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PERFORMANCE OF DISTANCE CLASSIFIER IN PATTERN RECOGNITION

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ABSTRACT

The criterion of error of misclassification of Distance linear classifier has not been fully investigated under the influence of intra class correlations in classifying two Multivariate Normal populations [c.f p 527 Johnson and Wrichen (2001)]. Here an attempt is made to study the performance of Distance classifier in classifying a new observation X_o into one of the two Multivariate Normal populations $N_p(\mu_1, \Sigma_1)$ and $N_p(\mu_2, \Sigma_2)$ in the above said context.

1. INTRODUCTION TO LINEAR CLASSIFIER

As it is proposed to study the performance of Distance classifier in case of two Multivariate Normal populations, the decision rule is presented here for ready reference:

a) ***Distance*** Rule *when the parameters are specified*

The decision rule is

$$\|X-\mu_1\|^2 - \|X-\mu_2\|^2 \lesseqgtr 2\ln\frac{P(\pi_1)}{P(\pi_2)} \rightarrow X \in \begin{cases} \pi_1 \\ \pi_2 \end{cases} \tag{1.1}$$

This decision rule has the geometrical interpretation of comparing the distances from X to μ_1 and μ_2 according to a Threshold. When $P(\pi_1)=P(\pi_2)$=0.5, the decision boundary is the perpendicular bisector of the line joining μ_1 and μ_2 .

b) When the parameters are unknown

When the parameters are unknown, then they are estimated based upon two samples of sizes n_1 and n_2 respectively from π_1 and π_2 , before the classification is done. The estimates are given by

$$\hat{\mu}_i = \overline{X}_i \text{ and } \hat{\Sigma} = S_{pooled} = \frac{(n_1-1)S_1 + (n_2-1)S_2}{n_1+n_2-2}$$

where $S_i^2 = \dfrac{1}{n_i-1}\sum_{i=1}^{n_i}\left(X_i - \overline{X}\right)^2$

Now, the rule described above can be used by replacing μ_1, μ_2, Σ respectively by $\overline{X}_1, \overline{X}_2$ and S_{pooled} .

2. Computation of Total Probability of Misclassification (TPM)

Computing probability of error of misclassification is somewhat difficult as it involves evaluation of multiple integrals [c.f Fukunaga (1990)]. However TPM can be estimated by means of the confusion matrix using simulation and the process is as under:

Total Probability of Misclassification (TPM) is defined as,

TPM = P(misclassifying a π_1 observation or misclassifying a π_2 observation)

Therefore the confusion matrix is of the form

		Predicted	
		π_1	π_2
Actual	π_1	n_{1C}	$n_{1M} = n_1 - n_{1C}$
	π_2	$n_{2M} = n_1 - n_{2C}$	n_{2C}

n_{1C} = Number of π_1 items correctly classified as π_1 items

n_{2C} = Number of π_2 items correctly classified as π_2 items

n_{1M} = Number of π_1 items misclassified as π_2 items

n_{2M} = Number of π_2 items misclassified as π_1 items

3. METHODOLOGY

Let $N_p(\mu_1, \Sigma)$ and $N_p(\mu_2, \Sigma)$ be the two normal populations to be discriminated for the specified parameters using above said methods. Without loss of generality μ_1 is taken as Null vector, μ_2 is taken as $k\left[1\right]^T$ (where $\left[1\right]^T$ is a vector with all components as unity), *k* varying from 0.5(0.5) 3.

Exploiting the relationship $V^{1/2}\rho V^{1/2} = \Sigma$, (where *V* and ρ are respectively the diagonal matrix of variances and intra class correlations), here it is considered $\Sigma = \rho$ and the dimensionality (p) ranging from 3(1)10 and the correlations in the correlation matrix ranging from 0.2 to 0.8 spreading with equidistant along the rows are taken in ascending fashion and in another case it is taken in the descending order. The priori probabilities $p(\pi_1) = p_1$ and $p(\pi_2) = p_2 = 1 - p_1$. Here we have taken $p_1 = 0.1(0.1)0.9$.

4. GENERATION OF MULTIVARIATE NORMAL DATA

The vectors of multivariate normal data with the specified parameters can be obtained starting from univariate standard normal data using Box and Muller (1958) technique and from this, we can generate multivariate normal data for any specified set of parameters μ , Σ [c.f Fukunaga (1990)].

When the parameters are not specified then the simulation study involves two phases - *Training* (estimation) and *Validation* (classification). In training phase, based on samples of size n_1 and n_2 drawn respectively from the specified populations, estimation of parameters is done, then the classifiers are constructed, while in validation phase another set of pseudo random vectors are drawn from Multivariate Normal and is used to study the performance of classifiers under consideration.

5. SALIENT OBSERVATIONS ON THE SIMULATION RESULTS

The tolerance limit for TPM is taken as 10% and following observations were made:

PARAMETERS ARE SPECIFIED

I. Decreasing pattern of correlations in $\sum$matrix.

1. Distance classifier performs better in all cases under orthogonal transformation.

II. Increasing pattern of correlations in $\sum$matrix

2. Only Orthogonal transformation satisfies the tolerance limit condition.

PARAMETERS ARE NOT SPECIFIED

I. *Decreasing pattern of correlations in $\sum$matrix.*

3. Distance classifier performs better under orthogonal transformation than given vector and Jackknifing.
4. Under *Jackknifing* Distance classifier performs better with the gradual increase in $p(\pi_1)$.

II. *Increasing pattern of correlations in $\sum$matrix.*

5. Distance classifier under given vector is performing better when compared to Orthogonal transformation and Jackknifing.
6. *Jackknifing* under Distance classifier is good with the increase in priori probability $p(\pi_1)$.

Performance of *jackknifing* very much depends on samples and priori probabilities. It is observed that *Jackknifing* gives better results with the increase in sample sizes and priori probabilities $p(\pi_1)$.

ANNEXURE

100*TPM OF DISTANCE CLASSIFIER IN CLASSIFYING

TWO POPULATIONS $\pi_1 : N_p(0,\Sigma)$ **and** $\pi_2 : N_p(\mu_2,\Sigma)$

TABLE 1.1: DECREASING ORDER OF CORRELATIONS IN Σ MATRIX

WHEN THE PARAMETERS ARE SPECIFIED

$\pi_1 : N_{10}(0,\Sigma)$ $\pi_2 : N_{10}(\mu_2,\Sigma)$

→ μ_2	**(3.0)1^T**		**(2.5)1^T**		**(2.0)1^T**		**(1.5)1^T**		**(1)1^T**		**(0.5)1^T**	
$p(\pi_1)$	❶	❷	❶	❷	❶	❷	❶	❷	❶	❷	❶	❷
0.1	5.8	0.0	10.1	0.0	13.7	0.3	19.8	1.5	27.9	8.4	40.3	36.1
0.2	8.4	0.0	9.2	0.0	11.5	0.0	18.6	0.9	28.7	6.7	38.8	29.3
0.3	6.8	0.0	11.5	0.0	15.2	0.1	20.5	0.9	30.5	8.0	36.7	24.3
0.4	7.9	0.0	11.2	0.0	13.5	0.1	20.8	0.9	29.7	5.4	37.6	22.5
0.5	7.9	0.0	11.9	0.0	13.4	0.2	22.2	1.3	30.1	5.3	39.7	21.7
0.6	7.7	0.0	11.2	0.0	15.5	0.0	21.4	0.9	30.6	5.3	40.0	23.3
0.7	7.4	0.0	9.6	0.0	14.6	0.0	21.6	1.3	29.8	5.8	41.5	25.3
0.8	7.1	0.0	11.8	0.0	15.1	0.1	25.2	1.1	29.5	6.3	40.9	28.8
0.9	8.8	0.0	12.4	0.0	16.6	0.0	22.5	1.3	34.5	10.0	41.9	38.0

$p(\pi_1)$ **: Priori Probability of** π_1 ❶ **For a Given Vector** ❷ **Using Orthogonal Transformation**

TABLE 1.2: INCREASING ORDER OF CORRELATIONS IN Σ MATRIX WHEN THE PARAMETERS ARE SPECIFIED

$\pi_1 : N_{10}(0,\Sigma)$ $\pi_2 : N_{10}(\mu_2,\Sigma)$

→ μ_2	$(3.0)1^T$		$(2.5)1^T$		$(2.0)1^T$		$(1.5)1^T$		$(1.0)1^T$		$(0.5)1^T$	
$p(\pi_1)$	❶	❷	❶	❷	❶	❷	❶	❷	❶	❷	❶	❷
0.1	6.7	0.0	11.1	0.0	13.9	0.4	23.6	1.2	30.3	8.4	41.4	37.7
0.2	6.7	0.0	9.9	0.0	16.8	0.4	24.6	0.8	30.5	8.1	41.0	27.4
0.3	8.0	0.0	9.6	0.0	15.2	0.2	22.7	0.6	32.7	7.4	39.6	24.6
0.4	6.9	0.0	10.5	0.0	17.2	0.2	23.8	1.5	31.6	7.6	42.1	23.3
0.5	6.8	0.0	10.5	0.0	14.2	0.1	19.9	1.0	29.6	8.3	40.4	23.6
0.6	7.2	0.0	10.9	0.0	16.7	0.2	21.9	1.9	30.6	5.5	42.5	25.0
0.7	6.6	0.0	11.7	0.0	17.5	0.4	24.6	1.4	30.5	8.2	39.4	25.1
0.8	6.9	0.0	14.7	0.1	15.0	0.6	25.8	2.0	31.5	9.0	42.2	29.2
0.9	8.3	0.0	10.6	0.0	15.4	0.1	25.4	2.4	30.9	14.4	40.4	40.9

$p(\pi_1)$ **: Priori Probability of** π_1 **❶ For a Given Vector ❷Using Orthogonal Transformation**

TABLE 1.3: DECEASING ORDER OF CORRELATIONS IN Σ MATRIX WHEN THE PARAMETERS ARE NOT SPECIFIED

$\pi_1 : N_{10}(0,\Sigma); \pi_2 : N_{10}(\mu_2,\Sigma)$ Size of First Sample: 10 ; Size of Second Sample: 20

→	$(3.0)1^T$			$(2.5)1^T$			$(2.0)1^T$			$(1.5)1^T$			$(1.0)1^T$			$(0.5)1^T$		
μ_2 $p(\pi_1)$	❶	❷	❸	❶	❷	❸	❶	❷	❸	❶	❷	❸	❶	❷	❸	❶	❷	❸
0.1	0	0.5	12.5	0.2	0.5	13.7	4.2	0.7	21.5	9	5.5	29	21.5	14.7	42.2	36	37.7	48.5
0.2	1	1	8.2	1.2	0.5	11.5	3.7	3.2	16.5	8.5	4.2	23.7	19	15	34.5	33	38	46.5
0.3	0	0.2	6.2	1.2	1.2	11.2	4.7	0.7	15	10.7	6.5	22	19.5	10.7	29	34	31.5	40.5
0.4	0	0.5	6.2	0.7	0.2	8.7	3.2	1.7	11.2	8.2	6.2	17.2	16.7	15	24.7	34.5	28.5	35.7
0.5	0.2	0	6.7	0	1.5	6.5	3	3	9.7	9.5	7.2	13.2	15.7	13.7	20.7	28	33.2	32.5
0.6	0.2	0.2	5.5	0.7	1.2	6.7	3.7	1.7	9.2	7.5	8	14.7	16.7	14.5	18	26.5	30	30.2
0.7	0.2	0.7	2.7	0.7	2.5	6	3	3.5	8.7	5.7	5.7	12	18.5	19	18	33.5	36.5	35.7
0.8	0.2	0.7	4.7	0.2	3	5.5	2.7	3.6	5.7	7.2	8.5	8.5	16.2	26	16.2	31	35.5	36.7
0.9	0.2	1	3.5	1.2	1	4.7	2	4.7	5.5	9	8.2	7.7	17.5	23	18.5	37.5	42.5	46

$p(\pi_1)$ **: Priori Probability of** π_1 **❶ For a Given Vector ❷ Using Orthogonal Transformation ❸ Jackknife Method**

TABLE 1.4: INCEASING ORDER OF CORRELATIONS IN Σ MATRIX WHEN THE PARAMETERS ARE NOT SPECIFIED

$\pi_1 : N_{10}(0,\Sigma); \pi_2 : N_{10}(\mu_2,\Sigma)$ Size of First Sample: 10 ; Size of Second Sample: 20

→	$(3.0)1^T$			$(2.5)1^T$			$(2.0)1^T$			$(1.5)1^T$			$(1.0)1^T$			$(0.5)1^T$		
μ_2 $p(\pi_1)$	❶	❷	❸	❶	❷	❸	❶	❷	❸	❶	❷	❸	❶	❷	❸	❶	❷	❸
0.1	0.7	0.2	10.7	3.0	0.0	14.5	6.2	2.0	20.2	15.5	4.0	29.2	25.0	11.7	38.5	36.0	35.7	47.0
0.2	1.5	0.2	9.0	2.7	1.7	12.5	5.5	2.2	18.5	11.5	7.0	22.5	19.2	17.7	30.5	46.0	37.5	46.0
0.3	1.0	0.2	7.5	1.7	0.7	13.2	6.0	3.5	16.2	10.7	5.7	20.5	23.0	11.7	25.2	37.5	25.2	40.5
0.4	1.2	1.2	8.5	0.5	1.2	8.5	4.2	2.0	12.5	17.5	8.2	22.2	18.5	11.2	24.7	35.5	28.5	34.2
0.5	0.7	0.5	9.5	2.7	2.2	11.2	3.7	6.2	13.0	15.7	4.5	20.0	19.7	13.0	24.0	37.2	26.5	35.0
0.6	0.5	0.0	8.0	3.0	2.0	9.0	5.7	4.7	11.5	12.7	5.0	14.5	19.2	11.7	23.0	33.0	27.7	33.0
0.7	0.2	0.5	7.7	2.7	1.0	7.5	4.2	2.7	12.0	9.7	7.2	11.7	21.5	15.7	21.7	38.2	29.7	35.7
0.8	0.5	2.0	6.0	2.5	0.7	9.5	4.2	7.0	9.2	11.5	8.5	11.0	25.0	18.5	23.5	34.0	38.7	37.2
0.9	1.2	0.5	6.2	2.5	3.0	4.5	4.0	4.0	6.5	9.7	16.2	11.7	24.5	22.2	25.2	37.2	41.7	45.7

$p(\pi_1)$ **: Priori Probability of** π_1 **❶ For a Given Vector ❷ Using Orthogonal Transformation ❸ Jackknife Method**

REFERENCES

[1]. Box GEP and Muller ME (1958) A Note on the generation of Normal Variates. Ann.Math.Statist,29, pp610-611.

[2]. Fukunaga, K. (1990). Introduction to Statistical Pattern Recognition. Academic Press, 2nd edition, New York.

[3]. Johnson Richard A., and Wichern Dean. W(2001). Applied Multivariate Statistical Analysis, Prentice Hall International Inc., Englewood Cliffs, New Jersey.

[4]. A.V.Dattatreya Rao, and N.Srinivasa Rao (2003) Performance of Bayes classifier in Pattern Recognition, Gujarat Statistical Review, Vol.30, pp.59-66.

[5]. A.V.Dattatreya Rao, and N.Srinivasa Rao (2007) Performance of Fishers discriminant function in classification problem – A simulation study, The Aligarh Journal of Statistics, vol. 27, pp1-9.