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INTEGER POINTS ON THE HYPERBOLA

$$x^2 - 10xy + y^2 + 8x = 0$$

K.MEENA,[1] S.VIDHYALAKSHMI,[2] M.A.GOPALAN,[3] G.AKILA[4]

[1]Former VC, Bharathidasan university, Trichy, Tamilnadu,India,

[2,3]Professor, Department of Mathematics, SIGC, Trichy, Tamilnadu, India

[4]M .Phil student, Department of Mathematics, SIGC,Trichy, Tamilnadu, India

*** G.AKILA**

Author for Correspondence

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ABSTRACT

The binary quadratic equation $x^2 - 10xy + y^2 + 8x = 0$ representing hyperbola is considered. Different patterns of solutions are obtained. A few interesting recurrence relations satisfied by x and y are exhibited.

Keywords: binary quadratic, hyperbola, integer solutions.

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INTRODUCTION

The binary quadratic equation offers an unlimited field for research because of their variety [1-5]. In this context one may also refer [6-20]. This communication concerns with yet another interesting binary quadratic equation $x^2 - 10xy + y^2 + 8x = 0$ for determining its infinitely many non-zero integral solutions. Also a few interesting relations are presented.

2. METHOD OF ANALYSIS: The hyperbola under consideration is

$$x^2 - 6xy + y^2 + 4x = 0 \qquad (1)$$

Different patterns of solutions for (1) are illustrated below:

2.1 PATTERN: 1

Treating (1) as a quadratic in x and solving for x, we get

$$x = (5y-4) \pm 2\sqrt{6y^2 - 10y + 4} \tag{2}$$

Let $\alpha^2 = 6y^2 - 10y + 4$ (3)

Substituting $y = \frac{Y+5}{6}$ (4)

in (3), we have

$$Y^2 = 6\alpha^2 + 1$$

whose general solution is given by,

$$Y_n = \frac{1}{2}\left[\left(5+2\sqrt{6}\right)^{n+1} + \left(5-2\sqrt{6}\right)^{n+1}\right] \tag{5}$$

$$\alpha_n = \frac{1}{2\sqrt{6}}\left[\left(5+2\sqrt{6}\right)^{n+1} - \left(5-2\sqrt{6}\right)^{n+1}\right] \tag{6}$$

From (4) and (5), we have

$$y_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n+1} + \left(5-2\sqrt{6}\right)^{n+1}\right] + \frac{5}{6} \tag{7}$$

Substituting (6) and (7) in (2) and taking the positive sign, the corresponding integer solutions to (1) are given by

$$x_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n+2} + \left(5-2\sqrt{6}\right)^{n+2}\right] + \frac{1}{6}, \quad n = 1,3,5,\ldots\ldots$$

$$y_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n+1} + \left(5-2\sqrt{6}\right)^{n+1}\right] + \frac{5}{6}, \quad n = 1,3,5,\ldots\ldots$$

PROPERTIES:

- $72x_{2n+2}$ is a Nasty Number
- $12x_{3n+4} + 36x_n - 8$ is a Cubical integer
- $12x_{4n+6} + 576x_n^2 - 192x_n + 12$ is a Bi-quadratic integer
- $3x_{2n} = (6y_n - 5)^2$

Some numerical example are presented below

N	x_n	y_n
1	81	9
3	7921	801
5	776161	78409
7	76055841	7683201
9	7452696241	752875209

Also, taking the negative sign in (5), the other set of solutions to (1) is given by

$$x_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n} + \left(5-2\sqrt{6}\right)^{n}\right] + \frac{1}{6}, \quad n = 1,3,5,\ldots\ldots$$

$$y_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n+1} + \left(5-2\sqrt{6}\right)^{n+1}\right] + \frac{5}{6}, \quad n = 1,3,5,\ldots\ldots$$

PROPERTIES:

- $72x_{2n}$ is a Nasty Number
- $12x_{3n} + 36x_n - 8$ is a Cubical integer
- $12x_{4n} + 576x_n^2 - 192x_n + 12$ is a Bi-quadratic integer

- x_{2n+1} is a perfect square

In addition, the above two sets of solutions satisfy the following properties:

1. $3y_{n+2} - 30y_{n+1} + 3y_n = -20$
2. $3y_{n+3} - 297y_{n+3} + 30y_n = -220$
3. $57y_{n+1} + 24y_{n+2} - 3y_{n+3} - 6y_n = 60$
4. $y_{n+4} - 98y_{n+2} + y_n = -80$
5. $267y_{n+3} - 3y_{n+5} + 267y_{n+2} - 3y_n = 440$
6. $3x_{n+2} - 3x_{n+4} - 30x_{n+3} = -20$
7. $3x_{n+1} - 30x_{n+1} + 12x_n = -4$
8. $6x_{n+2} - 594x_{n+1} + 6x_n = -88$
9. $1\grave{9}x_{n+1} + 8x_{n+2} - x_{n+3} - 2x_n = 4$
10. $72y_{2n+1} - 48$ is a Nasty Number
11. $12y_{3n+2} + 36y_n - 40$ is a Cubical integer
12. $12y_{4n+3} + 576y_n^2 - 960y_n - 402$ is a Bi-quadratic integer

2.2 PATTERN: 2

Treating (1) as a quadratic in y and solving for y, we get

$$y = 5x \pm 2\sqrt{6x^2 - 2x} \tag{8}$$

Let $\alpha^2 = 6x^2 - 2x$ (9)

Substituting $x = \frac{X+1}{6}$ (10)

in (9), we have

$$X^2 = 6\alpha^2 + 1$$

whose general solution is given by,

$$X_n = \frac{1}{2}\left[\left(5+2\sqrt{6}\right)^{n+1} + \left(5-2\sqrt{6}\right)^{n+1}\right] \tag{11}$$

$$\alpha_n = \frac{1}{2\sqrt{2}}\left[\left(5+2\sqrt{2}\right)^{n+1} - \left(5-2\sqrt{2}\right)^{n+1}\right] \tag{12}$$

From (10) and (11), we have

$$x_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n+1} + \left(5-2\sqrt{6}\right)^{n+1}\right] + \frac{1}{6} \tag{13}$$

Substituting (12) and (13) in (8) and taking the positive sign, the corresponding integer solutions to (1) are given by

$$x_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n+1} + \left(5-2\sqrt{6}\right)^{n+1}\right] + \frac{1}{6}, \quad n = 0,2,4,\ldots\ldots$$

$$y_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n+2} + \left(5-2\sqrt{6}\right)^{n+2}\right] + \frac{5}{6}, \quad n = 0,2,4,\ldots\ldots$$

PROPERTIES:

- $72y_{2n+2} - 48$ is a Nasty Number

- $12y_{3n+4} + 36y_n - 40$ is a Cubical integer
- $12y_{4n+6} + 576y_n^2 - 960y_n - 402$ is a Bi-quadratic integer

Also, taking the negative sign in (11), the other set of solutions to (1) is given by

$$x_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n+1} + \left(5-2\sqrt{6}\right)^{n+1}\right] + \frac{1}{6}, \quad n = 0,2,4,\ldots\ldots$$

$$y_n = \frac{1}{12}\left[\left(5+2\sqrt{6}\right)^{n} + \left(5-2\sqrt{6}\right)^{n}\right] + \frac{5}{6}, \quad n = 0,2,4,\ldots\ldots$$

PROPERTIES:

- $72y_{2n} - 48$ is a Nasty Number
- $12y_{3n} + 36y_n - 40$ is a Cubical integer
- $12y_{4n} + 576y_n^2 - 960y_n - 402$ is a Bi-quadratic integer

In addition, the above two sets of solutions satisfy the following properties:

- $72x_{2n+1}$ is a Nasty Number
- $12x_{3n+2} + 36x_n - 8$ is a Cubical integer
- $12x_{4n+3} + 576x_n^2 - 192x_n + 12$ is a Bi-quadratic integer

3. CONCLUSION:

As the binary quadratic equations are rich in variety, one may consider other choices of hyperbolas and search for their non-trivial distinct integral solutions along with the corresponding properties.

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