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FUZZY SHORTEST PATH PROBLEM BY INCLUSION MEASURE ON TYPE-2 FUZZY NUMBER

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ABSTRACT

Similarity measures of type-2 fuzzy sets are used to indicate the similarity degree between type-2 fuzzy sets. Inclusion measures for type-2 fuzzy sets are the degrees to which a type-2 fuzzy set is a subset of another type-2 fuzzy set. In this paper, we have considered the similarity and inclusion measures formulas between type-2 fuzzy sets based on the sugeno integral are proposed. To demonstrate our proposed approach an illustrative example also included.

Key words : Discrete type-2 fuzzy number Similarity Measure, Inclusion Measure, Extension Principle

INTRODUCTION

The shortest path problem is the problem of finding a path from source node to destination node on a network. The fuzzy shortest path problem was first analyzed by Dubois and Prade[2]. Okada and Soper [5] developed an algorithm based on the multiple labeling approach by which a number of non dominated paths can be generated.

Type-2 fuzzy set was introduced by Zadeh[7] as an extension of the concept of an ordinary fuzzy set. The type-2 fuzzy logic has gained much attention recently due to its ability to handle uncertainty and many advances appeared in both theory and applications.

The fuzzy measures was introduced by sugeno[6]. As an important tool for determining the similarity between two objects, Zadeh[8] initiated fuzzy similarity measure, and later on, various similarity measures for fuzzy set have been sequentially proposed.

When Zadeh[9] introduced fuzzy sets, he also defined the inclusion for fuzzy sests. Afterwards, the inclusion measure for fuzzy sets as to define the degree to which a fuzzy set is included in another fuzzy set had been studied in the literature. Zeng and Li[10] investigated the relations among inclusion measures, similarity measures, and the fuzziness of fuzzy sets.

The structure of paper is following: In Section 2, we have some basic concepts. Section 3, gives an algorithm to find shortest path and shortest path length combined with type-2 fuzzy number using inclusion measure. Section 4 gives the network terminology. To illustrate the proposed algorithm the numerical example is solved in section 5.

CONCEPTS

2.1 Type-2 Fuzzy Set:

A Type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$ where x $\in$ X and u $\in J_x \subseteq [0,1]$.

ie., $\tilde{A}$ = { ((x,u), $\mu_{\tilde{A}}(x,u)$) / $\forall$ x $\in$ X, $\forall$ u $\in J_x \subseteq [0,1]$ } in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$. $\tilde{A}$ can be expressed as $\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$ $J_x \subseteq [0,1]$, where $\int\int$ denotes union over all admissible x and u. For discrete universe of discourse $\int$ is replaced by $\sum$.

2.2 Type-2 Fuzzy Number:

Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. $\tilde{A}$ is normal,
2. $\tilde{A}$ is a convex set,
3. The support of $\tilde{A}$ is closed and bounded, then $\tilde{A}$ is called a type-2 fuzzy number.

2.3 Discrete Type-2 Fuzzy Number:

The discrete type-2 fuzzy number $\tilde{A}$ can be defined as follows:

$\tilde{A} = \sum_{x\in X} \mu_{\tilde{A}}(\mathrm{x}) / \mathrm{x}$ where $\mu_{\tilde{A}}(\mathrm{x}) = \sum_{u\in J_x} f_x(\mathrm{u}) / \mathrm{u}$ where J_x is the primary membership.

2.4 Extension Principle:

Let $A_1, A_2, \ldots, A_r$ be type-1 fuzzy sets in $X_1, X_2, \ldots, X_r$, respectively. Then, Zadeh's Extension Principle allows us to induce from the type-1 fuzzy sets $A_1, A_2, \ldots, A_r$ a type-1 fuzzy set B on Y, through f, i.e,B = f($A_1, \ldots, A_r$), such that

$$\mu_B(y) = \begin{cases} \sup\limits_{x_1,x_2,\ldots x_n \in f^{-1}(y)} \min\{\mu_{A_1}(x_1),\ldots,\mu_{A_n}(x_n) & iff^{-1}(y) \ne \phi \\ 0, & f^{-1}(y) = \phi \end{cases}$$

2.5 Addition On Type-2 Fuzzy Numbers:

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number be $\tilde{A} = \sum \mu_{\tilde{A}}(x) / \mathrm{x}$ and $\tilde{B} = \sum \mu_{\tilde{B}}(\mathrm{y}) / \mathrm{y}$ where $\mu_{\tilde{A}}(x) = \sum f_x(\mathrm{u}) / \mathrm{u}$ and $\mu_{\tilde{B}}(x) = \sum g_y(\mathrm{w}) / \mathrm{w}$. The addition of these two types-2 fuzzy numbers $\tilde{A} \oplus \tilde{B}$ is defined as

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z}) = \bigcup_{z=x+y} (\mu_{\tilde{A}}(\mathrm{x}) \bigcap \mu_{\tilde{B}}(\mathrm{y}))$$

$$= \bigcup_{z=x+y} ((\sum_i \mathrm{f}_x(\mathrm{u}_i) / \mathrm{u}_i) \bigcap (\sum_j g_y(\mathrm{w}_j) / \mathrm{w}_j))$$

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z}) == \bigcup_{z=x+y} ((\sum_{i,j} (\mathrm{f}_x(\mathrm{u}_i) \wedge g_y(\mathrm{w}_j)) / (\mathrm{u}_i \wedge \mathrm{w}_j))$$

2.6 Minimum of two discrete type-2 fuzzy number:

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then minimum of two type-2 fuzzy sets is denoted as Min($\tilde{A}$, $\tilde{B}$) is given by

$$Min(\tilde{A}, \tilde{B})(z) = \underset{z=Min(x,y)}{Sup} \left[(f_x(u_i) \wedge g_y(w_j)) / (\mathrm{u}_i \wedge w_j) \right]$$

Where $\tilde{A} = \sum f_x(\mathrm{u}) / \mathrm{u} / \mathrm{x}$ and $\tilde{B} = \sum g_y(\mathrm{w}) / \mathrm{w} / \mathrm{y}$.

2.7 Inclusion Measure:

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then the Inclusion measure is given by

$$\mathrm{I}(\tilde{A}, \tilde{B}) = \frac{1}{n} \left[\sum_{x \in X} \frac{\sum_{u \in J_x} \min\{\mathrm{u}.\mathrm{f}_x(u), u.g_x(u)\}}{\sum_{u \in J_x} \{\mathrm{u}.\mathrm{f}_x(u)\}} \right]$$

Where $\tilde{A} = \sum f_x(\mathrm{u}) / \mathrm{u} / \mathrm{x}$ and $\tilde{B} = \sum g_x(\mathrm{u}) / \mathrm{u} / \mathrm{x}$.

2.8 Relation between Similarity and Inclusion Measure:

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then

$$s(\tilde{A}, \tilde{B}) = Min\{I(\tilde{A}, \tilde{B}), I(\tilde{B}, \tilde{A})\}$$

ALGORITHM

Algorithm for Finding Shortest Path Length

Step 1 : Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

Step 2 : Set $\tilde{L}$ = $\tilde{L}_1$

Step 3 : Let i = 2

Step 4 : Compute $\tilde{L}$ = Min ($\tilde{L}$, $\tilde{L}_i$) using def 2.6.

Step 5 : Set i = i + 1

Step 6 : If i $\leq$ n goto step 4

Step 7 : The shortest path length is $\tilde{L}$

Algorithm for finding Shortest Path

Step 1 : Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

Step 2 : Let i =1

Step 3 : Compute $I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})$ using def 2.7

Step 4 : Find $S(\tilde{L},\tilde{L}_i) = Min\{I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})\}$

Step 5 : Print $S(\tilde{L},\tilde{L}_i)$

Step 6 : Put i = i + 1

Step 7 : If i $\leq$ n then go to Step 3

Step 8 : Find the Shortest path with highest similarity degree.

NETWORK TERMINOLOGY

Consider a directed network G(V,E) consisting of a finite set of nodes V = {1,2, . . .n} and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i,j), where i,j $\in$ V and i $\neq$ j. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence P_{ij} = {i = i_1, (i_1,i_2),i_2,. . . ., i_{l-1}, (i_{l-1},i_l), i_l = j} of alternating nodes and edges. The existence of at least one path P_{si} in G(V,E) is assumed for every node i $\in$ V – {s}.

$\tilde{d}_{ij}$ denotes a Type-2 Fuzzy Number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i,j\in P)} \tilde{d}_{ij}$

NUMERICAL EXAMPLE

The problem is to find the shortest path and shortest path length between source node and destination node in the network having 6 vertices and 8 edges with type-2 fuzzy number.

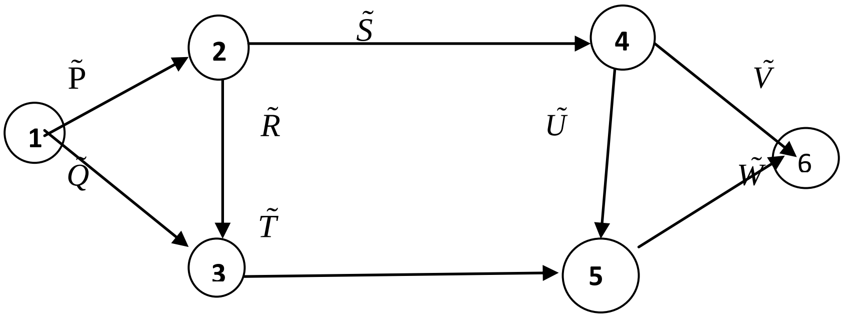

Fig 5.1

Solution:

The edge Lengths are

$\tilde{P}$ = (0.3/0.8+0.2/0.7)/2 + (0.3/0.9)/4

$\tilde{Q}$ = (0.5/0.8 + 0.3/0.6)/3

$\tilde{R}$ = (0.7/0.6)/1 + (0.5/0.7)/2

$\tilde{S}$ = (0.4/0.4 + 0.5/0.5)/2 + (0.2/0.9)/3

$\tilde{T}$ = (0.5/0.7)/2 + (0.7/0.4)/3

$\tilde{U}$ = (0.2/0.6)/1 + (0.3/0.5 + 0.4/0.4)/3

$\tilde{V}$ = (0.6/0.6)/2 +(0.7/0.5 + 0.4/0.4)/3

$\tilde{W}$ = (0.6/0.8)/1 + (0.4/0.5)/3

Algorithm for Finding Shortest Path Length

Step 1 : Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

$\tilde{P}_1$: 1 – 2 – 4 – 6

$\tilde{P}_2$: 1 – 2 – 4 – 5 – 6

$\tilde{P}_3$: 1 – 2 – 3 – 5 – 6

$\tilde{P}_4$: 1 – 3 – 5 – 6

$\tilde{L}_1$ = (0.3/0.4 + 0.3/0.5)/6+ (0.2/0.6)/7 + (0.2/0.5 + 0.2/0.4)/8 + (0.2/0.6)/9 +(0.2/0.5 + 0.2/0.4)/10

$\tilde{L}_2$ = (0.2/0.4 + 0.2/0.5)/6 + (0.2/0.6)/7 + (0.2/0.4+ 0.2/0.5)/8 + (0.2/0.6)/9 + (0.2/0.4 + 0.2/0.5)/10 + (0.2/0.4 + 0.2/0.5)/11 + (0.3/0.4 + 0.3/0.5)/12 + (0.2/0.4 + 0.2/0.5)/13

$\tilde{L}_3$ = (0.3/0.6)/6 + (0.3/0.7)/7 + (0.3/0.6)/8 + (0.3/0.7)/9 + (0.3/0.7)/10 + (0.3/0.5)/11 + (0.3/0.5)/12

$\tilde{L}_4$ = (0.5/0.7 + 0.3/0.6)/6 + (0.5/0.4)/7 + (0.4/0.5)/8 + (0.4/0.4)/9

Step 2 : Set $\tilde{L}$ = $\tilde{L}_1$

Step 3 : Let i = 2

Step 4 : Compute $\tilde{L}$ = Min ($\tilde{L}$, $\tilde{L}_i$) using def 2.6.

$\tilde{L}$ = (0.2/0.4 + 0.2/0.5)/6 + (0.2/0.4)/7 + (0.2/0.4 +0.2 /0.5)/8 + (0.2/0.4)/9 + (0.2/0.4 + 0.2/0.5)/10

Step 5 : Set i = 3

Step 6 : If 3 $\leq$ 4 goto step 4

Step 4 : Compute $\tilde{L}$ = Min ($\tilde{L}$, $\tilde{L}_i$) using def 2.6.

$\tilde{L}$ = (0.2/0.4 + 0.2/0.5)/6 + (0.2/0.4)/7 + (0.2/0.4 +0.2 /0.5)/8 + (0.2/0.4)/9 + (0.2/0.4 + 0.2/0.5)/10

Step 5 : Set i = 4

Step 6 : If 4 $\leq$ 4 goto step 4

Step 4 : Compute $\tilde{L}$ = Min ($\tilde{L}$, $\tilde{L}_i$) using def 2.6.

$\tilde{L}$ = (0.2/0.4 + 0.2/0.5)/6 + (0.2/0.4)/7 + (0.2/0.4 +0.2 /0.5)/8 + (0.2/0.4)/9

Step 5 : Set i = 5

Step 6 : If 5 $\leq$ 4 stop the procedure

Step 7 : The shortest path length is $\tilde{L}$

$\tilde{L}$ = (0.2/0.4 + 0.2/0.5)/6 + (0.2/0.4)/7 + (0.2/0.4 +0.2 /0.5)/8 + (0.2/0.4)/9

Algorithm for finding Shortest Path

Step 1 : Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

$\tilde{P}_1$: 1 – 2 – 4 – 6

$\tilde{P}_2$: 1 – 2 – 4 – 5 – 6

$\tilde{P}_3$: 1 – 2 – 3 – 5 – 6

$\tilde{P}_4$: 1 – 3 – 5 – 6

$\tilde{L}_1$ = (0.3/0.4 + 0.3/0.5)/6+ (0.2/0.6)/7 + (0.2/0.5 + 0.2/0.4)/8 + (0.2/0.6)/9 +(0.2/0.5 + 0.2/0.4)/10

$\tilde{L}_2$ = (0.2/0.4 + 0.2/0.5)/6 + (0.2/0.6)/7 + (0.2/0.4+ 0.2/0.5)/8 + (0.2/0.6)/9 + (0.2/0.4 + 0.2/0.5)/10 + (0.2/0.4 + 0.2/0.5)/11 + (0.3/0.4 + 0.3/0.5)/12 + (0.2/0.4 + 0.2/0.5)/13

$\tilde{L}_3$ = (0.3/0.6)/6 + (0.3/0.7)/7 + (0.3/0.6)/8 + (0.3/0.7)/9 + (0.3/0.7)/10 + (0.3/0.5)/11 + (0.3/0.5)/12

$\tilde{L}_4$ = (0.5/0.7 + 0.3/0.6)/6 + (0.5/0.4)/7 + (0.4/0.5)/8 + (0.4/0.4)/9

Step 2 : Let i =1

Step 3 : Compute $I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})$ using def 2.7

$I\ (\tilde{L},\tilde{L}_1)$ = 0.9861 $\quad I\ (\tilde{L}_1,\tilde{L})$ = 0.73613

Step 4 : Find $S(\tilde{L},\tilde{L}_i) = Min\{I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})\}$

$S(\tilde{L},\tilde{L}_1)$ = 0.73613

Step 5 : Print $S(\tilde{L},\tilde{L}_i)$

$S(\tilde{L},\tilde{L}_1)$ = 0.73613

Step 6 : Put i = 2

Step 7 : If 2 $\leq$ 4 then go to Step 3

Step 3 : Compute $I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})$ using def 2.7

$I\ (\tilde{L},\tilde{L}_2)$ = 0.9722 $\quad I\ (\tilde{L}_2,\tilde{L})$ = 0.8056

Step 4 : Find $S(\tilde{L},\tilde{L}_i) = Min\{I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})\}$

$S(\tilde{L},\tilde{L}_2)$ = 0.8056

Step 5 : Print $S(\tilde{L},\tilde{L}_i)$

$S(\tilde{L},\tilde{L}_2)$ = 0.8056

Step 6 : Put i = 3

Step 7 : If 3 $\leq$ 4 then go to Step 3

Step 3 : Compute $I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})$ using def 2.7

$I\ (\tilde{L},\tilde{L}_3)$ = 1 $\quad I\ (\tilde{L}_3,\tilde{L})$ = 0.4405

Step 4 : Find $S(\tilde{L},\tilde{L}_i) = Min\{I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})\}$

$S(\tilde{L},\tilde{L}_3)$ = 0.4405

Step 5 : Print $S(\tilde{L},\tilde{L}_i)$

$S(\tilde{L},\tilde{L}_3)$ = 0.4405

Step 6 : Put i = 4

Step 7 : If 4 $\leq$ 4 then go to Step 3

Step 3 : Compute $I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})$ using def 2.7

$I\ (\tilde{L},\tilde{L}_4)$ = 1 $\quad I\ (\tilde{L}_4,\tilde{L})$ = 0.4224

Step 4 : Find $S(\tilde{L},\tilde{L}_i) = Min\{I\ (\tilde{L},\tilde{L}_i)\,\text{and}\,I\ (\tilde{L}_i,\tilde{L})\}$

$S(\tilde{L},\tilde{L}_4)$ = 0.4224

Step 5 : Print $S(\tilde{L},\tilde{L}_i)$

$S(\tilde{L},\tilde{L}_4)$ = 0.4224

Step 6 : Put i = 5

Step 7 : If 5 $\leq$ 4 then Stop the procedure

Step 8 : Find the Shortest path with highest similarity degree.

The highest similarity degree is $S(\tilde{L},\tilde{L}_2)$ = 0.8056

Shortest path is $\tilde{P}_2$: 1 – 2 – 4 – 5 – 6

CONCLUSION

In this paper we have developed an algorithm for finding shortest path and shortest path length using inclusion measure with discrete type-2 fuzzy number. Similarity measure is expressed interms of Inclusion measure. Hence we get the same similarity degree for both the measures.So we conclude that for finding shortest path and shortest path length we can use either similarity measure or inclusion measure.

REFERENCES

[1]. V. Anusuya and R, Sathya, " Type-2 fuzzy shortest path", International Journal of fuzzy mathematical Archive, Vol.2, 2013, 36-42.

[2]. D. Dubois and H. Prade, " Theory and Applications:", Fuzzy sets and systems, Man and Cybernetics, Vol. 23, No.2, 1993, pp.403-405.

[3]. Miin – Shen Yang, Der – Chen Lin, " On similarity ad Inclusion Measures between type-2 fuzzy sets with ad application to Clustering ", An International Journal of Computers ad Mathematics with Applications, 57, 2009, 896 – 907.

[4]. Nagoorgani, A. and Anusuya, V. Fuzzy Shortest Path by Linear Multiple Objective Programming, Proceedings of the International Conference On Mathematics and Computer Science, , Loyola College, Chennai, Jan 10 and 11, 2005, pp.287 – 294.

[5]. S. Okada and T. Soper, " A shortest path problem on a network with fuzzy arc lengths", Fuzzy sets and systems, 109, 2000, 129-140.

[6]. Sugeno,M. " Fuzzy measures and Fuzzy integrals- a survey", In Gupta, saridis, and Gaines, 1977, pp.89-102.

[7]. L.A. Zadeh, " The Concept of a Linguistic Variable and its Application to Approximate Reasoning – 1", Inform. Sci. 8 (1975) 199 – 249.

[8]. L.A. Zadeh, " Similarity relations and Fuzzy ordering", Information sciences, 3 (1971), 177 – 200.

[9]. L. A. Zadeh, Fuzzy sets, Information and control 8, (1965), 338-356.

[10]. W. Zeng, H. Li, " Inclusion Measures, Similarity Measures and the fuzziness of fuzzy sets and their relations", International Journal of Intelligent systems, 21, (2006), 639 – 6