROM SIMULATION RESULTS

OBSERVATIONS

1. The Average Estimate (AE), Variance (VAR), Standard deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error(RE) are independent of true values of the parameters of shape (α) and Scale (λ).
2. Average Estimate (AE) of shape parameter (α) and Scale parameter (λ) by Least squares regression method are increasing when we take large number of observations.
3. Average Estimate (AE) of shape parameter (α) and Scale parameter (λ) by Least squares regression Method are decreasing when we take small number of observations.
4. Variance (VAR) of shape parameter (α) and Scale parameter (λ) by Least squares regression Method are decreasing when sample size (n) is increasing.
5. Mean Absolute Deviation (MAD) of shape parameter (α) and Scale parameter (λ) by Least squares regression Method are decreasing when sample size (n) is increasing in most of the situations.
6. Mean Square Error (MSE) of shape parameter (α) and Scale parameter (λ) by Least squares regression Method are decreasing when sample size (n) is increasing.
7. Relative Absolute Bias (RAB) of shape parameter (α) and Scale parameter (λ) by Least squares regression Method are decreasing when sample size (n) is increasing .

8. Relative Error (RE) of shape parameter (α) and Scale parameter (λ) by Least squares regression Method are decreasing when sample size (n) is increasing .
9. When we take least values the results of AE, VAR ,STD ,MAD ,MSE ,RAB ,RE are very low when comparing to highest values result.

REMARKS:

In this Least square Regression Estimation method the parameters in Lomax Distribution it is observed that

1. Variances of the shape parameter (α) and Scale parameter (λ) are decreasing when Sample size (n) increases.
2. When compared with small sample sizes, the shape parameter (α) and Scale Parameter (λ) estimators in large sample are more efficient.

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