Research Article

INTEGER SOLUTIONS OF TERNARY QUADRATIC DIAPHANTINE EQUATION

$$z^2 = 19x^2 + y^2$$

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ABSTRACT

The ternary quadratic homogeneous equation representing homogeneous cone given by $z^2 = 19x^2 + y^2$ is analyzed for its non-zero distinct integer points on it. Three different patterns of integer points satisfying the cone under consideration are obtained. A few interesting relations between the solutions and special number patterns, namely, Polygonal number , Pyramidal number , Octahedral number, Pronic number, Decagonal and Nasty number are presented. Also, knowing an integer solution satisfying the given cone, three triples of integers generated from the given solution are exhibited.

Keywords: Ternary homogeneous quadratic, integral solutions.
2010 Mathematics Subject Classification: 11D09

INTRODUCTION

The ternary quadratic Diophantine equations offer an unlimited field for research due to their variety [1,21]. For an extensive review of various problems, one may refer [2-20]. This communication concerns with yet another interesting ternary quadratic equation $z^2 = 19x^2 + y^2$ representing a cone for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions are presented.

Notations:

P_n^m - Pyramidal number of rank n with size m.

$T_{m,n}$ - Polygonal number of rank n with size m.

$\Pr_n$ - Pronic number of rank n

OH_n - Octahedral number of rank n

$T_{10,n}$ -Decagonal number of rank n

2. METHOD OF ANALYSIS

The ternary quadratic equation under consideration is $z^2 = 19x^2 + y^2$ (1)

To start with, it is seen that the triples $(k, 9k, 10k)$, $(2k+1, 2k^2+2k-9, 2k^2+2k+10)$ satisfy (1).

However, we have other choices of solutions to (1) which are illustrated below:
consider (1) as

$$19x^2 + y^2 = z^2 *1$$ (2)

Assume $z = a^2 + 19b^2$ (3)

Write 1 as

$$1 = \frac{[(9+2n-2n^2)+i\sqrt{19}(2n-1)][(9+2n-2n^2)-i\sqrt{19}(2n-1)]}{(10-2n+2n^2)^2}$$ (4)

Substituting (3) and (4) in (2) and employing the method of factorization define

$$y+i\sqrt{19}x = \frac{[(9+2n-2n^2)+i(2n-1)\sqrt{19}](a+i\sqrt{19}b)^2}{(10-2n+2n^2)}$$

Equating the real and imaginary parts in the above equation, we get

$$x = \frac{[2(9+2n-2n^2)ab+(a^2-19b^2)(2n-1)]}{(10-2n+2n^2)}$$

$$y = \frac{[(9+2n-2n^2)(a^2-19b^2)-38ab(2n-1)]}{(10-2n+2n^2)}$$

Replacing a by $(10-2n+2n^2)A$, b by $(10-2n+2n^2)B$ in the above equation corresponding integer solutions to (1) are given by

$x = (10-2n+2n^2)[(A^2-19B^2)(2n-1)+\{2AB(9+2n-2n^2)\}]$

$y = (10-2n+2n^2)[(A^2-19B^2)(9+2n-2n^2)-\{38AB(2n-1)\}]$

$z = (10-2n+2n^2)^2(A^2+19B^2)$

For simplicity and clear understanding, taking n=1 in the above equations, the corresponding integer solutions of (1) are given by

$$x = 10A^2 - 190B^2 + 180AB$$

$$y = 90A^2 - 1710B^2 - 380AB$$

$$z = 10^2(A^2 + 19B^2)$$

Properties:

$x(A,1) - t_{(22,A)} \equiv -1 (\text{mod } 189)$

$x(A,1) - t_{(14,A)} - t_{(10,A)} \equiv -2 (\text{mod } 188)$

$x(A+1, A^2) - t_{(22,A)} + 190 t_{(4,A^2)} - 360 p_A^5 \equiv 10 (\text{mod} 29)$

$$x(A+1,A) + t_{(362,A)} - 180pr_A \equiv 10 (\text{mod} 159)$$

$$y(A,1) - t_{(182,A)} \equiv -255 (\text{mod} 291)$$
$$y(1,B) + t_{(3422,B)} \equiv 90 (\text{mod} 2089)$$

$$x(A,2A^2 + 1) + 760t_{(4,A^2)} + t_{(1502,A)} - 540OH_A \equiv -190 (\text{mod} 749)$$

the following represents a nasty number

$$-60x(A,A) = 6(60A)^2$$

It is worth to note that 1 in (2) may also be represented as

$$1 = \frac{[(19 - 4n^2) + i(4n)\sqrt{19}][(19 - 4n^2) - i\sqrt{19}(4n)]}{(19 + 4n^2)^2}$$

Following the analysis presented above, the corresponding integer solutions to (1) are found to be

$$x = (19 + 4n^2)[(A^2 - 19B^2)(4n) + \{2AB(19 - 4n^2)\}]$$

$$y = (19 + 4n^2)[(A^2 - 19B^2)(19 - 4n^2) - \{152nAB\}]$$

$$z = (19 + 4n^2)^2(A^2 + 19B^2)$$

For the sake of simplicity, taking n=1 in the above equations, the corresponding integer solution of (1) are given by

$$x = 92A^2 - 1748B^2 + 690AB$$

$$y = 345A^2 - 6555B^2 - 3496AB$$

$$z = 23^2(A^2 + 19B^2)$$

Properties:

$$x(A,1) - t_{(186,A)} \equiv -186 (\text{mod} 781)$$

$$x(A,1) - t_{(178,A)} - t_{(10,A)} \equiv -188 (\text{mod} 780)$$

$$x(A+1,A^2) - t_{(186,A)} + 1748t_{(4,A^2)} - 1380p_A^5 \equiv 92 (\text{mod} 275)$$

$$x(A+1,A) + t_{(3314,A)} - 690pr_A \equiv 92 (\text{mod} 1471)$$

$$x(A,2A^2 + 1) + t_{(13802,A)} + 6992t_{(4,A^2)} - 2070OH_A \equiv -1748 (\text{mod} 6899)$$

3. Generation of integer solutions

Let (x_0, y_0, z_0) be any given integer solution of (1)

Then, each of the following triples of integers satisfies (1):

Triple 1 : (x_{n1}, y_{n1}, z_{n1})

$$x_{n1} = 19^n x_0$$

$$y_{n1} = \frac{1}{6}[\{8(3)^n - 2(-3)^n\}y_0 + \{-4(3)^n + 4(-3)^n\}z_0]$$

$$z_{n1} = \frac{1}{6}[\{4(3)^n - 4(-3)^n\}y_0 + \{-2(3)^n + 8(-3)^n\}z_0]$$

Triple 2 : (x_{n2}, y_{n2}, z_{n2})

$$x_{n2} = \frac{1}{18}[\{-(9)^n + 19(-9)^n\}x_0 + \{(9)^n - (-9)^n\}z_0]$$

$$y_{n2} = 9^n y_0$$

$$z_{n2} = \frac{1}{18}[\{-19(9)^n + 19(-9)^n\}x_0 + \{19(9)^n - (-9)^n\}z_0]$$

Triple 3: (x_{n3}, y_{n3}, z_{n3})

$$x_{n3} = \frac{1}{20}[\{(10)^n + 19(-10)^n\}x_0 + \{-(10)^n + (-10)^n\}y_0]$$

$$y_{n3} = \frac{1}{20}[\{-19(10)^n + 19(-10)^n\}x_0 + \{19(10)^n + (-10)^n\}y_0]$$

$$z_{n3} = 10^n z_0$$

4. CONCLUSION

In this paper, we have presented two different patterns of infinitely many non-zero distinct integer solutions of the homogeneous cone given by $19x^2 + y^2 = z^2$. To conclude, one may search for other patterns of solution and their corresponding properties.

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