Vol.2.Issue.2.2014

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

ISSN:2348-0580

RESEARCH ARTICLE

CONTROLLABILITY OF QUASILINEAR NEUTRAL MIXED INTEGRO-DIFFERENTIAL IMPULSIVE SYSTEMS WITH SOBOLEV TYPE

R.PRABAKARAN[1], V.VINOBA[2]

[1]Department of Science and Humanities, Sri Krishna College of Technology, Coimbatore-42, India.

[2]Department of Mathematics, K.N.Government Arts college for women, Thanjavur, India

*** R.PRABAKARAN**

Author for Correspondence

Article Info:
Article received :26/05/2014
Revised on:27/06/2014
Accepted on:29/06/2014

ABSTRACT

In this article, we study some sufficient conditions for the controllability results of nonlocal quasilinear mixed integro-differential impulsive systems with Sobolev type in Banach space. The results are obtained by using Schaefer's fixed point theorem.

Key Words: Controllability, neutral differential system, fixed point theorem, measure of noncompactness.

2000 AMS Subject Classification: 93 B 05, 47 D 06, 34 K 40, 74 H 20

INTRODUCTION

A large class of scientific and engineering problems is modelled by partial differential equations, integral equations or coupled ordinary and partial differential equations which can be described as differential equations in infinite dimensional spaces using semigroups. In general functional differential equations or evolution equations serve as an abstract formulation of many partial integrodifferential equations which arise in problems connected with heat-flow of materials with memory and many other physical phenomena. Neutral differential equations arise in many areas of applied mathematics and for this reason these equations received much attention during the last few decades [kban1, kban2, H5].

The notion of a measure of weak compactness was introduced by De Blasi [DB] and was subsequently used in numerous branches of functional analysis and the theory of differential and integral equations. Several authors have studied the measures of noncompactness in Banach spaces [ban2, ban1, ABA, BE]. Motivated by

[DL, BG], in this paper, we study the existence results for quasilinear equation represented by first-order neutral integrodifferential equations using the semigroup theory and the measure of noncompactness.

Controllability of linear and nonlinear systems represented by ordinary differential equations in finite-dimensional spaces has been extensively investigated. The problem of controllability of linear systems represented by differential equations in Banach spaces has been extensively studied by several authors [c2]. Several papers have appeared on finite dimensional controllability of linear systems [K1] and infinite dimensional systems in abstract spaces [c1]. Of late the controllability of nonlinear systems in finite-dimensional spaces is studied by means of fixed point principles [B1]. Several authors have extended the concept of controllability to infinite-dimensional spaces by applying semigroup theory [p1, y1]. Controllability of nonlinear systems with different types of nonlinearity has been studied by many authors with the help of fixed point principles [B2]. Naito [Na3] discussed the controllability of nonlinear Volterra integrodifferential systems.

PRELIMINARIES

Consider the class of Sobolev-type quasilinear neutral functional integrodifferential system with nonlocal conditions of the form

$$\frac{d}{dt}\left[Ex(t)-g(t,x(t))\right]=A(t,x(t))x(t)+Bu(t)+ \quad (1)$$

$$F(t,x(t),\int_0^t k(t,s,x(s))ds,\int_0^a h(t,s,x(s))ds) \quad t\in[0,b]$$

$$x(0)+q(x)=x_0 \quad (2)$$

Where the state variable $x(\cdot)$ takes values in a separable banch space X with norm $\|\cdot\|$ and the control function $u()$ is given in $L^2(I,U)$. A banach space of admissible control functions with U

As a banach space the interval $I=[0,b]$. E and B is a bounded linear operator from U into U and $A:[0,b]\times X\to X$ is a continous function in banach space U .the function $g:I\times C\to X, f:I\times C\times X\times X\to X, k:I\times I\times C\to X, h:I\times I\times C\to X, q:C(I,X)\to X$ are given functions .The norm of U is denoted by $\|\cdot\|$

Definition 2.1: A family of operators $\{U(t,s):0\le s\le t\le b\}\subset L(X)$ is called a evolution family of operators for(3) if the following properties hold:

(a) $U(t,s)U(s,T)=U(t,T) and U(t,t)x=x,$ for every $s\le T\le t \,\&\, all x\in X$

(b)for each $x\in X$,the function for $(t,s)\to U(t,s)x$ is continuous and $U(t,s)\in L(X)$ $forevery$ $t\ge s$ and

(c) for $0\le s\le t\le b$,the function $t\to U(t,s), for\ (s,t]\in L(X),$ is differentiable with $\frac{\partial}{\partial t}U(t,s)=A(t)U(t,s)$

Definition 2.2:[22]system(1)-(2)is said to be controllable on the interval J ,if for every intial functions $x_0\in X$ and $x_1\in X$,there exit a control $u\in L^2(J,U)$ such that the solution $x(\cdot)$ of (1)-(2) satisfies $x(0)=x_0$ and $x(b)=x_1$

Definition 2.3: A solution $x(\cdot)\in C([0,b],X)$ is said to be mild solution of (1)-(2), then the following integral equation satisfied

$$x(t)=E^{-1}U(t,0)\left[Ex_0-Eq(x)-g(0,x_0)\right]+E^{-1}g(t,x_t)$$

$t\in I$

We need the following fixed point theorem due to Schaefer [22]

Theorem: Let E be a normed linear space .Let $F : E \to E$ be a completely continuous operator ,that is it is continuous and the image of any bounded set is contained in a compact set and let

$\varsigma(F) = \{x \in E : x = \lambda Fx$ for some $0 < \lambda < 1\}$

Then either $\varsigma(F)$ is unbounded or F has a fixed point

To study the controllability problem ,We assume the following hypotheses:

(H1) $A(t)$ generates a strongly continuous semigroup of a family of evolution operators $U(t,s)$ and there exist constants $M_1 > 0$ such that

$\|U(t,s)\| \le M_1 for 0 \le s \le t \le b$

(H2) There exits a positive constant $0 < b_0 < b$ and ,for each $0 < t < b_0$,there is compact set $V_1 \subset X$ suchthat

$U(t,s) f(s, x_s, \int_0^s k(s,T,x_T)dT, \int_0^a h(s,T,x_T)dT), U(t,s)A(s,x(s))g(s,x_s), U(t,s)Bu(s) \in V_t \;\& all$

$0 \le t \le s \le b_0$

(H3) The linear operator $W : L^2(I,U) \to X$ defined by

$$W_u = \int_0^b U(b,s)Cu(s)ds$$

Has an inverse operator W^{-1} which takes values in $L^2(I,U)/\ker W$ and there exists a positive constant M_2 such that $\|CW^{-1}\| \le M_2$

(H4) (i) the function $g : I \times X \to X$ is continuous for a.e $t \in I$ and there exists a positive constant $M_g > 0, L_g > 0$ such that

$\|g(t,s_t)\| \le M_g \|x_t\|$

And

$\|g(0,x_0)\| \le L_g$

(ii)also there exists a constant $M_A > 0$ such that

$$\|A(t,x(t))g(t,x_t)\| \le M_A \|x_t\|.$$

Holds for $t \in I$

(H5) (i) for each $t \in I$, the function $k(t,s,\cdot) : C \to X$ is continuous and ,for each $x \in C$ the function $k(.,.,x) : I \to X$ is strongly measurable

(ii) there exists an integrable function $M_k : I \to [0,\infty)$ such that

$\|k(t,s,x)\| \le M_k(t,s)\Omega_1(\|x\|)$

Holds for $t \in I$, $x \in C$ Where $\Omega_1 :: [0,\infty) \to [0,\infty)$.is continuous non decreasing function

(H6) (i) for each $t \in I$,the function $h(t,s,\cdot) : C \to X$ is continuous and ,for each $x \in C$,

The function $h(.,.,x) : I \to X$ is strongly measurable

(ii) there exists an integrable function $M_h : I \to [0,\infty)$ such that

$\|h(t,s,x)\| \le M_h(t,s)\Omega_2(\|x\|)$

Holds for $t \in I$, $x \in C$ Where $\Omega_2 : : [0,\infty) \to [0,\infty)$.is continuous non decreasing function

(H7) the function $f : I \times C \times X \times X \to X$ satisfies the caratheodary conditions

(i) for each $t \in I$,the function $f(t,.,.,\cdot) : C \times X \times X \to X$ is continuous and ,for $(x, y, z) \in C \times X \times X$ The function $f(.,x,y,z) : I \to X$ is strongly measurable

(ii) there exists an integrable function $M_f : I \to [0,\infty)$such that

$$\|f(t,x,y,z)\| \le M_f(t)\Omega_3(\|x\| + \|y\| + \|z\|)$$

Holds for $t \in I$, $x \in C$ and $y, z \in X$ Where $\Omega_3 : [0,\infty) \to [0,\infty)$.is continuous non de creasing function

(H8) the function $q : C(I, X) \to X$ is continous and there exists a constant $M_q \ge 0$ such that $\|q(x)\| \le M_q for x \in X$

(H9) The following inequality holds , the function

$$\widehat{m}(t) = \max\{1, M_1\|E^{-1}\|M_f(t), M_k(t,s), M_h(t,s), \int_0^t \frac{\partial}{\partial t} M_k(t,s)ds, \int_0^a \frac{\partial}{\partial t} M_k(t,s)ds,\}$$

Satisfies

$$\int_0^b \widehat{m}(s)ds < \int_d^\infty \frac{ds}{s + 2\Omega_1(s) + 2\Omega_2(s) + \Omega_3(s)}$$

Where

$$d = \|E^{-1}\|M_1[\|E\phi(0)\| + \|EM_q\| + L_g]$$

And

$$d_2 = M_2\|E^{-1}\|\{\|x_1\| + \|E^{-1}\|[M_g + M_1L_g] + M_1\|E^{-1}\|M_A br$$
$$+ M_1\|E^{-1}\|\int_0^b M_f(s)\Omega_3[r + \int_0^s M_k(s,T)\Omega(T)dT + \int_0^a M_k(s,t)\Omega_2(T)dT]ds\}$$

CONTROLLABILITY RESULT

Theorem 3.1 If the hypotheses [H1] - [H9] are satisfiedd, then the system (1) - (2) is controllable on I

Proof. Using the hypothese [H3] for an arbitrary function define the control

$$u(t) = W^{-1}[x_1 - E^{-1}U(b,0)[Ex_0 - Eq(x) - g(0,x_0)] - E^{-1}g(b,x_b)$$
$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,x_s)ds - \int_0^b E^{-1}U(b,s)f(s,x_s,\int_0^s k(s,T,x_T)dT, \int_0^a h(s,T,x_T)dT)ds](t) \quad (3)$$

If $x(t) = y(t) + \widehat{\phi}(t), t \in [0,b]$, it is easy to see that y satisfied

$$y(t) = E^{-1}U(b,0)[Ex_0 - Eq(x)] + E^{-1}g(t, y_t + \widehat{\phi}_t)$$
$$- E^{-1}U(t,0)g(0,\widehat{\phi}_0) + \int_0^t E^{-1}U(t,s)A(s,x(s))g(s,y_s + \widehat{\phi}_s)ds$$
$$+ \int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[Ex_0 - Eq(x) - g(0,\widehat{\phi}_0)] - E^{-1}g(b, y_b + \widehat{\phi}_b)$$
$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,y_s + \widehat{\phi}_s)ds - \int_0^b E^{-1}U(b,s)f(s,y_s + \widehat{\phi}_s, \int_0^s k(s,\tau,y_\tau + \widehat{\phi}_\tau)d\tau, \int_0^a h(s,\tau,y_\tau + \widehat{\phi}_\tau)d\tau)ds](\eta)d\eta$$
$$+ \int_0^t E^{-1}U(t,s)f(s,y_s + \widehat{\phi}_s, \int_0^s k(s,\tau,y_\tau + \widehat{\phi}_\tau)d\tau, \int_0^a h(s,\tau,y_\tau + \widehat{\phi}_\tau)d\tau)ds, t \in I$$

If and only if x satisfies

$$x(t) = E^{-1}U(t,0)[Ex_0 - Eq(x)] + E^{-1}g(t,x_t)$$
$$-E^{-1}U(t,0)g(0,x_0) + \int_0^t E^{-1}U(t,s)A(s,x(s))g(s,x_s)ds$$
$$+\int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[Ex_0 - Eq(x) - g(0,x_0)] - E^{-1}g(b,x_b)$$
$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,x_s)ds - \int_0^b E^{-1}U(b,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,\int_0^a h(s,\tau,x_\tau)d\tau)ds](\eta)d\eta$$
$$+\int_0^t E^{-1}U(t,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,\int_0^a h(s,\tau,x_\tau)d\tau)ds, t\in I$$

Define $C_b^0 = \{y \in C_b : y_0 = 0\}$ and we now show that when using the control , the operator $F : C_b^0 \to C_b^0$, defined by

$$(Fy)(t) = E^{-1}U(b,0)[Ex_0 - Eq(x)] + E^{-1}g(t, y_t + \widehat{\phi}_t)$$
$$-E^{-1}U(t,0)g(0,\widehat{\phi}_0) + \int_0^t E^{-1}U(t,s)A(s,x(s))g(s, y_s + \widehat{\phi}_s)ds$$
$$+\int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[Ex_0 - Eq(x) - g(0,\widehat{\phi}_0)] - E^{-1}g(b, y_b + \widehat{\phi}_b)$$
$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s, y_s + \widehat{\phi}_s)ds$$
$$-\int_0^b E^{-1}U(b,s)f(s, y_s + \widehat{\phi}_s, \int_0^s k(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau, \int_0^a h(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau)ds](\eta)d\eta$$
$$+\int_0^t E^{-1}U(t,s)f(s, y_s + \widehat{\phi}_s, \int_0^s k(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau, \int_0^a h(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau)ds, t\in I$$

has a fixed point. This fixed point point is then a solution of equation (5):

Clearly $x(b) = x_1$ which means that the control u steers the system (1) - (2) from the intial function ϕ to x_1 in time b, provided we can obtain a fixed point of the nonlinear operator F.

In order to study the controllability problem of (1) - (2)we introduce a parameter $\lambda \in (0,1)$
And consider the following system

$$\frac{\mathrm{d}}{\mathrm{dt}}\left[Ex(t) - g(t,x_t)\right] = \lambda A(t,x(t))x(t) + \lambda Bu(t) + \lambda f(t,x_t,\int_0^t k(t,s,x_s)ds, \int_0^a h(t,s,x_s)ds) \qquad (5)$$

$$x(0) + q(x) = \lambda x_0 \qquad (6)$$

First we obtain a priori bounds for the mild solution of the equation (5) - (6). Then from

$$x(t) = \lambda E^{-1}U(t,0)[Ex_0 - Eq(x)] + \lambda E^{-1}g(t,x_t)$$
$$-\lambda E^{-1}U(t,0)g(0,x_0) + \lambda\int_0^t E^{-1}U(t,s)A(s,x(s))g(s,x_s)ds$$
$$+\lambda\int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[Ex_0 - Eq(x) - g(0,x_0)] - E^{-1}g(b,x_b)$$
$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,x_s)ds - \int_0^b E^{-1}U(b,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,\int_0^a h(s,\tau,x_\tau)d\tau)ds](\eta)d\eta$$
$$+\lambda\int_0^t E^{-1}U(t,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,\int_0^a h(s,\tau,x_\tau)d\tau)ds, t\in I$$

We have

$$\|x(t)\| \le E\|^{-1}U(t,0)[Ex_0 - Eq(x)]\| + \|E^{-1}g(t,x_t)\|$$

$$+\left\|E^{-1}U(t,0)g(0,x_0)\right\|+\int_0^t\left\|E^{-1}U(t,s)A(s,x(s))g(s,x_s)\right\|ds$$
$$+\int_0^t\left\|E^{-1}U(t,\eta)CW^{-1}[x_1-E^{-1}U(b,0)[Ex_0-Eq(x)-g(0,x_0)]-E^{-1}g(b,x_b)\right\|$$

$$-\int_0^b\left\|E^{-1}U(b,s)A(s,x(s))g(s,x_s)ds-\int_0^b E^{-1}U(b,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,\int_0^a h(s,\tau,x_\tau)d\tau)ds\right\|]\|(\eta)\|d\eta$$
$$+\int_0^t\left\|E^{-1}U(t,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,\int_0^a h(s,\tau,x_\tau)d\tau)\right\|ds$$
$$\leq\left\|E^{-1}\right\|M_1[\|Ex_0\|+\|E\|M_q+L_g]+\left\|E^{-1}\right\|M_g\|x_t\|+M_1\left\|E^{-1}\right\|\int_0^t M_A\|x_s\|ds$$
$$+M_1M_2\int_0^b\left\|E^{-1}\right\|[\|x_1\|+\left\|E^{-1}\right\|[M_1\|Ex_0\|+M_1\|E\|M_q+M_1\widehat{M}_g]$$
$$+M_1\left\|E^{-1}\right\|M_A br$$
$$+M_1\left\|E^{-1}\right\|\int_0^t M_f(s)\Omega_3\{r+\int_0^b M_k(s,\tau)\Omega_1(r)d\tau+\int_0^a M_k(s,\tau)\Omega_2(r)d\tau\}]ds$$
$$\leq\left\|E^{-1}\right\|M_1[\|Ex_0\|+\|E\|M_q+L_g]+\left\|E^{-1}\right\|M_g r+M_1\left\|E^{-1}\right\|\int_0^t M_A\|x_s\|ds+M_1bd_2$$
$$+M_1\left\|E^{-1}\right\|\int_0^t M_f(s)\Omega_3\left[\|x\|+\int_0^b M_k(s,\tau)\Omega_1(\|x\|)d\tau+\int_0^a M_k(s,\tau)\Omega_2(\|x\|)d\tau\right]ds$$

Let us take the right hand side of the above inequality as $\mu(t)$ then we have

$x(0)=\mu(0)=d\ \&\ \|x(t)\|\leq\mu(t)$ with

$\mu'(t)=$

$$M_1\left\|E^{-1}M_A\right\|\|x_t\|+M_1\left\|E^{-1}\right\|M_f(s)\Omega_3\left[\|x\|+\int_0^t M_k(t,s)\Omega_1(\|x\|)ds+\int_0^a M_h(t,s)\Omega_2(\|x\|)ds\right]$$
$$\leq d_1\mu(t)+M_1\left\|E^{-1}\right\|M_f(s)\Omega_3\left[\mu(t)+\int_0^b M_k(t,s)\Omega_1(\mu(s))ds+\int_0^a M_h(t,s)\Omega_2(\mu(s))ds\right]$$

Where $d1=M_1\left\|E^{-1}\right\|M_A$ .since μ is obviously increasing,let

$$w(t)=\mu(t)+\int_0^t M_k(t,s)\Omega_1(\mu(s))ds+\int_0^a M_h(t,s)\Omega_2(\mu(s))ds$$

Then $w(0)=\mu(0)=c$ and $\mu(t)\leq w(t)$

$$w'(t)=\mu'(t)+M_k(t,s)\Omega_1(\mu(t))+M_h(t,s)\Omega_2(\mu(t))+\int_0^t\frac{\partial}{\partial s}M_k(t,s)\Omega_1(\mu(s))ds+\int_0^a\frac{\partial}{\partial s}M_h(t,s)\Omega_2(\mu(s))ds$$

$$\leq w(t)+M1\|E-1\|M_f(t)\Omega_3(w(t))+\int_0^t\frac{\partial}{\partial s}M_k(t,s)\Omega_1(\mu(s))ds+\int_0^a\frac{\partial}{\partial s}M_h(t,s)\Omega_2(\mu(s))ds$$
$$\leq\widehat{m}\{w(t)+\Omega_3(w(t))+2\Omega_1(w(t))+2\Omega_2(w(t))\}$$

This implies

$$\int_{w(0)}^{w(t)}\frac{ds}{s+\Omega_3(s)+2\Omega_1(s)+2\Omega_2(s)}\leq\int_0^b\widehat{m}(s)ds\leq\int_d^\infty\frac{ds}{s+\Omega_3(s)+2\Omega_1(s)+2\Omega_2(s)}.$$

This inequality that there is a priori bound $r>0$ such that $w(t)\leq r$ and hence $\mu(t)\leq r, t\in[0,b]$

Since $\|x(t) \le r, t \in I\|$, we have

$\|x\|_1 = \sup\{\|x(t)\| : -p \le t \le b\} < r$

Where r depending only on b and the functions $M_f(.), M_k(.), M_h(.), \Omega_1(.), \Omega_2(.), \Omega_3(.)$

Next we must prove the operator Fis a completely continuous operator.let

$B_k = \{y \in C_b^0 : \|y\|_1 \le r\}$ for some $r \ge 1$

We first show that the set $\{F_y : y \in B_k\}$is equi continuous .let $y \in B_k \; \& t_1, t_2 \in [0,b]$,

Then if $0 < t_1 < t_2 \le b$,

$$\|(Fy)(t_1) - (Fy)(t_2)\|$$

$$\le \|E^{-1}\|\|U(t_1,0) - U(t_2,0)\|\|[Ex_0 - Eq(x)]\| + \|E^{-1}\|\|g(t_1, y_t + \widehat{\phi}_{t_1}) - (t_2, y_t + \widehat{\phi}_{t_2})\|$$

$$+ \|E^{-1}\|\|U(t_1,0) - U(t_2,0)\|\|g(0,\widehat{\phi}_0)\| + \left\|\int_0^{t_1} E^{-1}[U(t_1,s) - U(t_2,s)]A(s,x(s))g(s, y_s + \widehat{\phi}_s)ds\right\|$$

$$+ \left\|\int_{t_1}^{t_2} E^{-1}U(t_2,s)A(s,x(s))g(s, y_s + \widehat{\phi}_s)ds\right\|$$

$$+ \int_0^{t_1} E^{-1}[U(t_1,\eta) - U(t_2,\eta)]CW^{-1}[x_1 - E^{-1}U(b,0)[Ex_0 - Eq(x) - g(0,\widehat{\phi}_0)] - E^{-1}g(b, y_b + \widehat{\phi}_b)$$

$$- \int_0^b E^{-1}U(b,s)A(s,x(s))g(s, y_s + \widehat{\phi}_s)ds$$

$$- \int_0^b E^{-1}U(b,s)f(s, y_s + \widehat{\phi}_s, \int_0^s k(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau, \int_0^a h(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau)ds](\eta)d\eta$$

$$+ \left\|\int_{t_1}^{t_2} E^{-1}U(t_2,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[Ex_0 - Eq(x) - g(0,\widehat{\phi}_0)] - E^{-1}g(b, y_b + \widehat{\phi}_b)\right\|$$

$$- \left\|\int_0^b E^{-1}U(b,s)A(s,x(s))g(s, y_s + \widehat{\phi}_s)ds\right\|$$

$$- \left\|\int_0^b E^{-1}U(b,s)f(s, y_s + \widehat{\phi}_s, \int_0^s k(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau, \int_0^a h(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau)ds](\eta)d\eta\right\|$$

$$+ \left\|\int_0^{t_1} E^{-1}[U(t_1,s) - U(t_2,s)]f(s, y_s + \widehat{\phi}_s, \int_0^s k(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau, \int_0^a h(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau)ds\right\|$$

$$+ \left\|\int_{t_1}^{t_2} E^{-1}U(t_2,s)f(s, y_s + \widehat{\phi}_s, \int_0^s k(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau, \int_0^a h(s,\tau, y_\tau + \widehat{\phi}_\tau)d\tau)ds\right\|$$

$$\le \|E^{-1}\|\|U(t_1,0) - U(t_2,0)\|\|[Ex_0 - Eq(x)]\|$$

$$+ \|E^{-1}\|\|g(t_1, y_{t_1} + \widehat{\phi}_{t_1}) - g(t_2, y_{t_2} + \widehat{\phi}_{t_2})\| + \|E^{-1}\|\|U(t_1,0) - U(t_2,0)\|\|g(0,\widehat{\phi}_0)\|$$

$$+ \int_0^{t_1} \|E^{-1}\|\|[U(t_1,\varepsilon) - U(t_2,\varepsilon)]U(\varepsilon,s)A(s,x(s))g(s, y_s + \widehat{\phi}_s)\|ds + (t_2 - t_1)M_1\|E^{-1}\|M_A r'$$

$$+\int_0^{t_1}\|E^{-1}\|\|[U(t_1,\varepsilon)-U(t_2,\varepsilon)]U(\varepsilon,\eta)CW^{-1}[x_1-E^{-1}U(b,0)[Ex_0-Eq(x)-g(0,\hat{\phi}_0)]-E^{-1}g(b,y_b+\hat{\phi}_b)\|$$

$$-\left\|\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,y_s+\hat{\phi}_s)ds\right\|$$

$$-\left\|\int_0^b E^{-1}U(b,s)f(s,y_s+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau)ds](\eta)\right\|d\eta$$

$$+(t_2-t_1)\|E^{-1}\|M_1M_2[x_1+\|E^{-1}\|[\|Ex_0\|+\|E\|M_q+\hat{M}_g]+\|E^{-1}\|M_g r'$$

$$+M_1b\|E^{-1}\|M_A r'+M_1b\|E^{-1}\|M_f(t)(r'+bM_k r'+bM_h r')]$$

$$+\left\|\int_0^{t_1}E^{-1}[U(t_1,s)-U(t_2,s)]f(s,y_s+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau)ds\right\|$$

$$+(t_2-t_1)\|E^{-1}\|M_1M_f(t)(r'+bM_k r'+bM_h r')]$$

Where $r'=r+\|\hat{\phi}\|$ the right hand side is independent of $y\in B_k$ and tends to zero as $(t_2-t_1)\to 0$,since by assumption (H2)implies the continuity in the uniform operator topology

Thus the set $\{F_y : y\in B_k\}$ is equi continuous.

Notice that we considered here only the case $0<t_1<t_2$; since the other cases $t_1<t_2 0<$ or $t_1<0<t_2$ are very simple.

It is easy to see that the family FB_k is uniformly bounded. Next we show that $\overline{FB_k}$ is compact. Since we have shown that FB_k is an equi continuous collection, it suffices, by the Arzela-Ascoli theorem, to show that F maps B_k into a pre compact set in X .

Let $0<t\le b$ be fixed and ε a real number satisfying $0<\varepsilon<t$: For $y\in B_k$; we define

$$(F_{\varepsilon y})(t)=E^{-1}U(b,0)[Ex_0-Eq(x)]+E^{-1}g(t,y_t+\hat{\phi}_t)$$

$$-E^{-1}U(t,0)g(0,\hat{\phi}_0)+\int_0^{t_\varepsilon}E^{-1}U(t,s)A(s,x(s))g(s,y_s+\hat{\phi}_s)ds$$

$$+\int_0^{t_\varepsilon}E^{-1}U(t,\eta)CW^{-1}[x_1-E^{-1}U(b,0)[Ex_0-Eq(x)-g(0,\hat{\phi}_0)]-E^{-1}g(b,y_b+\hat{\phi}_b)$$

$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,y_s+\hat{\phi}_s)ds$$

$$-\int_0^b E^{-1}U(b,s)f(s,y_s+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau)ds](\eta)d\eta$$

$$+\int_0^{t_\varepsilon}E^{-1}U(t,s)f(s,y_s+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau)ds$$

Now by assumption (H2), the set $Y_\varepsilon(t)=\{(F_\varepsilon y)(t): y\in B_k\}$ is pre compactin X for e $0<\varepsilon<t$

More over for every $y\in B_k$ we have

$$\|(Fy)(t)-(F_\varepsilon y)(t)\|\le$$

$$\int_{t-\varepsilon}^{t}\left\|E^{-1}U(t,s)A(s,x(s))g(s,y_s+\widehat{\phi}_s)\right\|ds$$

$$+\int_{t-\varepsilon}^{t}\left\|E^{-1}U(t,\eta)CW^{-1}[x_1-E^{-1}U(b,0)[E\phi(0)-Eq(x)-g(0,\widehat{\phi}_0)]-E^{-1}g(b,y_b+\widehat{\phi}_b)\right\|$$

$$-\int_{0}^{b}\left\|E^{-1}U(b,s)A(s,x(s))g(s,y_s+\widehat{\phi}_s)\right\|ds$$

$$-\left\|\int_0^b E^{-1}U(b,s)f(s,y_s+\widehat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\widehat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\widehat{\phi}_\tau)d\tau)ds](\eta)\right\|d\eta$$

$$+\int_{t-\varepsilon}^{t}\left\|E^{-1}U(t,s)f(s,y_s+\widehat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\widehat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\widehat{\phi}_\tau)d\tau)\right\|ds$$

Therefore ,

$$\|(Fy)(t)-(F_\varepsilon y)(t)\|\to 0$$

As $\varepsilon\to 0$,and there are pre compact setsarbitrarily close to the set $\{(Fy)(t): y\in B_k\}$

Hence the set $\{(Fy)(t): y\in B_k\}$ is pre compact set in X .

It remains to be shown that $F: C_b^0\to C_b^0$ is continuous .let $\{y_n\|_0^\infty\subset C_b^0\| with y_n\to y in C_b^0$. Then there is an integer K such that $\|y_n(t)\|\le K$ for all n and $t\in I$ so $y_n\in B_k$ and $y\in B_k$ .by (H4),(H7), $g(t,y_n(t)+\widehat{\phi}_t)\to g(t,y(t)+\widehat{\phi}_t)$ for each $t\in I$ and since $\|g(t,y_n(t)+\widehat{\phi}_t)-g(t,y_n(t)+\widehat{\phi}_t)\|\le 2M_g r', A(t,x(t))g(t,y(t)+\widehat{\phi}_t)\to A(t,x(t))g(t,y(t)+\widehat{\phi}_t)$ fo r each $t\in I$ and since

$$\|A(t,x(t))g(t,y_n(t)+\widehat{\phi}_t)-A(t,x(t))g(t,y(t)+\widehat{\phi}_t)\|\le 2M_A r', \& f(t,y_n(t)+\widehat{\phi}_t,\int_0^t k(t,s,y_n(s)+\widehat{\phi}_s)ds,\int_0^a h(t,s,y_n(s)+\widehat{\phi}_s)ds)$$

$\to f(t,y(t)+\widehat{\phi}_t,\int_0^t k(t,s,y(s)+\widehat{\phi}_s)ds,\int_0^a h(t,s,y(s)+\widehat{\phi}_s)ds)$ for each $t\in I$ and since

$$\left\|f(t,y_n(t)+\widehat{\phi}_t,\int_0^t k(t,s,y_n(s)+\widehat{\phi}_s)ds,\int_0^a h(t,s,y_n(s)+\widehat{\phi}_s)ds)-f(t,y(t)+\widehat{\phi}_t,\int_0^t k(t,s,y(s)+\widehat{\phi}_s)ds,\int_0^a h(t,s,y(s)+\widehat{\phi}_s)ds)\right\|$$

$\le 2\mu_{r'}(t), r'=r+\|\widehat{\phi}\|$,we have, by dominated convergence theorem ,

$$\|(Fy_n)(t)-(Fy)(t)\|$$

$$=\sup_{t\in I}\left\|E^{-1}[g(t,y_n(t)+\widehat{\phi}_t)-g(t,y(t)+\widehat{\phi}_t)]\right\|$$

$$+\left\|\int_0^t E^{-1}U(t,s)[A(s,x(s))g(s,y_n(s)+\widehat{\phi}_s)-A(s,x(s))g(s,y(s)+\widehat{\phi}_s)]ds\right\|$$

$$+\int_0^t E^{-1}U(t,\eta)CW^{-1}[-E^{-1}[g(b,y_n(b)+\widehat{\phi}_b)-g(b,y(b)+\widehat{\phi}_b)]$$

$$-\int_0^b E^{-1}U(b,s)[A(s,x(s))g(s,y_n(s)+\widehat{\phi}_s)-A(s,x(s))g(s,y(s)+\widehat{\phi}_s)]ds$$

$$-\int_0^b E^{-1}U(b,s)[f(s,y_n(s)+\widehat{\phi}_s,\int_0^s k(s,\tau,y_n(\tau)+\widehat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_n(\tau)+\widehat{\phi}_\tau)d\tau)$$

$$-f(s,y(s)+\widehat{\phi}_s,\int_0^s k(s,\tau,y(\tau)+\widehat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y(\tau)+\widehat{\phi}_\tau)d\tau)]ds](\eta)d\eta$$

$$+\int_0^t E^{-1}U(t,s)[f(s,y_n(s)+\widehat{\phi}_s,\int_0^s k(s,\tau,y_n(\tau)+\widehat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_n(\tau)+\widehat{\phi}_\tau)d\tau)$$

$$-f(s,y(s)+\hat{\phi}_s,\int_0^s k(s,\tau,y(\tau)+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y(\tau)+\hat{\phi}_\tau)d\tau)]ds]$$

$$\leq \|E^{-1}\|\|g(t,y_n(t)+\hat{\phi}_t)-g(t,y(t)+\hat{\phi}_t)\|$$

$$+M_1\int_0^t\|E^{-1}\|\|A(s,x(s))g(s,y_n(s)+\hat{\phi}_s)-A(s,x(s))g(s,y(s)+\hat{\phi}_s)\|ds$$

$$+M_1M_2\int_0^t\|E^{-1}\|[\|E^{-1}\|\|g(b,y_n(b)+\hat{\phi}_b)-g(b,y(b)+\hat{\phi}_b)\|$$

$$+M_1\int_0^b\|E^{-1}\|\|A(s,x(s))g(s,y_n(s)+\hat{\phi}_s)-A(s,x(s))g(s,y(s)+\hat{\phi}_s)\|ds$$

$$+M_1\int_0^b\|E^{-1}\|\|f(s,y_n(s)+\hat{\phi}_s,\int_0^s k(s,\tau,y_n(\tau)+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_n(\tau)+\hat{\phi}_\tau)\|d\tau)$$

$$\|-f(s,y(s)+\hat{\phi}_s,\int_0^s k(s,\tau,y(\tau)+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y(\tau)+\hat{\phi}_\tau)d\tau)\|ds](\eta)d\eta$$

$$+M_1\int_0^t\|E^{-1}\|\|f(s,y_n(s)+\hat{\phi}_s,\int_0^s k(s,\tau,y_n(\tau)+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_n(\tau)+\hat{\phi}_\tau)d\tau)\|$$

$$\|-f(s,y(s)+\hat{\phi}_s,\int_0^s k(s,\tau,y(\tau)+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y(\tau)+\hat{\phi}_\tau)d\tau)\|ds]$$

$\to 0$ as $n\to\infty$

Thus F is continuous .this completes the proof that F is completely continuous

Finally the set $\zeta(F)=\{y\in C_b^0 : y=\lambda Fy,\lambda\in(0,1)\}$ is bounded .since for every solution y in $\zeta(F)$, the function $x=y+\phi$ is a mild solution of (5)-(6) for which we have proved that $\|x\|_1\leq r$ and hence $\|y\|_1\leq r+\|\hat{\phi}\|$

Consequently, by Schaefer's theorem , the operator F has a fixed point in C_0^b .This means that any fixed point of F is a mild solution of (1)-(2) on I satisfying $(Fx)(t)=x(t)$ .Thus the system (1)-(2) is controllable on I

SOBOLEV TYPE QUASILINEAR NEUTRAL IMPULSIVE SYSTEMS

However, one may easily visualize that abrupt changes such as shock, harvesting and disasters may occur in nature. These phenomena are short time perturbations whose durationis negligible in comparison with the duration of the whole evolution process. Consequently,it is natural to assume, in modeling these problems, that these perturbations act instantaneously, that is in the form of impulses. The theory of impulsive differential equation [14, 15, 16] is much richer than the corresponding theory of differential equations without impulsive effects. The impulsive condition

$$\Delta u(t_i)=u(t_i^+)-u(t_t^-)=I_i(u(t_i^-)),i=1,2,.......m,$$

is a combination of traditional initial value problems and short-term perturbations whose duration is negligible in comparison with the duration of the process. Liu [17] discussed the iterative methods for the solution of impulsive functional differential systems. Consider the class of sobolev-type quasilinear neutral functional impulsive integro differential system with nonlocal conditions of the form

$$\frac{d}{dt}\left[Ex(t)-g(t,x_t)\right]=A(t,x(t))x(t)+Bu(t)+ \qquad (1)$$
$$f(t,x(t),\int_0^t k(t,s,x_s)ds,\int_0^a h(t,s,x_s)ds) \quad t\in[0,b]$$
$$\Delta x(t_k)=I_k(x_{t_k}),k=1,2,...............m,$$
$$x(0)+q(x)=x_0$$

Where the state variable $x(\cdot)$ takes values in a separable banach space X with norm $\|\cdot\|$ and the control function $u()$ is given in $L^2(I,U)$. A banach space of admissible control functions with U

As a banach space the interval $I=[0,b]$. E and B is a bounded linear operator from U into U and the function

$g:I\times C\to X, f:I\times C\times X\times X\to X, k:I\times I\times C\to X, h:I\times I\times C\to X, q:C(I,X)\to X$ are given functions and $I_k:X\to X$ are appropriate functions and the symbol $\Delta x(t_k)$ represents the jump of the function $u\to t$, which is defined by $\Delta x(t_k)=x(t_k^+)-x(t_k^-)$ .The norm of X is denoted by $\|\cdot\|$ the solution of the (7)-(9) equation is given by

$$x(t)=E^{-1}U(t,0)[Ex_0-Eq(x)-g(0,x_0)]+E^{-1}g(t,x_t)$$
$$+\int_0^t E^{-1}U(t,s)Cu(s)ds+\int_0^t E^{-1}U(t,s)A(s,x(s))g(s,x_s)ds$$
$$+\int_0^t E^{-1}U(t,s)f(s,x_s,\int_0^s k(s,T,x_T)dT,\int_0^a h(s,T,x_T)dT)ds,$$
$$+\sum_{0<t_i<t}B^{-1}S(t-t_k)I_kx(t_k) \qquad t\in I$$
$$\Delta x(t_k)=I_k(x_{t_k}),k=1,2,...............m,$$

In order to prove the main result we shall assume some additional hypothesis:

(H10) the maps $I_k:X\to X$ are continuous and there exists constant $I>0$ suchthat

$$\|I_k(x)\|\le I\|(x_t)\|, fork\in\aleph\ \&\ x\in X$$

(H11) The following inequality holds , the function

$$\widehat{m}(t)=\max\{1,M_1\|E^{-1}\|M_f(t),M_k(t,t),M_h(t,t),\int_0^t\frac{\partial}{\partial t}M_k(t,s)ds,\int_0^a\frac{\partial}{\partial t}M_k(t,s)ds,\}$$

Satisfies

$$\int_0^b\widehat{m}(s)ds<\int_d^\infty\frac{ds}{s+2\Omega_1(s)+2\Omega_2(s)+\Omega_3(s)}$$

Where

$$d=\|E^{-1}\|M_1[\|E\phi(0)\|+\|EM_q\|+L_g]+M_1\sum_{k=1}^m I_k$$

And

$$d_2=M_2\|E^{-1}\|\{\|x_1\|+\|E^{-1}\|[M_g+M_1L_g]+M_1\|E^{-1}\|M_Abr$$
$$+M_1\|E^{-1}\|\int_0^b M_f(s)\Omega_3[r+\int_0^s M_k(s,\tau)\Omega(\tau)d\tau+\int_0^a M_k(s,t)\Omega_2(\tau)d\tau]ds\}+M_1\sum_{k=1}^m I_k$$

Theorem 4.1

If the hypotheses [H1] - [H8], [H10] - [H11] are satisfiedd, then the system (7) - (9) is controllable on I

Proof. Using the hypothese [H3] for an arbitrary function $x(.)$ define the control

$$u(t)=W^{-1}[x_1-E^{-1}U(b,0)[Ex_0-Eq(x)-g(0,x_0)]-E^{-1}g(b,x_b)$$

$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,x_s)ds-\int_0^b E^{-1}U(b,s)f(s,x_s,\int_0^s k(s,T,x_T)dT,\int_0^a h(s,T,x_T)dT)ds-\sum_{0<tk<t}U(b,t_k)I_k(x_k)](t) \quad (3)$$

If $x(t)=y(t)+\hat{\phi}(t), t\in[0,b]$, it is easy to see that y satisfied

$$y(t)=E^{-1}U(b,0)[Ex_0-Eq(x)]+E^{-1}g(t,y_t+\hat{\phi}_t)$$

$$-E^{-1}U(t,0)g(0,\hat{\phi}_0)+\int_0^t E^{-1}U(t,s)A(s,x(s))g(s,y_s+\hat{\phi}_s)ds$$

$$+\int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1-E^{-1}U(b,0)[Ex_0-Eq(x)-g(0,\hat{\phi}_0)]-E^{-1}g(b,y_b+\hat{\phi}_b)$$

$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,y_s+\hat{\phi}_s)ds-\int_0^b E^{-1}U(b,s)f(s,y_s+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau)ds$$

$$-\sum_{0<tk<t}U(b,t_k)I_k(y_k+\hat{\phi}_k)](\eta)d\eta$$

$$+\int_0^t E^{-1}U(t,s)f(s,y_s+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau)ds+\sum_{0<tk<t}U(t,t_k)I_k(y_k+\hat{\phi}_k)],t\in I$$

If and only if x satisfies

$$x(t)=E^{-1}U(t,0)[Ex_0-Eq(x)]+E^{-1}g(t,x_t)$$

$$-E^{-1}U(t,0)g(0,x_0)+\int_0^t E^{-1}U(t,s)A(s,x(s))g(s,x_s)ds$$

$$+\int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1-E^{-1}U(b,0)[Ex_0-Eq(x)-g(0,x_0)]-E^{-1}g(b,x_b)$$

$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,x_s)ds-\int_0^b \begin{matrix} E^{-1}U(b,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,\int_0^a h(s,\tau,x_\tau)d\tau)ds \\ -\sum_{0<tk<b}U(t,t_k)I_k(x_k)] \end{matrix}](\eta)d\eta$$

$$+\int_0^t E^{-1}U(t,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,\int_0^a h(s,\tau,x_\tau)d\tau)ds-\sum_{0<tk<t}U(t,t_k)I_k(x_k)],t\in I$$

Define $C_b^0=\{y\in C_b: y_0=0\}$ and we now show that when using the control , the operator $F: C_b^0\to C_b^0$, defined by

$$(Fy)(t)=E^{-1}U(b,0)[Ex_0-Eq(x)]+E^{-1}g(t,y_t+\hat{\phi}_t)$$

$$-E^{-1}U(t,0)g(0,\hat{\phi}_0)+\int_0^t E^{-1}U(t,s)A(s,x(s))g(s,y_s+\hat{\phi}_s)ds$$

$$+\int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1-E^{-1}U(b,0)[Ex_0-Eq(x)-g(0,\hat{\phi}_0)]-E^{-1}g(b,y_b+\hat{\phi}_b)$$

$$-\int_0^b E^{-1}U(b,s)A(s,x(s))g(s,y_s+\hat{\phi}_s)ds$$

$$-\int_0^b E^{-1}U(b,s)f(s,y_s+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau)ds-\sum_{0<tk<b}U(t,t_k)I_k(y_k+\hat{\phi}_k)]](\eta)d\eta$$

$$+\int_0^t E^{-1}U(t,s)f(s,y_s+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau,\int_0^a h(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau)ds+\sum_{0<tk<t}U(t,t_k)I_k(y_k+\hat{\phi}_k)],t\in I$$

has a fixed point.

Clearly $x(b)=x_1$ which means that the control u steers the system (7) - (9) from the intial function ϕ to x_1 in time b, provided we can obtain a fixed point of the nonlinear operator F^*.

REFERENCES

[1]. K. Balachandran and J. P. Dauer; Controllability of nonlinear systems via Fixed Point Theo-rems, J. Optim. Theory Appl. 53 (1987), 345-352.

[2]. K. Balachandran and J. P. Dauer; Controllability of nonlinear systems in Banach spaces: A survey, J. Optim. Theory Appl. 115 (2002), 7-28.

[3]. K. Balachandran and E. R. Anandhi, Neutral functional integrodifferential control systems in Banach spaces, Kybernetika, 39 (2003), 359-367.

[4]. K. Balachandran and E. R. Anandhi, Controllability of neutral functional integrodifferential infinite delay systems in Banach spaces, Taiwanese Journal of Mathematics, 8 (2004), 689-702.

[5]. E. Hernandez and H. R. Henriquez, Impulsive partial neutral differential equations, AppliedMathematics Letters, 19 (2006), 215-222.

[6]. J. Banas; Measure of noncompactness in the space of continuous tempered functions, Demon-stratio Mathematica. XIV (1991), 127-133.

[7]. J. Banas and K. Goebel, Measure of Noncompactness in Banach Space, in: Lecture Notes in Pure and Applied Matyenath, Dekker, New York, 1980.

[8]. E. N. Chukwu and S. M. Lenhart; Controllability questions for nonlinear systems in abstractspaces, J. Optim. Theory Appl. 68 (1991), 437-462.

[9]. R. F. Curtain and H.Zwart; An Introduction to Infinite Dimensional Linear Systems Theory,Springer-Verlag, Berlin, 1995.

[10]. F.S. De Blasi, On a property of the unit sphere in a Banach space, Bull. Math. Soc. Sci. Math R.S. Roumanie, 21 (1977), 259-262.

[11]. J. Klamka; Controllability of Dynamical Systems, Kluwer Academic, Dordrecht, 1993.

[12]. K. Naito; On controllability for a nonlinear Volterra equation, Nonlinear Anal.: Theory Methods Appl. 18 (1992), 99-108.

[13]. Q. Dong, G. Li, J. Zhang, Quasilinear nonlocal integrodifferential equations in Banach spaces, Electronic J. Diff. Equa. 19 (2008), 1-8.

[14]. D. Guo, X. Liu, External solutions of nonlinear impulsive integrodifferential equations in Ba nach spaces, J. Math. Anal. Appl. 177 (1993), 538-552.

[15]. J. Liang, J. H. Liu, T. J. Xiao, Nonlocal impulsive problems for nonlinear differential equationsin Banach spaces, Math. Computer Modelling. 49 (2009), 798-804.

[16]. A. M. Samoilenko, N. A. Perestyuk, Impulsive Differential Equations, World Scientific, Singa-pore, 1995.

[17]. Z. Luo, J. J. Nieto, New results for the periodic boundary value problem for impulsive integrodifferential equations, Nonlinear Anal. 70 (2009), 2248-2260.

[18]. J. Banas, K. Goebel, Measure of noncompactness in Banach spaces, Lecture Notes in Pure and Applied Mathematics, vol. 60, Marcle Dekker, New York, 1980.

[19]. J. M. Ayerbe, T. D. Benavides, G. L. Acedo, Measure of noncmpactness in in metric fixed point theorem, Birkhauser, Basel, 1997.

[20]. J. Banas, W.G. El-Sayed, Measures of noncompactness and solvability of an integral equation in the class of functions of locally bounded variation, J. Math. Anal. Appl. 167 (1992), 133-151.

[21]. [21] A. Pazy; Semigroups of Linear Operators and Applications to Partial Differential Equations, Springer-Verlag, New York, 1983.

[22]. B. Radhakrishnan and K. Balachandran; Controllability of impulsive neutral functional evolution integrodifferential systems with infinite delay, Nonlinear Anal.: Hybrid Sys. 5 (2011),655-670.

[23]. M. Yamamoto and J. Y. Park; Controllability for parabolic equations with uniformly bounded nonlinear terms, J. Optim. Theory Appl. 66 (1990), 515-532