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REMARKABLE OBSERVATIONS ON THE HYPERBOLA $y^2 = 24x^2 + 1$

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ABSTRACT

The binary quadratic equation $y^2 = 24x^2 + 1$ is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration, a few special pythogorean triangles are obtained.

Keywords: binary quadratic, hyperbola, integral points.

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INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1,2,3,4]. In [6] infinitely many pythogorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2 = 3x^2 + 1$. In [7], a special pythogorean trinangle is obtained by employing the integral solutions of $y^2 = 10x^2 + 1$. In [13],different patterns of infinitely many pythogorean triangles are obtained by employing the non-integral solutions of $y^2 = 12x^2 + 1$. In this context one may also refer[8 – 14]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 24x^2 + 1$ representing a hyperbola.

Notations Used:

$t_{m,n}$ = Polygonal number of rank n with size m.

P_n^m = Pyramidal number of rank n with size m.

CP_n^m = Centered pyramidal number or rank n with size m.

$CP_{m,n}$ = Centered polygonal number or rank n with size m.

GNO_n = Gnomonic number of rank n.

S_n = Star number of rank n.

METHOD OF ANALYSIS

The binary quadratic equation representing hyperbola is

$$y^2 = 24x^2 + 1 \qquad \textbf{(1)}$$

Whose general solution (x_n, y_n) is given by $x_n = \frac{g}{4\sqrt{2}},, y_n = \frac{f}{2}$ where $f = (5+2\sqrt{6})^{n+1} + (5-2\sqrt{6})^{n+1}$ and $g = (5+2\sqrt{6})^{n+1} - (5-2\sqrt{6})^{n+1}$,n = 0,1,2…

The recurrence relations satisfied by x and y are given by

$$y_{n+2} - 10y_{n+1} + y_n = 0, y_0 = 5, y_1 = 49$$

$$x_{n+2} - 10x_{n+1} + x_n = 0, x_0 = 1, x_1 = 10$$

Some numerical examples of x and y satisfying (1) are given in the following table:

n	x_n	y_n
0	1	5
1	10	49
2	99	485
3	980	4801
4	9701	47525
5	96030	470449
6	950599	4656965
7	9409960	46099201
8	93149001	456335045
9	922080050	4517251249
10	9127651499	44716177445

From the above table we observe some interesting properties:

1. y_n and x_{2n} are always odd.
2. $y_{2n} \equiv 0 (\mathrm{mod}\, 5)$
3. $x_{2n+1} \equiv 0 (\mathrm{mod}\, 5)$
4. $x_{2n+1} \equiv 0 (\mathrm{mod}\, 10)$
5. $x_{n+2} = 2y_{n+1} + x_n$

A few interesting properties between the solutions and special numbers are given below:

1. $10y_{2n+2} - 48x_{2n+2} + 2$ is a Perfect Square.
2. $6(10y_{2n+2} - 48x_{2n+2} + 2)$ is a Nasty number.
3. $(10y_{3n+3} - 48x_{3n+3}) + 3(10y_{n+1} - 48x_{n+1})$ is a Cubic integer.
4. $6(10y_{2n+2} - 48x_{2n+2} + 48x_{n+1} - 10y_{n+1}) + 1 = Sf$
5. $2(10y_{n+1} - 48x_{n+1}) - 1 = GfOf$
6. $2t_{m,f} = (m-2)[10y_{2n+2} - 48x_{2n+2} + 2] + (4-m)[10y_{n+1} - 48x_{n+1}], m \geq 3$
7. $2CP_{m,f} = m[(m-2)(10y_{2n+2} - 48x_{2n+2} + 2) + (4-m)(10y_{n+1} - 48x_{n+1})] + 2, m \geq 3$
8. $6CP_f{}^m = m(10y_{3n+3} - 48x_{3n+3}) + 2(m+3)(10y_{n+1} - 48x_{n+1}), m \geq 3$
9. $6P_f{}^m = (m-2)(10y_{3n+3} - 48x_{3n+3}) + 3(10y_{2n+2} - 48x_{2n+2} + 2) + (2m-1)(10y_{n+1} - 48x_{n+1}), m \geq 3$
10. Let $y = 10y_{n+1} - 48x_{n+1}, x = (5x_{n+1} - y_{n+1})$ .Then the pair (x, y) satisfies the hyperbola

$y^2 = 96x^2 + 4$.

REMARKABLE OBSERVATIONS:

1. Let α be any non-zero positive integer such that $\alpha_s = \dfrac{y_s - 1}{2}$.

   It is seen that $2t_{3,\alpha_s}$ is a Nasty number.

2. Let m and n be any two non-zero distinct positive integers such that $m_s = x_s, n_s = \dfrac{y_s - 1}{6}, s = 1,3,5....$ Note that $m_s > n_s > 0$. Therefore, taking m_s and n_s as the generators of a Pythagorean triangle, then its leg $(m_s^{\ 2} - n_s^{\ 2})$ is represented by the triangular number of rank n_s.

3. Let p,q be the generators of the pythagorean triangle T(α, β, γ) with $\alpha = 2pq, \beta = p^2 - q^2, \gamma = p^2 + q^2, p > q > 0.$ Let $p_s = x_s + y_s$ and $q_s = x_s$, then the Pythagorean triangle T satisfies the relations

(i) $\alpha - 12\beta + 11\gamma + 1 = 0$

(ii) $\dfrac{4A}{P} + 1 = 13\beta - 12\gamma$

(iii) $13\alpha - \gamma = 48\dfrac{A}{P} - 1$

where A and P represent the Area and Perimeter of the Pythagorean triangle T.

4. Let m, n be any two non-zero distinct positive integers.

   (i) Let $m_s = \dfrac{x_{2s} - 1}{2}, n_s = \dfrac{y_{2s} + 3}{8}, s = 1,2,3.....$

   Note that $t_{10,n_s} = 12t_{3,m_s} + 1$

   (ii) Let $m_s = \dfrac{x_{2s} - 1}{2}, n_s = \dfrac{y_{2s} - 1}{2}, s = 1,2,3.....$

   Note that $CP_{8,n_s} = 198t_{3,m_s} + 25$

   (iii) Let $m_s = \dfrac{x_{2s} - 1}{2}, n_s = \dfrac{y_{2s} + 1}{3}, s = 1,2,3.....$

   Note that $t_{12,n_s} + t_{6,n_s} = 192t_{3,m_s} + 24$

   (iv) Let $m_s = \dfrac{x_{2s} - 1}{2}, n_s = \dfrac{y_{2s} + 1}{2}, s = 1,2,3...$

   Note that $t_{12,n} + t_{24,n} - GNO_n = 768t_{3,m} + 97$

CONCLUSION

To conclude, one may search for other pattern of solutions and their corresponding properties.

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